

# Static Study of Piezoelectric Bimorph Beams with Delamination Zone

A. Zemirline, M. Ouali, A. Mahieddine

**Abstract**—The FOSDT (the First Order Shear Deformation Theory) is taking into consideration to study the static behavior of a bimorph beam, with a delamination zone between the upper and the lower layer. The effect of limit conditions and lengths of the delamination zone are presented in this paper, with a PVDF piezoelectric material application. A FEM “Finite Element Method” is used to discretize the beam. In the axial displacement, a displacement field appears in the debonded zone with inverse effect between the upper and the lower layer was observed.

**Keywords**—Beam, Delamination, Piezoelectricity, Static.

## I. INTRODUCTION

THE actuation using the piezoelectric bimorph beams were studied before [1]. Now, we are going to view one of the most problems recognized in the multi layered structures, which it's called delamination. It is located in the interface between the upper and the lower layer.

The dynamic behavior of a bimorph beam with the effects of structural damping, shear deformation and a finite element model has been developed by [1] for a beam composed by tow piezoelectric layers subjected to a sinusoidal electric field. We can make several form of trajectory by changing dephasing value (or the phase angel) and we obtain better actuation since the thickness of beam is as well as smaller.

A. Nikkho [2] studied the constitutive equation of motion which is derived by employing Hamilton's principle for an Euler–Bernoulli beam, with a number of piezoelectric patches bonded on the bottom and top surfaces, and subjected (the beam) to arbitrary boundary conditions. A classical linear optimal control algorithm with displacement–velocity and velocity–acceleration feedbacks is used. The beam is composed by several linear springs with high stiffness as intermediate supports. a moving load and a moving mass were supposed to be the external excitations. The results signified the remarkable increase of the load inertial effects as the span number increased. However, it was revealed that the maximum response for beams with more spans occurs in larger values of the moving force velocity.

A. Mahieddine and M. Ouali [3] presented a model of beams with delamination using Euler–Bernoulli beam theory

and numerical results are presented to study the influence of delamination. It is shown that the axial displacement increases with the increasing of the thickness of layers. The frequencies computed with the model based on the formulation resented are in good agreement with the exact results

M. Ouali, A. Zemirline and A. Mahieddine [4] study and analyses a dynamic behavior of a beam modeled taking into account several parameters, like structural damping, shear deformation and piezoelectricity the results are compared using Ansys code with (BEAM188) and (SOLID226) element type the results obtained are in perfect agreement.

In this paper we study the static case of a bimorph beam subjected to a delamination zone between its upper and lower layers. The FOSDT (First Order Shear Deformation Theory) is taking into account and applied to a clamped-clamped and a clamped-free beam with several location of delamination zone.

## II. THE THEORETICAL FORMULATION

Combining the deformation relations (1) relating to the FOSDT and the piezoelectric equations applied to the beams (2), we obtain the relation of deformation energy (noticed by “ $U$ ”) in (3). With considering some assumptions related to, the beams theory, the PVDF materials, the electric field orientation, the series or anti-parallel configuration of the bimorph [1]-[6].

$$\begin{cases} \varepsilon_x = u_{,x} - z\psi_{,x} \\ \gamma_{xz} = w_{,x} - \psi \end{cases} \quad (1)$$

$$\begin{cases} \sigma_p = Q_{pq}\varepsilon_q - e_{pk}E_k \\ D_i = e_{iq}\varepsilon_q + \bar{\varepsilon}_{ik}E_k \end{cases} \quad (2)$$

where  $\varepsilon_x, \gamma_{xz}$  are strains,  $u, \psi$  and  $w$  axial, rotation and vertical displacement of the median fiber. And  $\sigma, Q, e, E, D$  and  $\bar{\varepsilon}$  are respectively the stress, elastic coefficients, piezoelectric coefficients, electric field, dielectric displacement and permittivity.

$$U = \frac{1}{2} \int_V (\varepsilon\sigma - DE) dV = \frac{1}{2} \int_V ((\varepsilon_x\sigma_x + \gamma_{xz}\tau_{xz}) - D_zE_z) dV \quad (3)$$

$$U = \frac{1}{2} \int_V \left( \underbrace{\varepsilon_x \bar{Q}_{11} \varepsilon_x + \gamma_{xz} \bar{Q}_{55} \gamma_{xz}}_{\text{Elastic}} - \underbrace{(\bar{e}_{31} \varepsilon_x E_z + \bar{e}_{31} E_z \varepsilon_x)}_{\text{Piezoelectric}} - \underbrace{(\bar{\varepsilon}_{33} E_z E_z)}_{\text{Dielectric}} \right) dV \quad (4)$$

with:

$$\begin{aligned} \bar{Q}_{11} &= \left( Q_{11} - \frac{Q_{12}^2}{Q_{22}} \right); \quad \bar{e}_{31} = \left( e_{31} - \frac{Q_{21}}{Q_{22}} e_{32} \right) \\ \bar{Q}_{55} &= \left( Q_{55} - \frac{Q_{45}^2}{Q_{44}} \right); \quad \bar{\varepsilon}_{33} = \left( \bar{\varepsilon}_{33} + \frac{e_{32}^2}{Q_{22}} \right) \end{aligned}$$

