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Stabilization of Rotational Motion of Spacecrafts Using Quantized Two Torque Inputs Based on Random Dither

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***Abstract*—The control problem of underactuated spacecrafts has attracted a considerable amount of interest. The control method for a spacecraft equipped with less than three control torques is useful when one of the three control torques had failed. On the other hand, the quantized control of systems is one of the important research topics in recent years. The random dither quantization method that transforms a given continuous signal to a discrete signal by adding artificial random noise to the continuous signal before quantization has also attracted a considerable amount of interest. The objective of this study is to develop the control method based on random dither quantization method for stabilizing the rotational motion of a rigid spacecraft with two control inputs. In this paper, the effectiveness of random dither quantization control method for the stabilization of rotational motion of spacecrafts with two torque inputs is verified by numerical simulations.

***Keywords*—Spacecraft control, quantized control, nonlinear control, random dither method.

I. Introduction

THE first theoretical investigation of the equations of the rotational motion of a rigid body is due to the paper [1], where the necessary and sufficient conditions have been provided for the controllability of a rigid body in the case of one, two and three independent control torques. This paper sparked a renewed interest in the area of control of rigid spacecraft equipped with less than three control torques. Fig. 1 shows the case when one of the nominal three thruster jets had failed. In [2], it was shown that the angular velocity equations can be made asymptotically stable about the origin by means of two torques, each applied along a principal axis. The problem of state feedback stabilization of the angular velocity of a rigid body with two control torques was studied and solved using various approaches. The design method proposed in [3] relies on the center manifold theory, whereas the solution in [4] makes use of control Lyapunov functions and La Salle invariance principle. Exponential stabilization is achieved by homogeneous feedback proposed in [5]. The attitude equations were addressed in [6]. It was proved in [6] that there is no smooth state variable feedback law that locally asymptotically stabilizes a rigid spacecraft about a desired reference attitude with two control torques. Time-varying control methods were proposed in [7], [8] for the attitude stabilization with two control torques to circumvent the topological obstruction to smooth stabilizability due to Brockett's necessary condition [2].

The thruster jets known as discrete-level actuators are often used for the attitude control of spacecrafts. Then, the control inputs with thruster jets are considered as quantized control inputs. The stabilization problem of rotational motion of spacecrafts using quantized control is one of the interesting research topics.

Recently, the so-called random dither quantization that transforms a given continuous signal to a discrete signal by adding artificial random noise to the continuous signal before the quantization has attracted much amount of interest. Model predictive control [9]-[11], also called as receding horizon control [12]-[17], is a type of optimal feedback control and the stochastic model predictive control [18]-[21] has been applied to the quantized control of systems with random dither quantizer in [22].

Motivated by the fact that the actuators such as thruster jets that are used for the attitude control of spacecrafts yield discrete-level inputs, the random dither quantization method was applied in [23] to the nonlinear feedback control for the attitude stabilization of spacecraft using three control torque inputs. However, the design problem of quantized control systems for the stabilization of rotational motion of spacecrafts using two control torque inputs has not yet been studied. Therefore, the objective of this study is to propose a control method based on the random dither quantization method for the stabilization of rotational motion of spacecrafts using two control torque inputs. The effectiveness of the proposed method is verified by numerical simulations.

This paper is organized as follows. In Section II, we introduce some notations and the system model of a spacecraft. In Section III, we consider the control design problem of spacecraft attitude with quantized control inputs. The main results are provided in Section IV. Finally, some concluding remarks are given in Section V.

II. Notations and System Model

First, we introduce the system model of a spacecraft with two control torque inputs. Let us consider a rigid spacecraft in an inertial reference frame and let $\omega_1(t)$, $\omega_2(t)$, and $\omega_3(t)$ denote the angular velocity components along a body fixed

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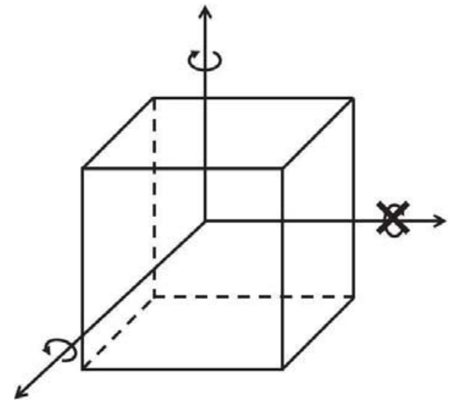

