

State Estimation Solution with Optimal Allocation of Phasor Measurement Units Considering Zero Injection Bus Modeling

M. Ravindra, R. Srinivasa Rao, V. Shanmukha Naga Raju

Abstract—This paper presents state estimation with Phasor Measurement Unit (PMU) allocation to obtain complete observability of network. A matrix is designed with modeling of zero injection constraints to minimize PMU allocations. State estimation algorithm is developed with optimal allocation of PMUs to find accurate states of network. The incorporation of PMU into traditional state estimation process improves accuracy and computational performance for large power systems. The nonlinearity integrated with zero injection (ZI) constraints is remodeled to linear frame to optimize number of PMUs. The problem of optimal PMU allocation is regarded with modeling of ZI constraints, PMU loss or line outage, cost factor and redundant measurements. The proposed state estimation with optimal PMU allocation has been compared with traditional state estimation process to show its importance. MATLAB programming on IEEE 14, 30, 57, and 118 bus networks is implemented out by Binary Integer Programming (BIP) method and compared with other methods to show its effectiveness.

Keywords—Observability, phasor measurement units, synchrophasors, SCADA measurements, zero injection bus.

I. INTRODUCTION

IN the recent development of synchrophasor measurements for power system protection, combination of PMU and conventional measurements showed to provide accurate measurements. Synchrophasor measurements allow direct measurement of phase angles associated with current and voltage measurements [1], [2]. State Estimation (SE) acts as key role in power system control and operation, providing the security and reliability of power network. SE program should be designed in such a way that network should remain observable during abnormal conditions besides detecting and identifying gross errors in measurement set. However this process relies on PMU locations to make network completely observable. PMU allocation on every bus of network is not feasible as it leads to cost factor. Cost factor and redundant measurements are considered as constraints to determine optimal PMU allocation in network.

In this paper a method based on ZI bus constraints is proposed and analyzed. The problem of optimal PMU allocation is considered with modeling ZI constraints, PMU loss or line outage. A Genetic Algorithm (GA) [3] for PMU

allocation considering observability is proposed in which observability is well-maintained in case of single loss of PMU. In this procedure the group of functions built for optimization takes long time for computation. In [4], the author proposed an upgraded Binary Harmony Search Algorithm for optimal location of PMUs with branch weight limitation. The redundancy of measurements obtained with these locations is very low which affects the reliability of the system. A Binary Cuckoo Optimization Algorithm (MBCOA) [5] for optimal PMU problem which is tested on different networks under normal conditions and single event contingency considering observability of system is presented but in this paper global optimal solution cannot be obtained. In [6] the author proposed a Binary Semi-definite Programming (BSDP) with binary decision variables to minimize linear objective function. The drawback with numerical observability in this paper is that observability and redundancy at all the buses cannot be obtained.

Many Meta Heuristic Optimization methods are introduced [6]-[10] but main drawback is global optimal solution. A hybrid constrained SE [11] is proposed in which conventional and synchrophasor measurements are incorporated simultaneously in problem without using any transformation of measurements and shown that solution converges faster with small uncertainty in measurements. Multistage SE procedure [12] is presented to include synchrophasor measurements without disturbing existing SCADA system. This procedure requires more number of PMUs which is against the economic criteria. Three different methods are investigated to include PMU measurements into SE problem [13].

In this paper SE Algorithm is developed in such a way that it measures accurate states of network with optimal phasor measurements with complete observability. The accuracy and performance of SE are measured with measurement of error considering true values of network. For optimal measurements of network a BIP method with ZIB modeling is formulated for optimal location of PMUs.

This paper is organized as follows: Section II discusses the problem of formulation for optimal allocation of PMUs with complete observability; Section III presents SE algorithm; Section IV shows results with discussion regarding the problem and Section V concludes problem.