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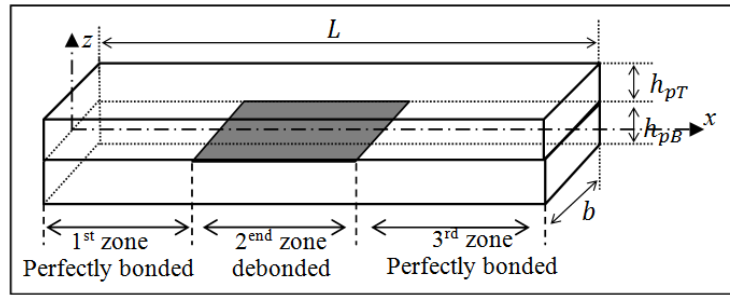


Fig. 1 Beam Configuration

Finally we obtain the following relations:

$$U_{Elas} = \frac{1}{2} \int \{u\}^t [B]^t [H] [B] \{u\} dx \quad (5)$$

$$U_{Piezo} = \frac{1}{2} b \int \left( \int_{-h_{pB}}^0 [\phi_z^B (u_x + z\psi_x) \bar{e}_{31}^B + (u_x + z\psi_x) \bar{e}_{31}^B \phi_z^B] dz - \int_0^{h_{pT}} [\phi_z^T (u_x + z\psi_x) \bar{e}_{31}^T + (u_x + z\psi_x) \bar{e}_{31}^T \phi_z^T] dz \right) dx \quad (6)$$

$$U_{Diele} = -\frac{1}{2} b \int \left( \int_{-h_{pB}}^0 \phi_z^B \bar{\epsilon}_{33}^B \phi_z^B dz + \int_0^{h_{pT}} \phi_z^T \bar{\epsilon}_{33}^T \phi_z^T dz \right) dx \quad (7)$$

where  $X^T, X^B$  denote Top and bottom layer. And  $X_{,x}$  denotes the  $x$  derivation.  $L, b, h_{pT}$  and  $h_{pB}$  are respectively the length, the width, and the thickness of the top and bottom layer.

$$E_z = -\frac{\partial \phi}{\partial z}$$

where  $\phi$  is the electrical potential on the electrodes.  $[H]$  and  $[B]$  are the elastic and derivation matrices.

### III. FINITE ELEMENT FORMULATION

Using the F. E. M. "Finite Element Method" to discrete the beam we find the following elementary matrices:

$$U_{Elas} = \frac{1}{2} \{q\}^t [K_{uu-b}]^e \{q\} \quad (8)$$

$$U_{Piezo} = \frac{1}{2} \{q\}^t [K_{u\phi-b}]^e \{\phi\} + \frac{1}{2} \{\phi\}^t [K_{\phi u-b}]^e \{q\} \quad (9)$$

$$U_{Diele} = \frac{1}{2} \{\phi\}^t [K_{\phi\phi}] \{\phi\} \quad (10)$$

$[K_{uu}]^e, [K_{u\phi}]^e, [K_{\phi u}]^e$  and  $[K_{\phi\phi}]$  are respectively elastic, elastic-piezoelectric coupling and dielectric elementary matrices.

### IV. RESULTS

We used MATLAB software to calculate the axial and vertical displacement. The results obtained for a PVDF material are presented below:

The physical properties are:

$$E_1 = E_2 = 2e9 \text{ N/m}^2; G_{12} = 7.75e8 \text{ N/m}^2; \nu = 0.29; \\ d_{31} = 2.2e-11 \text{ C/N}; \bar{\epsilon}_{11} = \bar{\epsilon}_{22} = \bar{\epsilon}_{33} = 1.62e-11 \text{ F/m};$$

and the geometrical properties are:

$$L = 0.1 \text{ m}; \quad b = 0.05 \text{ m}; \quad h_{pT} = h_{pB} = 0.05 \text{ m}$$

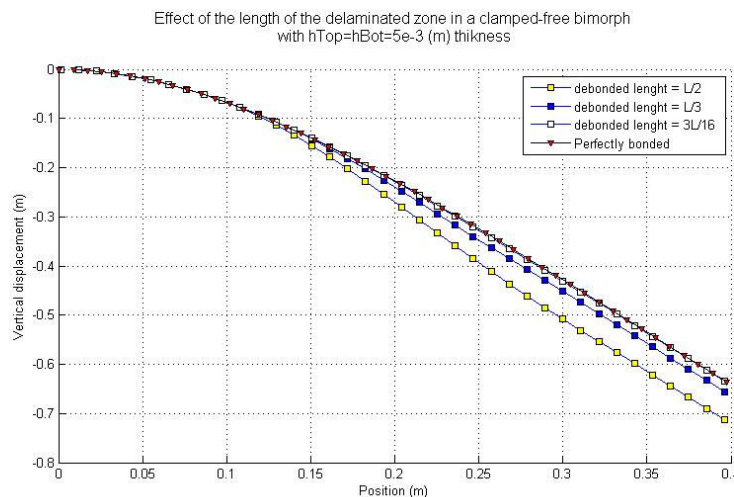


Fig. 2 Vertical displacement of a clamped-free beam under A uniformly distributed load  $Q = -100\text{N}$ , with several lengths of delaminated zone