Fig. 1 A schematic view of underactuated spacecrafts

reference frame having the origin at the center of gravity and consisting of three principal axes. The Euler's equations for the rigid body with two independent controls aligned with two principal axes are

$$J_1\dot{\omega}_1(t) = (J_2 - J_3)\omega_2(t)\omega_3(t) + u_1(t) \tag{1a}$$
$$J_2\dot{\omega}_2(t) = (J_3 - J_1)\omega_3(t)\omega_1(t) + u_2(t) \tag{1b}$$
$$J_3\dot{\omega}_3(t) = (J_1 - J_2)\omega_1(t)\omega_2(t) \tag{1c}$$

where $J_1 > 0$, $J_2 > 0$, and $J_3 > 0$ denote the principal moments of inertia and $u_1(t)$ and $u_2(t)$ denote the control torques. Let us introduce the inertia ratios I_1, I_2, I_3 defined as follows:

$$I_1 = \frac{J_2 - J_3}{J_1}$$
$$I_2 = \frac{J_3 - J_1}{J_2}$$
$$I_3 = \frac{J_1 - J_2}{J_3}$$

Using inertia ratios I_1, I_2, and I_3, the system model (1) can be rewritten as follows:

$$\dot{\omega}_1(t) = I_1\omega_2(t)\omega_3(t) + \frac{u_1(t)}{J_1} \tag{2a}$$
$$\dot{\omega}_2(t) = I_2\omega_3(t)\omega_1(t) + \frac{u_2(t)}{J_2} \tag{2b}$$
$$\dot{\omega}_3(t) = I_3\omega_1(t)\omega_2(t) \tag{2c}$$

III. Design of Feedback Control System

In this section, we design the feedback control system for stabilizing the rotational motion of a spacecraft with two independent torque inputs.

First, let the dummy output y be defined as follows:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \omega_1 - \frac{\omega_3}{I_3} \\ \omega_2 + k_3\omega_3^2 \end{bmatrix}, \tag{3}$$

where let $k_3 > 0$ denote the feedback gain. It follows from system equations (2) that the time derivative of y is given by

$$\dot{y}_1 = I_1\omega_2\omega_3 - \omega_1\omega_2 + \frac{u_1}{J_1}, \tag{4a}$$
$$\dot{y}_2 = I_2\omega_3\omega_1 + 2k_3I_3\omega_1\omega_2\omega_3 + \frac{u_2}{J_2}. \tag{4b}$$

Choosing the following control inputs

$$u_1 = J_1(-I_1\omega_2\omega_3 + \omega_1\omega_2 - k_1y_1), \tag{5a}$$
$$u_2 = J_2(-I_2\omega_3\omega_1 - 2k_3I_3\omega_1\omega_2\omega_3 - k_2y_2) \tag{5b}$$

yields

$$\dot{y}_1 = -k_1y_1, \tag{6a}$$
$$\dot{y}_2 = -k_2y_2, \tag{6b}$$

where let $k_1 > 0$ and $k_2 > 0$ denote the feedback gains. It follows from (6) that $y = 0$ is asymptotically stable and y exponentially converges to zero. The trajectory of solutions on the zero output $y = 0$ is called the zero dynamics.

It is seen from (3) that the following relations are satisfied on the zero dynamics $y = 0$.

$$\omega_1 = \frac{\omega_3}{I_3}, \tag{7a}$$
$$\omega_2 = -k_3\omega_3^2. \tag{7b}$$

Substituting (7a) and (7b) into (2c) yields

$$\dot{\omega}_3 = -k_3\omega_3^3. \tag{8}$$

Then, it is seen from (8) that $\omega_3 = 0$ is asymptotically stable and ω_3 converges to zero. Therefore, it follows from (7) that y_1 and y_2 also converge to zero when ω_3 converges to zero. Consequently, we see that the rotational motion of a spacecraft can be stabilized using the two control torque inputs (5a) and (5b).

For an example, the time responses of the angular velocities of a spacecraft using control inputs (5a) and (5b) is shown in Fig. 2. The initial angular momentum is set as $\|\omega(0)\| = 3$. The principal moments of inertia are set as $J_1 = 10$, $J_2 = 5$, and $J_3 = 15$. The feedback gains are set as $k_1 = k_2 = 10$.