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II. PROBLEM FORMULATION

A. Optimal Allocation of PMUs

A BIP method is utilized for optimal allocation of PMU. The optimization problem is designed as

$$\text{Min} \sum_{i=1}^n F_i x_i \tag{1}$$

Subjected to observability constraints

$$Ax_i \geq b_i \tag{2}$$

where n is number of buses in the system, x_i is a binary decision variable which is defined as

$$x_i = \begin{cases} 1 & \text{if PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases}$$

and A is binary connectivity matrix or bus incidence matrix formed from line connectivity data and is defined as

$$A = \begin{cases} 1 & \text{if } i = j \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

F is cost vector and is defined as diagonal unit matrix with order $n \times n$ b_i is vector of observability constraints defined as [1 1 1 1...n]

B. Modeling of ZI Bus Constraints

The buses in which flow of current is zero are known as ZI buses. With modeling of ZI buses in the PMU placement problem, number of PMUs required to measure states of network can be reduced.

Modeling of ZI bus constraints in BIP frame work has remained a challenge. Here we propose a method to model these ZI bus constraints with in a linear frame of work. The constraints involved in matrix formulation are buses linked to ZI buses and unobservable buses. For every ZI bus-j, let f_i denote set of buses associated with bus-j. Let $g_i = f_i \cup \{j\}$, then set g_i denote number of unobservable buses is at most one which include both ZI bus-j and adjacent buses.

While considering buses of network, the inequality associated with power flow and power injections need to be analyzed. Let us consider the power flow measurements on line p, q of bus the inequality constraint defined as

$$Z_p + Z_q \geq 1$$

Let bus 'm' connects the bus l, p and q Suppose an injection measurement is at bus 'm' as shown in Fig. 1 then following inequality constraints are defined as

$$Z_l + Z_p + Z_q + Z_m \geq 3$$

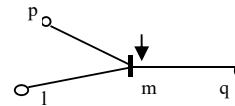


Fig. 1 Buses p,q,l connected to injection bus m

The power injection measurements and power flow measurements are associated such that inequalities are represented as

$$Z_l + Z_p + Z_q + Z_m - Z_p - Z_q \geq 3 - 1$$

$$Z_l + Z_m \geq 2$$

Due to single injection I_m the right hand side of equation is to be reduced by one. Thus inequality constraints to be satisfied are

$$Z_l + Z_m \geq 1$$

$$Z_p + Z_q \geq 1$$

In ZI buses, no current flows so therefore ZI bus constraints are modeled with an optimization function to minimize number of PMUs for allocation in bus network to calculate accurate SE. The objective function is formulated as:

$$\text{Min} \sum_{i=1}^n F_i x_i \tag{3}$$

Subjected to observability constraints

$$Z_{pmu} x_i \geq b_{con} \tag{4}$$

where Z_{pmu} is bus connectivity matrix formulated in this paper and b_{con} is vector of observability constraints defined as [1 1 1 1...n].

The constraints included in the formulations are unobservable buses if any, and buses associate with ZI buses. For every ZI bus-j, let f_i denote the set of buses associated with bus-j. Let $g_i = f_i \cup \{j\}$, then set g_i denote number of unobservable buses is at most one which include both ZI bus-j and adjacent buses.

C. Procedure for Formulation of ZI Matrix for Optimal allocation of PMUs

Step1. A vector of buses including ZI buses and buses associated with ZI buses are represented as

$$Z_a = [Z_b \quad Z_{as}] \tag{5}$$

while buses that are not associated with ZI-buses Z_{ns}

Step2. A new vector formed combining (1) and (2) is written as

$$b_{in} = [Z_{ns} \quad Z_a] \quad (6)$$

Step3. Permutation matrix P is established utilizing vector b_{in}

Step4. ZI Connectivity matrix Z_m with buses associated with ZI buses is established

Step5. ZI constraint matrix is created as

$$Z_{con} = \begin{bmatrix} I & 0 \\ 0 & Z_m \end{bmatrix} \quad (7)$$

where I - Identity matrix; Z_m - Matrix which represents buses associated with ZI buses.

Step6. The final connectivity matrix for optimal allocation is formulated as

$$Z_{pmu} = A * P * Z_{con} \quad (8)$$

A = binary connectivity matrix; P = permutation matrix; Z_{con} = ZI constraint matrix.

Step7. The constraint vector b_{con} is formed from number of constraints in Z_{pmu} to check observability.

Step8. Matrix Z_{pmu} and vector b_{con} are constraints that are modeled in BIP method for optimal allocation of PMUs.

