Solving Transient Conduction and Radiation Using Finite Volume Method

Ashok K. Satapathy, Prerana Nashine

Abstract—Radiative heat transfer in participating medium was carried out using the finite volume method. The radiative transfer equations are formulated for absorbing and anisotropically scattering and emitting medium. The solution strategy is discussed and the conditions for computational stability are conferred. The equations have been solved for transient radiative medium and transient radiation incorporated with transient conduction. Results have been obtained for irradiation and corresponding heat fluxes for both the cases. The solutions can be used to conclude incident energy and surface heat flux. Transient solutions were obtained for a slab of heat conducting in slab and by thermal radiation. The effect of heat conduction during the transient phase is to partially equalize the internal temperature distribution. The solution procedure provides accurate temperature distributions in these regions. A finite volume procedure with variable space and time increments is used to solve the transient radiation equation. The medium in the enclosure absorbs, emits, and anisotropically scatters radiative energy. The incident radiations and the radiative heat fluxes are presented in graphical forms. The phase function anisotropy plays a significant role in the radiation heat transfer when the boundary condition is non-symmetric.

Keywords—Participating media, finite volume method, radiation coupled with conduction, heat transfer.

I. INTRODUCTION

IN many combustion devices, radiant heat transfer are the predominant mode of energy transfer. Authentic solutions of the performance of these devices are strongly linked to the solutions of the radiative transport equations. Although a variety of computational tools have been proposed in the past several decades, recent advances in combustion modeling have re-emphasized several essential needs for radiative heat transfer calculation. Modern numerical modeling schemes require calculations tools that are efficient and integrate easily with the other transport equations based on finite volume approaches. In large scale industrial and utility pulverized coal combustion furnace, heat transfer is mainly by radiation. Radiation is attenuated by absorbing gases such as water vapour, carbon-di-oxide and by suspended particulates (coal, char, ash and soot).Accurate simulation of radiative processes requires analysis of combined absorption and scattering of radiant energy. An extensive survey of multi-dimensional radiative models has been reported elsewhere. Exact formulations for absorbing, emitting and scattering media in an enclosure have been documented. In conjugate mode

problems the FVM for the radiative heat transfer utilizes the same concept as that of the FVM of the CFD, its computational grids are compatible with the FVM grids that are used in the solution of the energy and momentum equations [1], [2].

The finite volume method (FVM) [3], [4] is widely used to compute the radiative information in radiative heat transfer. The discrete ordinate method suffers from the false-scattering [3] and this difficulty we did not face in FVM. In this method, the ray-effect is also less noticeable [3]. Using the conventional CFD based methods such as the FDM and the FVM for solving the energy equations of the combined radiation, conduction and convection heat transfer problems, strength of the FVM in providing radiative information is well employed [3], [4]. Thus, even though the FVM for the radiative heat transfer is a new method, it is more popular than the DTM, and the DOM. The present article is thus proposed at extending the application of the FVM to relatively new problems. To provide the radiative information required for the energy equation, there are many methods such as discrete ordinates method, collapsed dimension method, discrete transfer method, FVM and many more. Among all the available methods FVM is the most robust of all .The discrete ordinate method (DOM) [5]-[10] is one of the oldest and still the most extensively favored methods by physicists and engineers. Heat-transfer problems involving radiation are computationally expensive. Among all the modes of heat transfer, calculation of radiative information is the most time consuming. The efforts have been directed towards developing new radiative transfer methods [5] as well as improving the computational efficiencies of the existing ones [11]-[19] so as to reduce the computational expense.

The objective of the present work is to establish the compatibility of the FVM solution for the determination of radiative information. One other objective is also to see how the FVM performs against the radiation-conduction combination in which the RTE is solved using the FVM of radiative heat transfer and the conductive information is computed using the conduction equation. Towards this objective, transient conduction and radiation heat transfer problems in 1-D planar are examined. For different parameters like the scattering albedo, the conduction–radiation parameter and the boundary emissivity, results of the FVM are presented.

A. Formulation of Radiative Transfer Equation

The transient radiative transfer equation at any point *P* for direction *s* is given by

Ashok K. Satapathy is with the National Institute of Technology, Rourkela, India (phone: +919438575810; e-mail: aksatapathy2010@gmail.com).

