International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:7, 2010

Some solitary wave solutions of generalized Pochhammer-Chree equation via Exp-function method

Kourosh Parand, Jamal Amani Rad

Abstract—In this paper, Exp-function method is used for some exact solitary solutions of the generalized Pochhammer-Chree equation. It has been shown that the Exp-function method, with the help of symbolic computation, provides a very effective and powerful mathematical tool for solving nonlinear partial differential equations. As a result, some exact solitary solutions are obtained. It is shown that the Exp-function method is direct, effective, succinct and can be used for many other nonlinear partial differential equations.

Keywords—Exp-function method, Generalized Pochhammer-Chree equation, solitary wave solution, ODE's

I. Introduction

THE study of exact solutions of nonlinear partial differential equations (NPDE) plays an important role in mathematical physics, engineering and the other sciences. In the past several decades, various methods for obtaining solutions of NPDE's and ODE's have been presented, such as, tanh-function method [1], [2], [3], Adomian decomposition method [4], [5], Homotopy perturbation method [6], [7], [8], variational iteration method [9], [10], [11], spectral method [12], [13], [14], sine-cosine method [15], [16], radial basis method [17], [18] and so on. Recently, Ji-Huan He and Xu-Hong Wu [19] proposed a novel method, so called Exp-function method, which is easy, succinct and powerful to implement to nonlinear partial differential equations arising in mathematical physics. The Exp-function method has been successfully applied to many kinds of NPDEs, such as, KdV equation with variable coefficients [20], Maccari's system [21], Kawahara equation [22], Boussinesq equations [23], Burger's equations [24], [25], [26], Double Sine-Gordon equation [27], [28], Fisher equation [29], Jaulent-Miodek equations [30] and the other important nonlinear partial differential equations [31], [32], [33]. In this paper we apply the Exp-function method [19] to obtain exact solitary wave solution of a nonlinear partial differential equation, namely, generalized Pochhammer-Chree equation (GPC) given by

$$u_{tt} - u_{ttxx} - (\alpha u + \beta u^{n+1} + \gamma u^{2n+1})_{xx} = 0 \ , \qquad n \geq 1 \ .$$

where α, β and γ are constants. GPC equation represents a nonlinear model of longitudinal wave propagation of elastic rods [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46]. The model for $\alpha = 1$, $\beta = \frac{1}{n+1}$ and $\gamma = 0$ was studied in [40], [41] where solitary wave solutions for this model was obtained for $n = 1, 2$ and 4. A second model for $\alpha = 0$, $\beta = -\frac{1}{2}$ and $\gamma = 0$ was studied by [42] and solitary wave solutions were obtained as well. However, a third model was investigated in [37], [43], [44], [45], [46] for $n = 1$ and $n = 2$ where explicit solitary wave solutions and kinks solutions were derived.

The rest of the paper is organized as follows: Section 2 describes exp-function method for finding exact solutions to the NPDEs. The applications of the proposed analytical scheme presented in Section 3. The conclusions are discussed in the section 4. Exp-function calculations are provided in the end.

K. Parand is with Department of Computer Sciences, Shahid Beheshti University, Tehran, Iran (Phone:+98 21 22431653; Fax:+98 21 22431650; e-mail: (k_parand@sbu.ac.ir)).

J. Amani Rad is with Department of Computer Sciences, Shahid Beheshti University, Tehran, Iran (e-mail: (j.amanirad@gmail.com)).

II. Basic idea of Exp-function method

We consider a general nonlinear PDE in the following form

$$N(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, ...) = 0 \ , \tag{1}$$

where N is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformation. We introduce a complex variation as

$$u(x,t) = U(\eta) \ , \qquad \eta = k(x - ct) + \varphi_0 \ . \tag{2}$$

where k and c are constants and φ_0 is an arbitrary constant. We can rewrite Eq.(1) in the following nonlinear ordinary differential equations

$$N(U, kU', -kcU', k^2U'', ...) = 0 \ ,$$

where the prime denotes the derivation with respect to η. According to the Exp-function method [19], we assume that the solution can be expressed in the form

$$U(\eta) = \frac{\sum_{i=-d}^{c} a_i \exp(i\eta)}{\sum_{j=-q}^{p} b_j \exp(j\eta)} \ , \tag{3}$$

where c, d, p and q are positive integers which can be freely chosen, a_i and b_j are unknown constants to be determined. To determine the values of c and p, we balance the highest order linear term with the highest order nonlinear term in Eq.(3). Similarly to determine the values of d and q. So by means of the exp-function method, we obtain the generalized solitary solution and periodic solution for nonlinear evolution equations arising in mathematical physics.