D. Modeling of ZI Bus Constraints with Single PMU Outage

In real time control of power systems to enhance system control, each bus of system should be observed by at least two buses as it will not lead to loss of observability even with single PMU outage. The problem formulation is as:

$$\text{Min} \sum_{i=1}^n F_i x_i \quad (9)$$

Subjected to observability constraints

$$Z_{pmu} x_i \geq 2b_{con} \quad (10)$$

E. Redundancy for Optimal Allocation of PMUs

To measure quality and performance of optimization, we utilized System Observability Redundancy Index (SORI) which is defined as sum of observability of each bus of network [15].

$$SORI = \sum_{i=1}^n \beta_i \quad (11)$$

β_i is defined as Bus Observability Index (BOI) of i^{th} bus.

Maximum BOI is limited to maximum connectivity (χ_i) of

the bus plus one i.e.

$$\beta_i \leq \chi_i + 1 \quad (12)$$

For bus i , BOI is defined as no PMUs which can observe given bus. Maximum redundancy of bus can be defined as

$$\text{Max} \sum_{i=1}^n b_i^T A x_i \quad (13)$$

Subjected to the following constraints

$$\sum_{i=1}^n x_i = \gamma_0, \quad (14)$$

$$A x_i \geq b_i \quad (15)$$

where γ_0 is minimum number of PMUs obtained for system complete observability.

III. SE ALGORITHM DEVELOPMENT

A. Fusing of Phasor Measurements with Conventional Measurements

The SE problem involving measurements such as power injections and flows are nonlinear. The weighted least squares (WLS) estimates [14] of states are found by iteration process as follows

$$x^{k+1} = x^k + (H^T R^{-1} H)^{-1} H^T R^{-1} (Z - h(x^k)) \quad (16)$$

x^k is vector of state variables at k^{th} iteration. H is Jacobian matrix of mixed PMU and conventional measurements, R is measurement error covariance matrix, Z is measurement vector of PMU and conventional measurements, h is vector of measurement functions. The formulation Jacobian matrix for WLS SE involving both conventional and PMU measurements is illustrated as follows

$$H_{PMU} = \begin{bmatrix} \frac{\partial P_{in}}{\partial \theta} & \frac{\partial P_{in}}{\partial v} \\ \frac{\partial Q_{in}}{\partial \theta} & \frac{\partial Q_{in}}{\partial v} \\ \frac{\partial \theta}{\partial P_{fl}} & \frac{\partial v}{\partial P_{fl}} \\ \frac{\partial \theta}{\partial Q_{fl}} & \frac{\partial v}{\partial Q_{fl}} \\ \frac{\partial \theta}{\partial \theta} & \frac{\partial v}{\partial v_{meas}} \\ 0 & \frac{\partial v}{\partial v} \\ \frac{\partial \theta_{meas}}{\partial \theta} & 0 \\ \frac{\partial I_{rpmu}}{\partial \theta} & \frac{\partial I_{rpmu}}{\partial v} \\ \frac{\partial \theta}{\partial I_{ipmu}} & \frac{\partial v}{\partial I_{ipmu}} \\ \frac{\partial \theta}{\partial \theta} & \frac{\partial v}{\partial v} \end{bmatrix} \quad (17)$$

where P_{in} and Q_{in} are active and reactive power injections, P_{fl} and Q_{fl} are active and reactive power flows, v_{meas} and θ_{meas} are voltage magnitude and phase angles calculated from SCADA data, I_{rpmu} and I_{ipmu} are real and imaginary current magnitudes measured through PMU.

The weights R_{ii} in the form of diagonal matrix is derived from the variance of the individual measurements as:

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - u)^2} \quad (18)$$

Variance of the measurement is σ^2 where N is total number of measurements, x_i is individual i^{th} measurement, and u is mean of measurements. $R_{ii} = 1/\sigma^2$ are weights of measurements represented in diagonal matrix. These weights are used in WLS SE to obtain accurate estimated value of measurement. Fusing of all PMU units at all the branches in network is not feasible as it leads to economical factor.

SE method which includes optimal allocation of PMUs with ZI modeling in bus network is designed in such way that system is observable and redundant to measurements taken which improves accuracy and performance of network. The BIP method with ZI modeling is followed for optimal allocation of PMU in network.