Prerana Nashine is with the National Institute of Technology, Rourkela, India (e-mail: prerana.mech08@gmail.com).

$$\frac{1}{c}\frac{\partial I^l}{\partial t}+\frac{\partial I^l}{\partial s}=-\beta_m^l I^l + S_m^l \qquad (1)$$

where superscript l represents the angular direction. Transient radiative transfer equation is being integrated over a one dimensional control volume, control angle and over a time interval from t to $t+\Delta t$, the above equation can be discretized as follows

$$\iiint_{\Delta\Omega\,\Delta x\,\Delta t}\frac{1}{c}\frac{\partial I}{\partial t}dtdxd\Omega+\iiint_{\Delta t\,\Delta\Omega\,\Delta x}\frac{\partial I}{\partial x}dxd\Omega dt=\iiint_{\Delta t\,\Delta\Omega\,\Delta x}(-\beta_m I+S_m)dxd\Omega dt \quad (2)$$

where β_m^l is the modified extinction coefficient and S_m^l is modified source term and can be written as in (3a) and (3b) respectively. Control angle is elaborated as in (3c).

$$\beta_m^l = k+\sigma_s - \frac{\sigma_s}{4\pi}\Phi^{i'l}\Delta\Omega^l \qquad (3a)$$

$$S_m^l = kI_b + \frac{\sigma_s}{4\pi}\sum_{l'=1,l'\neq 1}^{M} I^{l'}\Phi^{i'l}\Delta\Omega^{l'} \qquad (3b)$$

$$\Delta\Omega^{'} = \int_{\phi^-}^{\phi^+}\int_{\theta^-}^{\theta^+}\sin\theta\,d\theta\,d\phi \qquad (3c)$$

The subscript l has been dropped from the above equation for the compactness. The order of integration is chosen according to the nature of terms. The first term of left hand side can be discretized as

$$\iiint_{\Delta\Omega\,\Delta x\,\Delta t}\frac{1}{c}\frac{\partial I}{\partial t}dtdxd\Omega=\frac{1}{c}(I_P - I_P^0)\Delta x\Delta\Omega \qquad (4)$$

where I_p^0 shows the intensity at previous time step and I_p at present time step. Pursuing the discretization practice of Chai et al. [20], [21], the second term in the left hand side and the term of right hand side can be written as

$$\iiint_{\Delta t\,\Delta\Omega\,\Delta x}\frac{\partial I}{\partial x}dxd\Omega dt = D_{cx}\int_{t}^{t+\Delta t}(I_e - I_w)dt \qquad (5)$$

where I_e and I_w are the intensities at east and west control volume faces.

$$\iiint_{\Delta t\,\Delta\Omega\,\Delta x}(-\beta_m I+S_m)dxd\Omega dt = \int_{t}^{t+\Delta t}[(-\beta_m I_P + S_m)\Delta x\Delta\Omega]dt \quad (6)$$

Using the fully implicit scheme (5) can be written as

$$D_{cx}\int_{t}^{t+\Delta t}(I_e - I_w)dt = D_{cx}(I_e - I_w)\Delta t \qquad (7)$$

Integrating (6) over the time interval result in following equation

$$\int_{t}^{t+\Delta t}[(-\beta_m I_P + S_m)\Delta x\Delta\Omega]dt = (-\beta_m I_P + S_m)\Delta x\Delta\Omega\Delta t \quad (8)$$

Combining (4), (6) and (7) gives

$$\frac{1}{c}(I_P - I_P^0)\frac{\Delta x}{\Delta t}\Delta\Omega + D_{cx}(I_e - I_w) = (-\beta_m I_P + S_m)\Delta x\Delta\Omega \quad (9)$$

Intensity I_p and I_p^0 are the centre node intensity at the end and at the beginning of time step respectively. To relate the boundary intensities to the nodal intensity spatial differencing scheme are needed and one such available scheme is the step scheme. The detailed formulation is shown for control angles $D_{cx} > 0$ and the boundary intensities I_e and I_w can be written as follows

$$I_e = I_P + S_{ce+} + S_{pe+}I_P \qquad (10a)$$

$$I_w = I_W + S_{cw+} + S_{pw+}I_P \qquad (10b)$$

Applying the step scheme

$$S_{ce+} = S_{pe+} = S_{cw+} = S_{pw+} = 0 \qquad (11a)$$

For $0 \leq \phi^+_w \leq 1$

$$S_{cw+} = -\Phi_w^+ I_W, \; S_{pw+} = -\Phi_w^+ \qquad (11b)$$

Otherwise

$$S_{cw+} = 0, \; S_{pw+} = 0 \qquad (11c)$$

For $0 \leq \phi^+_e \leq 1$

$$S_{ce+} = -\Phi_e^+ I_E, \; S_{pe+} = -\Phi_e^+ \qquad (11d)$$

Otherwise

$$S_{ce+} = 0, \; S_{pe+} = 0 \qquad (11e)$$

Here S_{ce+}, S_{pe+}, S_{cw+}, S_{pw+} are the additional source terms for high resolution scheme. The scattering phase function is described as