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:7, 2010

III. Applications of the Exp-function method

In this section, we show the detailed steps of the Exp-function method to construct exact solitary wave solutions of generalized Pochhammer-Chree equations (GPC)

$$u_{tt} - u_{ttxx} - (\alpha u + \beta u^{n+1} + \gamma u^{2n+1})_{xx} = 0 \ , \tag{4}$$

where α, β and γ are constants. Making the travelling wave transformation

$$u(x,t) = U(\eta), \qquad \eta = k(x - ct) + \varphi_0 \ ,$$

and integrating twice, here k and c are constants to be determined later, then Eq.(4) becomes an ordinary differential equation in the form

$$k^2(c^2-\alpha)U - k^4c^2U'' - k^2\beta U^{n+1} - k^2\gamma U^{2n+1} = 0 \ ,$$

where the prime denotes the derivative with respect to η and also where the integration constants are chosen as zero. We now use the transformation

$$U^n = v \ , \tag{5}$$

which we find

$$U'' = \frac{1-n}{n^2} v^{\frac{1}{n}-2}(v')^2 + \frac{1}{n} v^{\frac{1}{n}-1} v'' \ ,$$

substituting the transformations (5) into the GPC equation gives the ODE,

$$\begin{aligned} &n^2k^2(c^2-\alpha)v^2 - k^4c^2(1-n)(v')^2 - k^4c^2nvv'' \\ &-k^2\beta n^2v^3 - k^2\gamma n^2v^4 = 0 \ , \end{aligned} \tag{6}$$

We have the following cases:

I. $\beta \neq 0$

According to the Exp-function method [28], [47], [48], we assume that the solution of Eq.(6) can be expressed in the form

$$v(\eta) = \frac{a_c \exp(c\eta) + ... + a_{-d}\exp(-d\eta)}{b_p \exp(p\eta) + ... + b_{-q}\exp(-q\eta)} \ ,$$

where c, d, p and q are positive integers which are unknown to be determined later. In order to determine values of c and p, we balance the linear term of the highest order with the highest order nonlinear terms in Eq.(6), i.e. vv'' and v^4. By simply calculation, we have

$$vv'' = \frac{c_1\exp[(2c+3p)\eta] + ...}{c_2\exp[5p\eta] + ...} \ , \tag{7}$$

and

$$v^4 = \frac{c_3\exp[(4c+p)\eta] + ...}{c_4\exp[5p\eta] + ...} \ , \tag{8}$$

where c_i are coefficients only for simplicity. By balancing highest order of exp-function in Eqs.(7) and (8), we have

$$4c + p = 2c + 3p \ ,$$

which leads to the result

$$p = c \ .$$

Similarly to determine values of d and q, we balance the linear term of lowest order in Eq.(6)

$$vv'' = \frac{... + d_1\exp[-(3q+2d)\eta]}{... + d_2\exp[-5q\eta]} \ , \tag{9}$$

and

$$v^4 = \frac{... + d_3\exp[-(q+4d)\eta]}{... + d_4\exp[-5q\eta]} \ , \tag{10}$$

where d_i are determined coefficients only for simplicity, we have

$$-(3q+2d) = -(q+4d) \ ,$$

which leads to results

$$q = d \ .$$

For simplicity, we set $p = c = 1$ and $q = d = 1$, so Eq.(3) reduces to

$$v(\eta) = \frac{a_1\exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{\exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \ . \tag{11}$$

Substituting Eq.(11) into Eq.(6), equating to zero the coefficients of all powers of $\exp(n\eta)$ yields a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and c (see Appendix A). By solving the system of algebraic equations with a professional mathematical software, we obtain

$$\begin{aligned} &a_1 = 0 \ , \quad a_0 = \frac{b_0}{\beta}(c^2-\alpha)(n+2) \ , \quad a_{-1} = 0 \ , \\ &b_0 = b_0 \ , \quad k = \frac{n}{c}\sqrt{c^2-\alpha}, \quad c = c \ , \\ &b_{-1} = \frac{b_0^2}{4\beta^2(n+1)}\left[\gamma(c^2-\alpha)(n+2)^2 + \beta^2(n+1)\right] \ . \end{aligned}$$