Fig. 2 shows that all the trajectories of solutions enter into zero dynamics $y = 0$, then all angular velocities converge to zero.

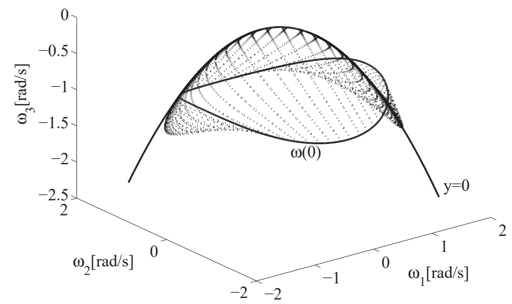

Fig. 2 A schematic view of the trajectories of solutions

Taking the fact into consideration that the actuators such as thruster jets used for the attitude control of spacecrafts yield discrete-level inputs, we next consider the quantized control system.

Here, we introduce the simple uniform quantizer (SUQ) defined by

$$v(t) = q\left(u(t)\right), \tag{9}$$

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where q denotes the static nearest-neighbor quantizer toward $-\infty$ with the quantization interval d as shown in Fig. 1 of [22].

On the other hand, we introduce the random dither quantizer (RDQ) defined by

$$v(t) = q\left(u(t) + \eta(t)\right), \tag{10}$$

where $\eta(t)$ is an independent and identically distributed random variable with the uniform probability distribution on $[-d/2, d/2)$.

Hereafter, we consider the stabilization problem of rotational motion of a spacecraft with quantized control inputs governed by the following equations:

$$\dot{\omega}_1(t) = I_1\omega_2(t)\omega_3(t) + \frac{v_1(t)}{J_1} \tag{11a}$$

$$\dot{\omega}_2(t) = I_2\omega_3(t)\omega_1(t) + \frac{v_2(t)}{J_2} \tag{11b}$$

$$\dot{\omega}_3(t) = I_3\omega_1(t)\omega_2(t) \tag{11c}$$

IV. Main Results

In this section, we show the control performances of spacecrafts with quantized controls governed by (11) for both cases of the SUQ (simple uniform quantizer) and the RDQ (random dither quantizer). The parameters employed in the numerical simulations are as follows: $J_1 = 1$, $J_2 = 2$, $J_3 = 3$, $k_1 = 1$, $k_2 = 1$, $k_3 = 10$, $d = 1$. Time responses of the angular velocities ω_1, ω_2, and ω_3 of quantized control system (11) for both cases of the SUQ and the RDQ are shown in Figs. 3-5. Fig. 6 shows the time response of the Euclid norm of angular velocity. Those figures verify the effectiveness of the proposed RDQ method. We can see from Figs. 3-6 that the proposed RDQ method exhibits much better performance than the SUQ method for the stabilization of rotational motion of a spacecraft. Figs. 7 and 8 show the difference between the quantized control inputs using the SUQ method and the RDQ method. We can see from Figs. 7 and 8 that the quantized control inputs using the proposed RDQ method are well adjusted to decrease the quantization errors rather than the ones using the SUQ method.