$$\Phi_w^+ = \frac{I_W - I_{WW}}{I_P - I_{WW}} \qquad (12a)$$

$$\Phi_e^+ = \frac{I_P - I_W}{I_E - I_W} \qquad (12b)$$

By solving (7) and (8), a final discretization equation (for $D_{cx} > 0$) can be reduced in a standard form of finite volume method as

$$a_P I_P = a_W I_W + b \qquad (13)$$

For the solution of (13) the initial intensity is prescribed according to initial boundary condition and the terms a_p the coefficient of discretization equation and b the source term in the discretization equation can be written in a discretized form as follows

$$a_w = D_{cx} \qquad (14a)$$

$$a_P = D_{cx}(1 + S_{pe+} - S_{pw+}) + \beta_{m,p} \Delta x \Delta \Omega + \frac{1}{c}\frac{\Delta x}{\Delta t}\Delta \Omega \qquad (14b)$$

$$b = D_{cx}(S_{cw+} - S_{ce+}) + S_{m,P}\Delta x \Delta \Omega + \frac{1}{c}I_p^0 \frac{\Delta x}{\Delta t}\Delta \Omega \qquad (14c)$$

$$D_{cx} = \int_{\Delta \Omega}(\hat{s}.\hat{n}_x)d\Omega \qquad (14d)$$

The problem of TRTE coupled with conduction in a one dimensional slab investigated in this article. The one dimensional transient conduction equation which will be coupled by RTE is given by (15)

$$\frac{\partial T}{\partial t} = \alpha_t \frac{\partial^2 T}{\partial x^2} \qquad (15)$$

and the solution of conduction equation is expressed in terms of the error function given below in (16)

$$T(x,t) = T_0(x)\left[1 - erf\left(\frac{x}{2\sqrt{(\alpha_t t)}}\right)\right] \qquad (16)$$

where erf is the error function, α_t is the thermal conductivity of slab, t is time in seconds. $T(x,t)$ is temperature in Kelvin, $T_0(x)$ is temperature at previous time step.

II. PROBLEM DESCRIPTION

The finite volume method has been applied to a problem that involves one dimensional radiative heat transfer in a slab with absorbing, emitting, scattering media. The slab is maintained at cold temperature i.e. at *0 K* and the space is divided into control volumes of unit depth in the x direction. At time *t=0*, the left boundary temperature is swiftly raised to an emissive power of value equal to *Phi* for all accompanying time. The medium has scattering coefficient and absorption coefficient of 0.5 each resulting in the extinction coefficient of one. Forty (40 polar and 1 azimuthal) control angles are used to discretize the angular space. The spatial domain is discretized into 100 uniform control volumes. According to initial conditions, and initial intensity field is imposed for the

solving transient radiative transfer equation. The solution within each time step is converged when it satisfies the following condition

$$\frac{\left|I_P - I_P^0\right|}{I_P} \leq \varepsilon \qquad (17)$$

III. RESULTS

A formulation of the finite volume method was proposed. After collapsing the radiative information to the 1-D solution plane, formulation procedure of the conduction equation was incorporated. The FVM is developed to study transient radiative transfer in one-dimensional (1D) anisotropic scattering, absorbing, and emitting medium. The transient formulation is feasible for both transient and steady-state radiative transfer. It is found that the present method is accurate and efficient. It is found that the ballistic component of the laser propagates with the speed of light in the medium, and its value is reduced dramatically with the advance of propagation.