Substituting these result into Eq.(11), we obtain

$$v(\eta) = \frac{\frac{b_0}{\beta}(c^2-\alpha)(n+2)}{\exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \ . \tag{12}$$

where b_0 and c are free parameters and

$$b_{-1} = \frac{b_0^2}{4\beta^2(n+1)}\left[\gamma(c^2-\alpha)(n+2)^2 + \beta^2(n+1)\right] \ .$$

To compare our results with those obtained in [43], [45], if we set

$$b_0 = \frac{2\sqrt{3}\beta}{\sqrt{3\beta^2 + 16\gamma(c^2-\alpha)}} \ , \qquad n = 2 \ ,$$

Eq(12) becomes

$$v(\eta) = \frac{\frac{8\sqrt{3}}{\sqrt{3\beta^2+16\gamma(c^2-\alpha)}}(c^2-\alpha)}{\exp(\eta) + \frac{2\sqrt{3}\beta}{\sqrt{3\beta^2+16\gamma(c^2-\alpha)}} + \exp(-\eta)} \ . \tag{13}$$

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:7, 2010

where $\eta = \frac{2}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0$. We re-write Eq.(13) and use of Eq.(5) in the form

$$u(x,t) = \pm\Bigg\{\Bigg(4\sqrt{\frac{3(\alpha-c^2)^2}{3\beta^2-16\gamma(\alpha-c^2)}}\sec h^2\left[\frac{1}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\Bigg)\Big/\Bigg(2+\Bigg(-1+\frac{\sqrt{3}\beta}{\sqrt{3\beta^2-16\gamma(\alpha-c^2)}}\Bigg)\sec h^2\left[\frac{1}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\Bigg)\Bigg\}^{1/2}$$

which is the traveling wave solution obtained in [43], [45]. Also, by the choice $\gamma=0$ in our solution (12) gives

$$v(\eta) = \frac{\frac{b_0}{\beta}(c^2-\alpha)(n+2)}{\exp(\eta)+b_0+\frac{1}{4}b_0^2\exp(-\eta)} \ . \tag{14}$$

where $\eta = \frac{n}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0$. To compare our results with those obtained in [43], [45], [46], we present the following discussion

(I) At $c^2>\alpha$ and $b_0=2$.
We can obtain from Eq.(14) and Eq.(5) that

$$u(x,t) = \Bigg\{\frac{(c^2-\alpha)(n+2)}{2\beta}\sec h^2\left[\frac{n}{2c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\Bigg\}^{1/n}$$

(II) At $c^2>\alpha$ and $b_0=-2$.
We can obtain from Eq.(14) and Eq.(5) that

$$u(x,t) = \Bigg\{-\frac{(c^2-\alpha)(n+2)}{2\beta}\csc h^2\left[\frac{n}{2c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\Bigg\}^{1/n}$$

or equivalently

$$u(x,t) = \Bigg\{\frac{(c^2-\alpha)(n+2)}{8\beta}\Bigg(2-\tanh^2\left[\frac{n}{4c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]-\coth^2\left[\frac{n}{4c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\Bigg)\Bigg\}^{1/n}$$

(III) At $c^2<\alpha$ and $b_0=2$.
We can obtain from Eq.(14) and Eq.(5) that

$$u(x,t) = \Bigg\{\frac{(c^2-\alpha)(n+2)}{2\beta}\sec^2\left[\frac{n}{2c}\sqrt{\alpha-c^2}(x-ct)+\varphi_0\right]\Bigg\}^{1/n}$$

(IV) At $c^2<\alpha$ and $b_0=-2$.
We can obtain from Eq.(14) and Eq.(5) that

$$u(x,t) = \Bigg\{\frac{(c^2-\alpha)(n+2)}{2\beta}\csc^2\left[\frac{n}{2c}\sqrt{\alpha-c^2}(x-ct)+\varphi_0\right]\Bigg\}^{1/n}$$

or equivalently

$$u(x,t) = \Bigg\{\frac{(c^2-\alpha)(n+2)}{8\beta}\Bigg(2+\tanh^2\left[\frac{n}{4c}\sqrt{\alpha-c^2}(x-ct)+\varphi_0\right]+\coth^2\left[\frac{n}{4c}\sqrt{\alpha-c^2}(x-ct)+\varphi_0\right]\Bigg)\Bigg\}^{1/n}$$

which are the traveling wave solutions obtained in [43], [45], [46].