B. SE Algorithm with Optimal Allocation of PMUs

Among load flow methods developed so far, fast decoupled method is most successful. This method is efficient, faster and needs less storage compared with NR method, but it fails to converge under certain nonlinear constraints as it does not hold basic assumptions. Therefore NR load flow method is chosen since it is robust and extensively used in practice. The state variables, voltage and phase angle (v, θ) generated by NR load flow method are considered as true states of network. The flow chart for the proposed SE is shown in Fig. 2.

The phasor measurement data have been calculated using KCL. To alter the measurement data obtained from load flow and Phasor measurements to realistic data, definite weights which are computed by standard deviation (18) of particular measurements are added. Finally the realistic measurement data (perturbed), measurement noise and nominal network parameters are inputs to WLS state estimator which results an output of (V_{est}, θ_{est}) that are compared with NR states to obtain SE error.

IV. RESULTS AND ANALYSIS

A. Optimal Allocation of PMUs with ZI Constraint Modeling

The optimal PMU allocation problem is programmed in MATLAB and it is run on Intel(R) core(TM), the i3 processor at 2.20 GHz with 4 GB of RAM. Table I shows the ZI buses of IEEE test systems. ZI modeling with BIP method of optimization decreases the number of PMU locations in the

network. PMU locations with and without ZI modeling is shown in Tables II and III. PMU locations with and without ZI modeling for single line outage or PMU outage are shown in Tables IV and V. Single line outage issue is subset of PMU outage problem.

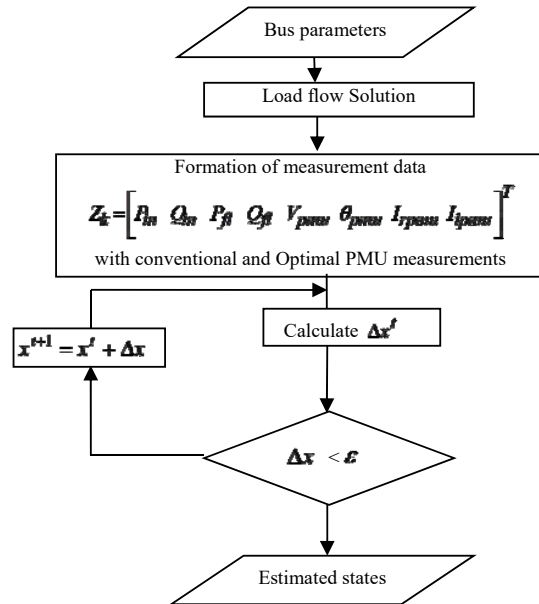


Fig. 2 SE flow chart with optimal allocation of PMUs

TABLE I
ZI BUSES

IEEE Test systems	No. of ZI buses	ZI Bus Locations
14bus	1	7
30 bus	6	6,9,22,25,27,28
57 bus	15	4,7,11,21,22,24,26,34,36,37,39,40,45,46,48
118 bus	10	5,9,30,37,38,63,64,68,71,81

TABLE II
WITHOUT ZI MODELING

IEEE Test systems	No. of PMUs	PMU locations
14bus	4	2,6,7,9
30 bus	10	1,7,9,10,12,18,24,25,27,28
57 bus	17	1,4,6,13,19,22,25,27,29,32,36,39,41,45,47,51,54
118 bus	32	3,7,9,11,12,17,21,25,28,34,37,41,45,53,56,62,63,68,70,71,76,79,85,86,89,92,96,100,105,110,105,114

TABLE III
WITH ZI MODELING

IEEE Test systems	No of PMUs	PMU locations
14bus	3	2,6,9
30 bus	7	1,7,10,12,19,24,27.
57 bus	13	1,6,9,14,20,25,27,32,37,38,50, 53, 56.
118 bus	27	1,5,12,15,17,21,25,28,34,40,45,49,53,56,62,64,70,77,80,85,87,90,94,101,105,110,114

SORI is a performance indicator on quality of optimization. Comparison of SORI with and without ZI modeling for single line outage and no line outage is shown in Table VI. The PMU

placement with maximum SORI is chosen for final placement with particular number of allocations.