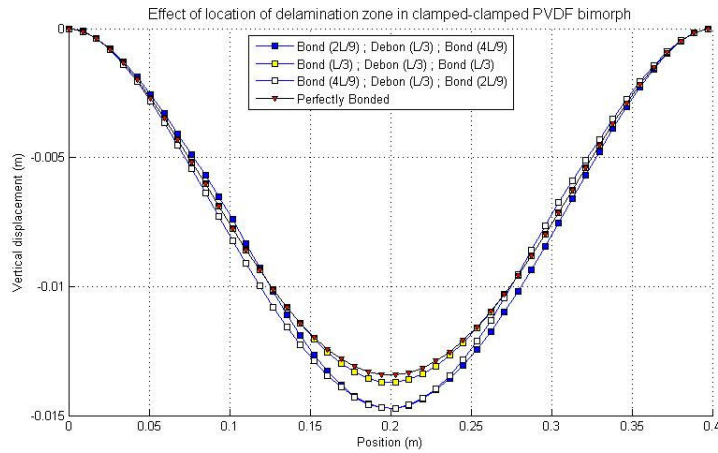


Fig. 3 Vertical displacement of a clamped-clamped beam under a uniformly distributed load  $Q = -100\text{N}$ , with location of delaminated zone

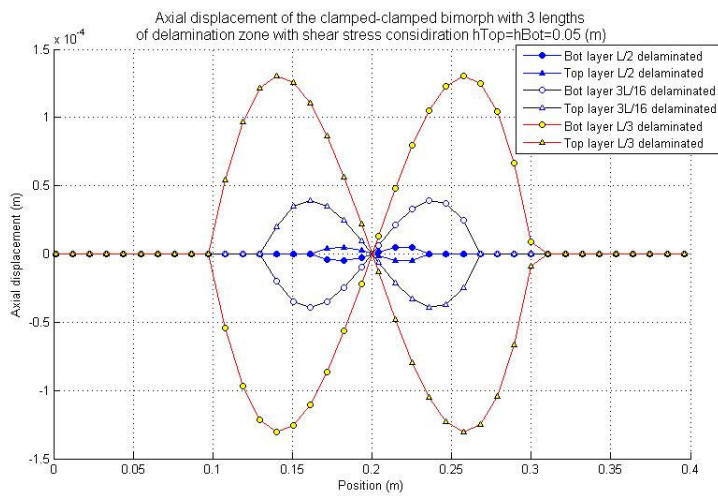


Fig. 4 Axial displacement of a clamped-clamped beam under a uniformly distributed load  $Q = -1000\text{N}$ , with several lengths of delaminated zone

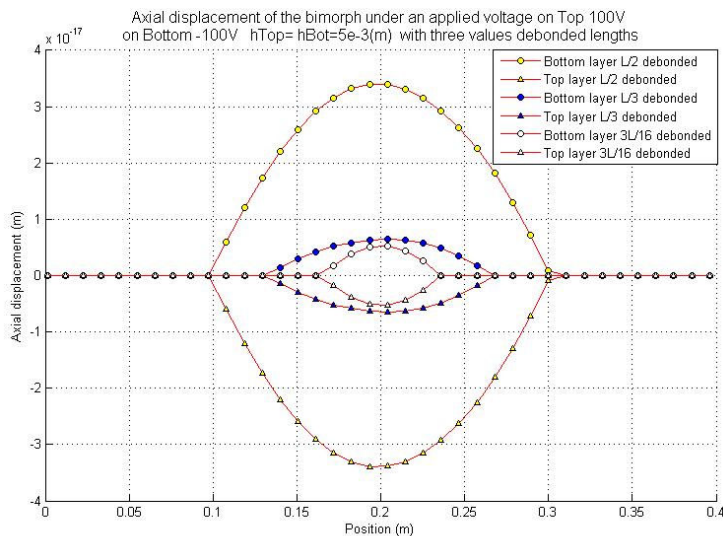


Fig. 5 Axial displacement of a clamped-free beam under an applied voltage on the Top and Bottom, with several lengths of delaminated zone

## V. CONCLUSIONS

A bimorph beam made of two piezoelectric layers, with delamination zone in the middle, is considered to study the static behaviour of a delaminated beam under two limit conditions. The FOSDT is taking in to account to establish a mathematical model; the model was discretized by the finite element method "F. E. M." A Matlab code was used to solve the system of equations. The following observations are noticed:

The length of delamination zone affect the vertical displacement, while the length of the DZ is important induces an increase in the vertical displacement. The beam support less loads.

In the Fig. 3, it is clearly seen the distortion in the vertical displacement due to the DZ. So the DZ can be located easily through the length of the beam.

A symmetric response is seen on the clamped-clamped beam due to the location of the DZ, with the same lengths, the DZ's generates the same deformed shape of the beam.

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