II. $\beta=0$
In the this case, Eq.(6) convert to

$$n^2k^2(c^2-\alpha)v^2-k^4c^2(1-n)(v')^2 - k^4c^2nvv''-k^2\gamma n^2v^4=0 \ , \tag{15}$$

According to the Exp-function method [28], [47], [48], we assume that the solution of Eq.(6) can be expressed in the form

$$v(\eta) = \frac{a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)}{\exp(\eta)+b_0+b_{-1}\exp(-\eta)} \ . \tag{16}$$

Substituting Eq.(16) into Eq.(15), equating to zero the coefficients of all powers of $\exp(n\eta)$ yields a set of algebraic equations for a_0, b_0, a_{-1}, a_1, b_1, k and c (see Appendix B). By solving the system of algebraic equations with a professional mathematical software, we obtain

$$a_1=0 \ , \quad a_0=a_0 \ , \quad a_{-1}=0 \ , \quad b_0=0 \ ,$$
$$b_{-1}=\frac{1}{4}\frac{\gamma a_0^2}{(c^2-\alpha)(n+1)} \ ,$$
$$k=\frac{n}{c}\sqrt{c^2-\alpha} \ , \quad c=c \ .$$

Substituting these result into Eq.(16), we obtain

$$v(\eta) = \frac{a_0}{\exp(\eta)+\frac{1}{4}\frac{\gamma a_0^2}{(c^2-\alpha)(n+1)}\exp(-\eta)} \ . \tag{17}$$

where a_0 and c are free parameters. To compare our results with those obtained in [37], [43], [45], [46], we present the following discussion

(I). At $a_0=2\sqrt{\frac{(n+1)(\alpha-c^2)}{\gamma}}$, $c^2>\alpha$ and $\gamma<0$.

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
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We can obtain from Eq.(17) and Eq.(5) that

$$u(x,t)=\left\{-\frac{1}{2}\sqrt{\frac{(n+1)(\alpha-c^2)}{\gamma}}\left(\tanh\left[\frac{n}{2c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]-\coth\left[\frac{n}{2c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\right)\right\}^{1/n}$$

(II). At $a_0=2\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}$, $c^2>\alpha$ and $\gamma>0$.
We can obtain from Eq.(17) and Eq.(5) that

$$u(x,t)=\left\{\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}\sec h^2\left[\frac{n}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\right\}^{1/n}$$

(III). At $a_0=2i\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}$, $c^2>\alpha$ and $\gamma>0$.
We can obtain from Eq.(17) and Eq.(5) that

$$u(x,t)=\left\{i\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}\csc h^2\left[\frac{n}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\right\}^{1/n}$$

(IV). At $a_0=2\sqrt{\frac{(n+1)(\alpha-c^2)}{\gamma}}$, $c^2<\alpha$ and $\gamma<0$.
We can obtain from Eq.(17) and Eq.(5) that

$$u(x,t)=\left\{\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}\sec^2\left[\frac{n}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\right\}^{1/n}$$

(V). At $a_0=2i\sqrt{\frac{(n+1)(\alpha-c^2)}{\gamma}}$, $c^2<\alpha$ and $\gamma<0$.
We can obtain from Eq.(17) and Eq.(5) that

$$u(x,t)=\left\{\sqrt{\frac{(n+1)(c^2-\alpha)}{\gamma}}\csc^2\left[\frac{n}{c}\sqrt{c^2-\alpha}(x-ct)+\varphi_0\right]\right\}^{1/n}$$

which are the traveling wave solutions obtained in [37], [43], [45], [46].