TABLE IV
WITHOUT ZI MODELING FOR SINGLE LINE OUTAGE

IEEE Test systems	No. of PMUs	PMU locations
14bus	9	2,4,5,6,7,8,9,11,13
30 bus	21	1,3,5,7,8,9,10,11,12,13,15,17,19,20,22,24,25,26,28,29,30
57 bus	17	1,3,4,6,9,12,15,19,20,22,24,26,28,29,30,31,32,33,35,36,38,39,41,43,45,46,47,50,51,53,54,56,57,2,3,5,7,9,10,11,12,15,17,19,21,22,24,25,26,27,29,31,32,34,36,37,40,42,44,45,46,49,52,53,56,57,58,
118 bus	32	59,62,64,65,67,68,70,71,73,75,77,79,80,84,85,86,87,89,91,92,94,96,100,102,105,107,109,110,111,112,115,116,117,118

TABLE V
WITH ZI MODELING FOR SINGLE LINE OUTAGE

IEEE Test systems	No. of PMUs	PMU locations
14bus	7	2,3,5,6,9,11,13
30 bus	16	2,3,4,7,10,12,13,15,17,19,20,21,24,25,27,29
57 bus	29	1,2,4,6,9,12,15,19,20,22,24,25,28,29,30,32,33,35,36,38,41,45,48,49,50,51,53,54,56
118 bus	61	1,2,5,6,11,12,15,17,18,20,21,23,27,29,30,31,32,35,36,37,41,42,43,44,47,48,49,52,53,56,57,58,59,62,64,66,67,70,75,77,79,80,84,85,86,87,89,91,92,94,96,100,102,105,107,101,110,111,115,117,118

TABLE VI
SORI WITH AND WITHOUT SINGLE LINE OUTAGE

IEEE Test systems	No Line outage		Single Line outage	
	SORI with ZI Modeling	SORI without ZI Modeling	SORI with ZI Modeling	SORI without ZI Modeling
14 bus	15	19	30	39
30 bus	31	43	64	74
57 bus	57	67	117	127
118 bus	144	157	272	299

B. SE with Optimal Allocation of PMU

The convergence tolerance value for developed SE method is set to 10^{-5} . Weights for each measurement are formed from their respective variances.

From the line data and bus data of the SCADA systems, true state variables are determined performing NR load flow method. WLS SE which is popular as Traditional State Estimation (TSE) is performed with the conventional measurements determined from load flow analysis. For optimal allocation PMU, we derived a binary connectivity matrix which is used in BIP method. WLS SE is performed to define accurate state variables. Comparison between state variables that are derived from NR method, traditional SE with conventional measurements, and proposed SE method with PMU and conventional measurements are shown in Table VII. Voltage magnitude error and phase angle error are calculated as

$$V(t)_{er} = V(t)_{est} - V(t)_{meas} \tag{19}$$

$$\theta(t)_{er} = \theta(t)_{est} - \theta(t)_{meas} \tag{20}$$

TABLE VII
PROPOSED SE SOLUTION FOR 14-BUS SYSTEM

Bus	True State N-R load flow Method		Traditional SE Method		SE Method with optimal allocation of PMU	
	V[p.u.]	θ [rad]	V[p.u.]	θ [rad]	V[p.u.]	θ [rad]
1	1.0600	0	1.0600	0	1.0600	0
2	1.0450	-0.0870	1.0440	-0.0885	1.0449	-0.0870
3	1.0100	-0.2223	1.0085	-0.2293	1.0100	-0.2223
4	1.0166	-0.1798	1.0158	-0.1845	1.0166	-0.1799
5	1.0183	-0.1529	1.0177	-0.1574	1.0183	-0.1529
6	1.0700	-0.2482	1.0695	-0.2531	1.0700	-0.2482
7	1.0610	-0.2330	1.0606	-0.2380	1.0610	-0.2332
8	1.0900	-0.2330	1.0897	-0.2381	1.0900	-0.2332
9	1.0555	-0.2606	1.0551	-0.2655	1.0555	-0.2608
10	1.0506	-0.2634	1.0502	-0.2683	1.0506	-0.2635
11	1.0567	-0.2581	1.0562	-0.2630	1.0567	-0.2582
12	1.0552	-0.2631	1.0546	-0.2680	1.0552	-0.2631
13	1.0503	-0.2645	1.0498	-0.2694	1.0503	-0.2644
14	1.0352	-0.2797	1.0348	-0.2847	1.0352	-0.2798