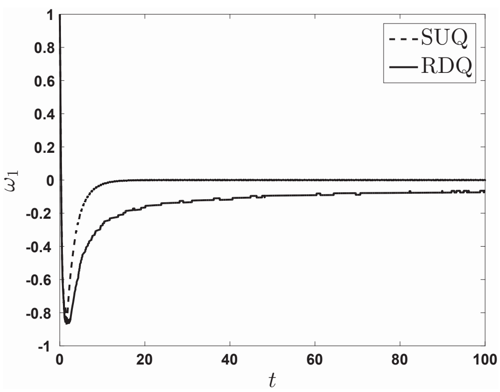

Fig. 3 Time responses of ω_1 for both cases of SUQ and RDQ

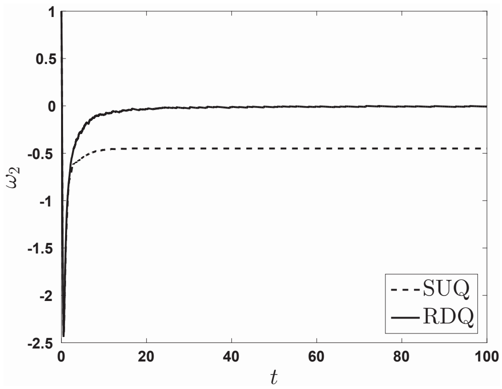

Fig. 4 Time responses of ω_2 for both cases of SUQ and RDQ

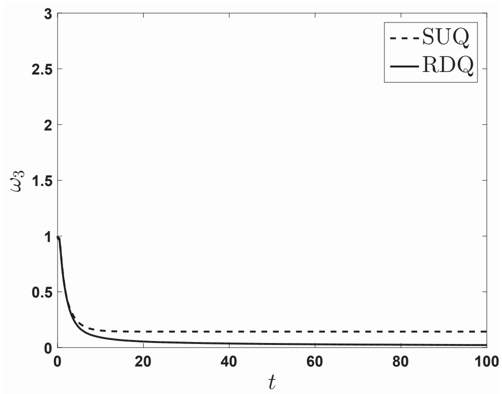

Fig. 5 Time responses of ω_3 for both cases of SUQ and RDQ

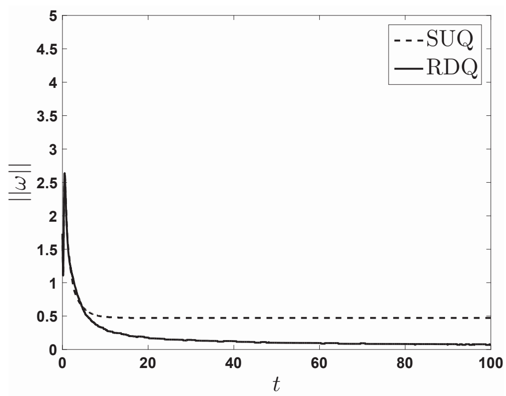

Fig. 6 Time responses of $||\omega||$ for both cases of SUQ and RDQ

V. Conclusion

In this study, we have examined the stabilization problem of rotational motion of a spacecraft with two control torque inputs. The control method for a spacecraft equipped with two control torques is useful to recover the case when one of the nominal three thruster jets had failed or to reduce the consumption of propulsive fuel. Furthermore, motivated by the fact that the actuators such as thruster jets used for the attitude control of spacecrafts yield discrete-level inputs, we have studied the control problem of quantized system for stabilizing

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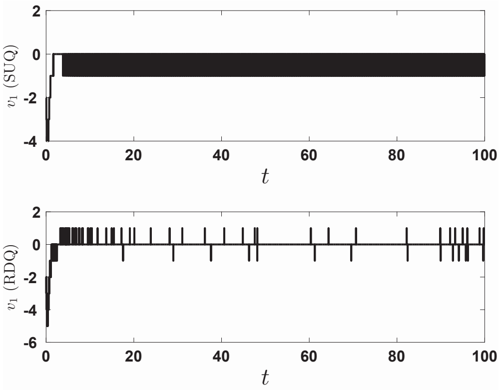

Fig. 7 Time responses of v_1 for both cases of SUQ and RDQ

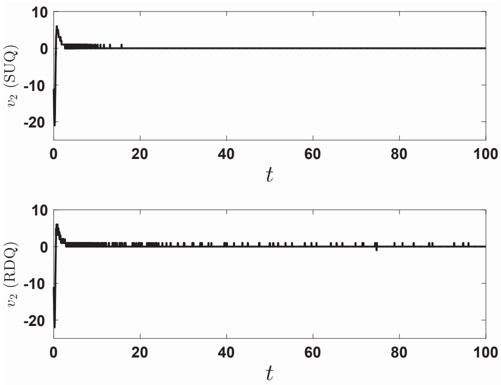

Fig. 8 Time responses of v_2 for both cases of SUQ and RDQ

the rotational motion of a spacecraft with two torque inputs. The random dither quantization method that transforms a given continuous-valued signal to a discrete-valued signal by adding artificial random noise to the continuous-valued signal before quantization was applied to the control method for stabilizing the rotational motion of a spacecraft with two torque inputs. Consequently, this paper proposed a control method based on the random dither quantization method for the stabilization of rotational motion of a spacecraft using two control torque inputs. The effectiveness of the proposed method is verified by numerical simulations. It is known that not only quantization errors but also time delays may cause performance deterioration of control systems [24]-[29]. The control problem of random dither quantized systems with time delays is a possible future work.

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