IV. Conclusion

In this paper, Exp-function method is used to obtain some exact solitary solutions of the generalized Pochhammer-Chree equation. Generalized Pochhammer-Chree equation represents a nonlinear model of longitudinal wave propagation of elastic rods. Exp-function method changes the problem from solving nonlinear partial differential equations to solving a ordinary differential equations by chosen free parameters and with the help of symbolic computation, provides a very effective and powerful mathematical tool for solving nonlinear partial differential equations. The obtained result clarify that the Exp-function method is direct, effective, succinct and can be used for many other nonlinear partial differential equations.

Appendix A

$$\begin{aligned}
&-n^2c^2a_{-1}^2b_{-1}^2+n^2\alpha a_{-1}^2b_{-1}^2+n^2\beta a_{-1}^3b_{-1}+n^2\gamma a_{-1}^4=0\ ,\\
&-2n^2c^2a_{-1}^2b_0b_{-1}+2n^2\alpha a_0a_{-1}b_{-1}^2+2n^2\alpha a_{-1}^2b_0b_{-1}\\
&\quad+3n^2\beta a_0a_{-1}^2b_{-1}-2n^2c^2a_0a_{-1}b_{-1}^2+n^2\beta a_{-1}^3b_0\\
&\quad+4n^2\gamma a_0a_{-1}^3+nk^2c^2a_{-1}a_0b_{-1}^2-nk^2c^2a_{-1}^2b_0b_{-1}=0\ ,\\
&-2n^2c^2a_1a_{-1}b_{-1}^2+2n^2\alpha a_1a_{-1}b_{-1}^2-4nk^2c^2a_{-1}^2b_{-1}\\
&\quad+3n^2\beta a_1a_{-1}^2b_{-1}+3n^2\beta a_0^2a_{-1}b_{-1}+3n^2\beta a_0a_{-1}^2b_0\\
&\quad-n^2c^2a_0^2b_{-1}^2-n^2c^2a_{-1}^2b_0^2+n^2\alpha a_0^2b_{-1}^2+n^2\alpha a_{-1}^2b_0^2\\
&\quad+k^2c^2a_0^2b_{-1}^2+k^2c^2a_{-1}^2b_0^2+6n^2\gamma a_0^2a_{-1}^2+4n^2\gamma a_1a_{-1}^3\\
&\quad-2n^2c^2a_{-1}^2b_{-1}+2n^2\alpha a_{-1}^2b_{-1}+n^2\beta a_{-1}^3+4n^2\alpha a_0a_{-1}b_0b_{-1}\\
&\quad-4n^2c^2a_0a_{-1}b_0b_{-1}-2k^2c^2a_0a_{-1}b_0b_{-1}\\
&\quad+4nk^2c^2a_{-1}a_1b_{-1}^2=0\ ,\\
&-6k^2c^2na_{-1}a_0b_{-1}+6k^2c^2na_1b_{-1}a_{-1}b_0+n^2\beta a_0^3b_{-1}\\
&\quad+4k^2c^2a_1b_{-1}^2a_0-4k^2c^2a_{-1}a_0b_{-1}+3n^2\beta a_0a_{-1}^2\\
&\quad+nk^2c^2a_0a_{-1}b_0^2-nk^2c^2a_0^2b_{-1}b_0-4k^2c^2a_1b_{-1}a_{-1}b_0\\
&\quad-4n^2c^2a_1a_{-1}b_0b_{-1}+6n^2\beta a_1a_0a_{-1}b_{-1}+4n^2\alpha a_1a_{-1}b_0b_{-1}\\