SE with PMUs gives error free measurements which are closer to zero. The performance of SE algorithm with PMU allocation is effective for accurate measurement of states in power network. The threshold value of SE accuracy is of order 10^{-6} . From the results it shows that accuracy of SE remains below threshold value. Voltage and phase angle error plots are shown in Figs. 4 and 5. The single line diagram of 14-bus system with optimal allocation of PMUs is shown in Fig. 3.

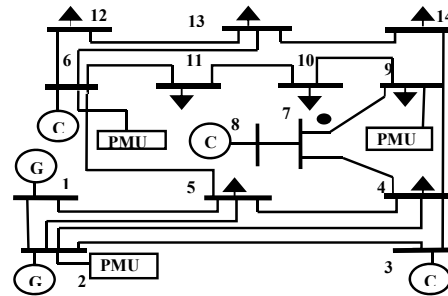


Fig. 3 Optimal Allocation of PMU for 14 bus system with ZI modeling

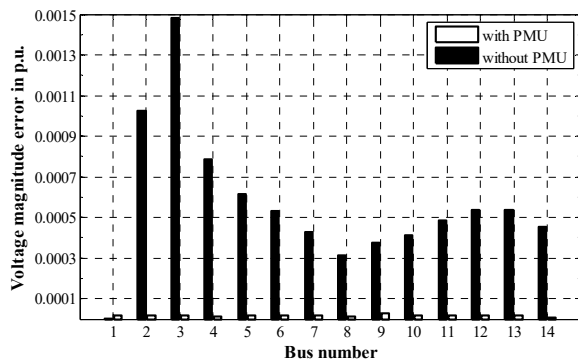


Fig. 4 Voltage magnitude error for 14 -bus system

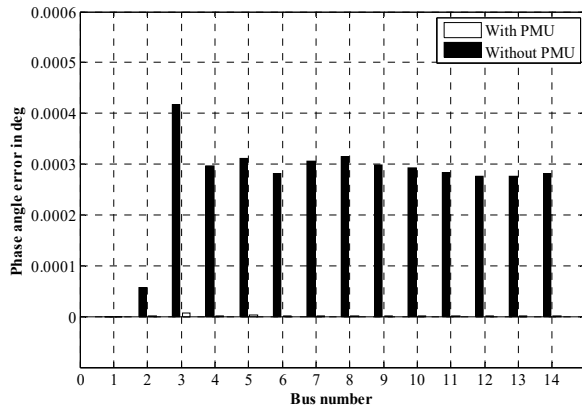


Fig. 5 Phase angle error for 14 –bus system

When compared with other methods proposed SE shows very accurate measurements. Here BIP method is chosen for optimal allocation of PMUs because of its global optimal solution with complete observability. When compared with other methods as shown in Table VIII proposed BIP method with ZI bus constraint modeling decreases the optimal location of PMUs with complete observability.

TABLE VIII
COMPARISON OF PROPOSED METHOD WITH OTHER METHODS

Methods	14-bus system	30-bus system	57-bus system	118 bus system
UBHA[4]	4	10	-	32
MBCOA[5]	3	7	11	28
BGO[7]	4	10	-	-
ACA[8]	4	-	-	0
BPSO[9]	4	10	17	
CRO[10]	4	10	17	
Multisheduling ILP[15]	4	10	17	32
Proposed BIP	3	7	13	27

V. CONCLUSION

This paper presented the modeling of ZI buses for optimal allocation of PMU in bus network and developed a SE algorithm with optimal allocation of PMU measurements including conventional measurements with complete observability. The SE accuracy with ZI constraint modeling remains below the acceptable variance of errors in estimated states. BIP method, when applied to optimal allocation of PMU with ZI modeling, is able to minimize number of PMUs, increase redundancy and improve accuracy of SE. The states of the network that are voltage and phase angle are obtained with optimal allocation PMU and conventional measurements. The error obtained with and without PMU is compared to show the effectiveness of the allocation of PMU.

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