&\quad-k^2c^2na_{-1}^2b_0+3n^2\beta a_0^2a_{-1}b_0-2n^2c^2a_0a_{-1}b_0^2\\
&\quad+2n^2\alpha a_0a_{-1}b_0^2-2n^2c^2a_1a_0b_{-1}^2-4n^2c^2a_0a_{-1}b_{-1}\\
&\quad+2n^2\alpha a_1a_0b_{-1}^2+4n^2\alpha a_0a_{-1}b_{-1}+3n^2\beta a_1a_{-1}^2b_0\\
&\quad+12n^2\gamma a_1a_0a_{-1}^2+4k^2c^2a_{-1}^2b_0-2n^2c^2a_{-1}^2b_0+2n^2\alpha a_{-1}^2b_0\\
&\quad+4n^2\gamma a_0^3a_{-1}+2n^2\alpha a_0^2b_0b_{-1}+k^2c^2na_1b_{-1}^2a_0\\
&\quad-2n^2c^2a_0^2b_0b_{-1}=0\ ,\\
&4k^2c^2a_{-1}^2+n^2\gamma a_0^4-n^2c^2a_0^2b_0^2+n^2\alpha a_0^2b_0^2+4k^2c^2a_1^2b_{-1}^2\\
&\quad+n^2\beta a_0^3b_0+3n^2\beta a_1a_{-1}^2+3n^2\beta a_0^2a_{-1}+6n^2\gamma a_1^2a_{-1}^2\\
&\quad-2k^2c^2a_0^2b_{-1}+2n^2\alpha a_0^2b_{-1}-2n^2c^2a_0^2b_{-1}-8k^2c^2a_1b_{-1}a_{-1}\\
&\quad+2k^2c^2a_{-1}a_0b_0-2k^2c^2a_1b_0^2a_{-1}+2k^2c^2a_1b_{-1}a_0b_0\\
&\quad+4k^2c^2na_1b_0^2a_{-1}-n^2c^2a_{-1}^2+n^2\alpha a_{-1}^2-4n^2c^2a_1a_{-1}b_{-1}\\
&\quad-2n^2c^2a_1a_{-1}b_0^2-4n^2c^2a_0a_{-1}b_0+4n^2\alpha a_1a_{-1}b_{-1}\\
&\quad+2n^2\alpha a_1a_{-1}b_0^2+4n^2\alpha a_0a_{-1}b_0-4nk^2c^2a_0^2b_{-1}\\
&\quad+3n^2\beta a_1^2a_{-1}b_{-1}+3n^2\beta a_1a_0^2b_{-1}+12n^2\gamma a_1a_0^2a_{-1}+n^2\alpha a_1^2b_{-1}^2\\
&\quad-n^2c^2a_1^2b_{-1}^2-4n^2c^2a_1a_0b_0b_{-1}+4n^2\alpha a_1a_0b_0b_{-1}\\
&\quad+6n^2\beta a_1a_0a_{-1}b_0=0\ ,\\
&n^2\beta a_0^3+nk^2c^2a_0a_1b_0^2-k^2c^2na_1^2b_0b_{-1}+6k^2c^2na_1b_0a_{-1}\\
&\quad+4k^2c^2a_{-1}a_0+2n^2\alpha a_0^2b_0+2n^2\alpha a_0a_{-1}-4k^2c^2a_1b_{-1}a_0\\
&\quad+4n^2\gamma a_1a_0^3+k^2c^2na_{-1}a_0+6n^2\beta a_1a_0a_{-1}-2n^2c^2a_0a_{-1}\\
&\quad-2n^2c^2a_0^2b_0+4k^2c^2a_1^2b_0b_{-1}-4k^2c^2a_1b_0a_{-1}+3n^2\beta a_1a_0^2b_0\\
&\quad+2n^2\alpha a_1a_0b_0^2-2n^2c^2a_1a_0b_0^2-nk^2c^2a_0^2b_0+3n^2\beta a_1^2a_0b_{-1}\\
&\quad+12n^2\gamma a_1^2a_0a_{-1}-2n^2c^2a_1^2b_0b_{-1}-4n^2c^2a_1a_0b_{-1}\\
&\quad-4n^2c^2a_1a_{-1}b_0+2n^2\alpha a_1^2b_0b_{-1}+4n^2\alpha a_1a_0b_{-1}\\
&\quad+4n^2\alpha a_1a_{-1}b_0+3n^2\beta a_1^2a_{-1}b_0-6k^2c^2na_1b_{-1}a_0=0\ ,
\end{aligned}$$

$$
\begin{aligned}
&4n^2\alpha a_1a_0b_0-4n^2c^2a_1a_0b_0+4k^2c^2na_1a_{-1}-4k^2c^2na_1^2b_{-1}\\
&\quad+3n^2\beta a_1^2a_0b_0-2k^2c^2a_1a_0b_0-n^2c^2a_0^2+n^2\alpha a_0^2+k^2c^2a_0^2\\
&\quad-n^2c^2a_1^2b_0^2+n^2\alpha a_1^2b_0^2+k^2c^2a_1^2b_0^2+3n^2\beta a_1a_0^2+6n^2\gamma a_1^2a_0^2\\
&\quad+3n^2\beta a_1^2a_{-1}+4n^2\gamma a_1^3a_{-1}+2n^2\alpha a_1^2b_{-1}+2n^2\alpha a_1a_{-1}\\
&\quad-2n^2c^2a_1^2b_{-1}-2n^2c^2a_1a_{-1}+n^2\beta a_1^3b_{-1}=0\ ,\\
&-k^2c^2na_1^2b_0+nk^2c^2a_0a_1+4n^2\gamma a_1^3a_0-2n^2c^2a_1^2b_0\\
&\quad-2n^2c^2a_1a_0+2n^2\alpha a_1^2b_0+2n^2\alpha a_1a_0+3n^2\beta a_1^2a_0\\
&\quad+n^2\beta a_1^3b_0=0\ ,\\
&-n^2c^2a_1^2+n^2\alpha a_1^2+n^2\beta a_1^3+n^2\gamma a_1^4=0\ .
\end{aligned}
$$

APPENDIX B

$$
\begin{aligned}
&-n^2c^2a_{-1}^2b_{-1}^2+n^2\alpha a_{-1}^2b_{-1}^2+n^2\gamma a_{-1}^4=0\ ,\\
&4n^2\gamma a_0a_{-1}^3+2n^2\alpha a_0a_{-1}b_{-1}^2+2n^2\alpha a_{-1}^2b_0b_{-1}\\
&\quad-2n^2c^2a_0a_{-1}b_{-1}^2-2n^2c^2a_{-1}^2b_0b_{-1}\\
&\quad+nk^2c^2a_{-1}a_0b_{-1}^2-nk^2c^2a_{-1}^2b_0b_{-1}=0\ ,\\
&-4n^2c^2a_0a_{-1}b_0b_{-1}+4n^2\alpha a_0a_{-1}b_0b_{-1}\\
&\quad-2k^2c^2a_0a_{-1}b_0b_{-1}+4nk^2c^2a_{-1}a_1b_{-1}^2+4n^2\gamma a_1a_{-1}^3\\
&\quad+2n^2\alpha a_{-1}^2b_{-1}-2n^2c^2a_{-1}^2b_{-1}-n^2c^2a_0^2b_{-1}^2\\
&\quad-n^2c^2a_{-1}^2b_0^2+n^2\alpha a_0^2b_{-1}^2+n^2\alpha a_{-1}^2b_0^2+k^2c^2a_0^2b_{-1}^2\\
&\quad+k^2c^2a_{-1}^2b_0^2+6n^2\gamma a_0^2a_{-1}^2-2n^2c^2a_1a_{-1}b_{-1}^2\\
&\quad+2n^2\alpha a_1a_{-1}b_{-1}^2-4nk^2c^2a_{-1}^2b_{-1}=0\ ,\\
&k^2c^2na_1b_{-1}^2a_0-6k^2c^2na_{-1}a_0b_{-1}-4n^2c^2a_1a_{-1}b_0b_{-1}\\
&\quad+4n^2\alpha a_1a_{-1}b_0b_{-1}+6k^2c^2na_1b_{-1}a_{-1}b_0\\
&\quad+nk^2c^2a_0a_{-1}b_0^2-nk^2c^2a_0^2b_{-1}b_0-4k^2c^2a_1b_{-1}a_{-1}b_0\\
&\quad-nk^2c^2a_{-1}^2b_0+2n^2\alpha a_0a_{-1}b_0^2-2n^2c^2a_0^2b_0b_{-1}\\
&\quad-2n^2c^2a_0a_{-1}b_0^2+2n^2\alpha a_0^2b_0b_{-1}+4k^2c^2a_1b_{-1}^2a_0\\
&\quad-4k^2c^2a_{-1}a_0b_{-1}-2n^2c^2a_1a_0b_{-1}^2-4n^2c^2a_0a_{-1}b_{-1}\\
&\quad+2n^2\alpha a_1a_0b_{-1}^2+4n^2\alpha a_0a_{-1}b_{-1}+12n^2\gamma a_1a_0a_{-1}^2\\
&\quad+4k^2c^2a_{-1}^2b_0+4n^2\gamma a_0^3a_{-1}+2n^2\alpha a_{-1}^2b_0\\
&\quad-2n^2c^2a_{-1}^2b_0=0\ ,\\
&4k^2c^2a_{-1}^2+n^2\gamma a_0^4+n^2\alpha a_{-1}^2-n^2c^2a_{-1}^2\\
&\quad+2k^2c^2a_1b_{-1}a_0b_0+4k^2c^2na_1b_0^2a_{-1}+n^2\alpha a_0^2b_0^2\\
&\quad-n^2c^2a_0^2b_0^2+12n^2\gamma a_1a_0^2a_{-1}+2k^2c^2a_{-1}a_0b_0\\
&\quad-2n^2c^2a_0^2b_{-1}+6n^2\gamma a_1^2a_{-1}^2+4k^2c^2a_1^2b_{-1}^2\\
&\quad-4n^2c^2a_1a_{-1}b_{-1}-2k^2c^2a_1b_0^2a_{-1}-2n^2c^2a_1a_{-1}b_0^2\\
&\quad-4n^2c^2a_0a_{-1}b_0+n^2\alpha a_1^2b_{-1}^2+2n^2\alpha a_1a_{-1}b_0^2\\
&\quad+4n^2\alpha a_1a_{-1}b_{-1}+4n^2\alpha a_0a_{-1}b_0-4nk^2c^2a_0^2b_{-1}\\
&\quad-2k^2c^2a_0^2b_{-1}+2n^2\alpha a_0^2b_{-1}-n^2c^2a_1^2b_{-1}^2\\
&\quad-4n^2c^2a_1a_0b_0b_{-1}-8k^2c^2a_1b_{-1}a_{-1}\\
&\quad+4n^2\alpha a_1a_0b_0b_{-1}=0\ ,\\
&-2n^2c^2a_1^2b_0b_{-1}-4n^2c^2a_1a_0b_{-1}-4n^2c^2a_1a_{-1}b_0\\
&\quad+2n^2\alpha a_1^2b_0b_{-1}+4n^2\alpha a_1a_0b_{-1}+4n^2\alpha a_1a_{-1}b_0\\
&\quad+12n^2\gamma a_1^2a_0a_{-1}+nk^2c^2a_0a_1b_0^2-2n^2c^2a_0^2b_0\\
&\quad+2n^2\alpha a_0^2b_0+4k^2c^2a_{-1}a_0+4n^2\gamma a_1a_0^3+2n^2\alpha a_0a_{-1}\\
&\quad-2n^2c^2a_0a_{-1}-nk^2c^2a_1^2b_0b_{-1}+6nk^2c^2a_1b_0a_{-1}\\
&\quad-6nk^2c^2a_1b_{-1}a_0+nk^2c^2a_{-1}a_0-2n^2c^2a_1a_0b_0^2\\
&\quad+2n^2\alpha a_1a_0b_0^2-nk^2c^2a_0^2b_0+4k^2c^2a_1^2b_0b_{-1}\\
&\quad-4k^2c^2a_1b_0a_{-1}-4k^2c^2a_1b_{-1}a_0=0\ ,\\
&4n^2\gamma a_1^3a_{-1}-2n^2c^2a_1^2b_{-1}-2n^2c^2a_1a_{-1}+2n^2\alpha a_1^2b_{-1}\\
&\quad+2n^2\alpha a_1a_{-1}-n^2c^2a_1^2b_0^2-4n^2c^2a_1a_0b_0\\
&\quad+4n^2\alpha a_1a_0b_0-2k^2c^2a_1a_0b_0+4nk^2c^2a_1a_{-1}-4nk^2c^2a_1^2b_{-1}\\
&\quad+n^2\alpha a_1^2b_0^2+k^2c^2a_1^2b_0^2+6n^2\gamma a_1^2a_0^2+k^2c^2a_0^2-n^2c^2a_0^2\\
&\quad+n^2\alpha a_0^2=0\ ,\\
&-nk^2c^2a_1^2b_0+nk^2c^2a_0a_1-2n^2c^2a_1^2b_0-2n^2c^2a_1a_0+2n^2\alpha a_1^2b_0\\
&\quad+2n^2\alpha a_1a_0+4n^2\gamma a_1^3a_0=0\ ,\\
&n^2\gamma a_1^4-n^2c^2a_1^2+n^2\alpha a_1^2=0\ .
\end{aligned}
$$

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