A. Variation of Heat Flux

Heat flux is maximum at the hot surface and reduces with time because a thick medium absorbs nearly all the radiation. Transient radiative heat transfer is accompanied with transient conduction and the variation of heat flux along *x* coordinate is shown in Fig. 1, and the heat flux is maximum at the left boundary and reduces while radiation reaches the opposite wall (*x=L*) of the slab. Fig. 2 shows the variation of heat flux without conduction along *x*-direction. Due to back scattering the radiative heat flux near the hot surface reduces with time. The effect in increase of heat flux can be seen when conduction is incorporated with radiative properties. The effect of radiative energy is at its peak at the left boundary because it has been provided with an emissive power and the radiative energy decreases while reaching the extreme end of the slab.
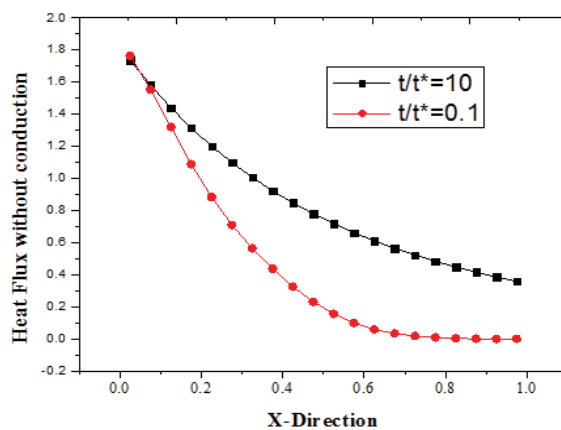
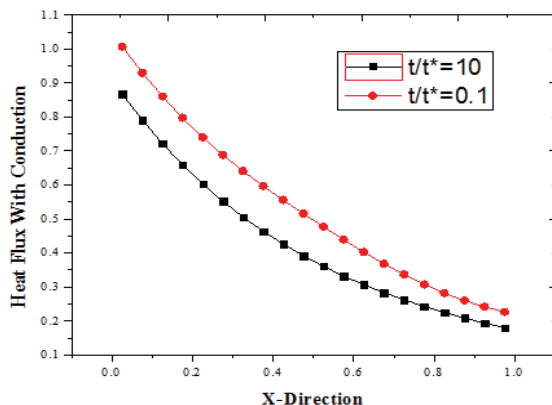

Fig. 1 Variation of heat flux with conduction with x-axis

Fig 2 Heat flux without conduction along *x*-direction
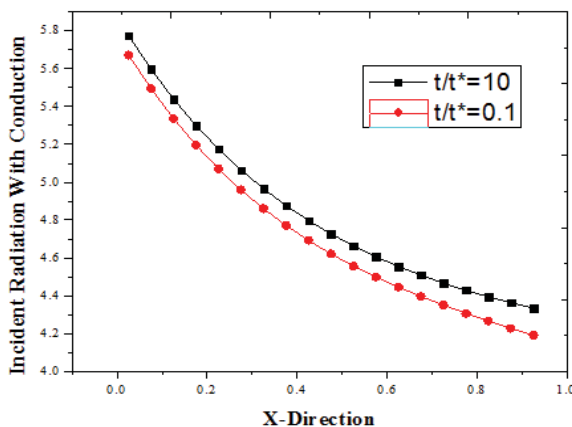
B. Variation of Incident Radiation Energy

Incident radiation energy is accompanied with and without transient conduction, and the results are plotted in Figs. 3 and 4. The incident radiation energy without conduction near the hot surface decreases with time due to back scattering effect before reaching steady state values. The variation of incident radiation energy without conduction along *x* coordinate is shown in Fig. 3. The incident radiation with conduction is maximum in the left boundary and decreases while reaching the extreme right of the slab at (*x=L*). Radiation coupled with conduction has been plotted in Fig. 4. The value of irradiation is more when radiation is coupled with conduction and incident radiation energy is at its peak at the left boundary and decreases while reaching the extreme right boundary.
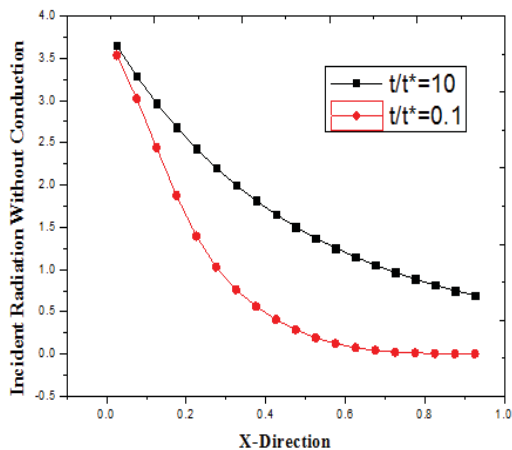
Fig. 3 Variation of incident radiation without conduction along x - axis

IV. Conclusion

A new finite volume method has been proposed for the prediction of radiative heat transfer that can be employed to predict radiative properties. The theory is advanced for an absorbing, emitting and scattering medium in an enclosure, but extensions to a broad bandwidth model and to include walls with other reflective properties are straightforward. The method has been demonstrated for a one-dimensional

problem, for which benchmark solutions and other approximate solution are available. Boundary conditions are more strongly a function of the incident energy and not the emissive power therefore incident energy at boundary propagate in to boundary conditions and finally into field of radiant intensities. We get the increased value of heat flux and incident radiation energy when radiation is coupled with conduction as compared to radiative properties without conduction.

Fig. 4 Variation of incident radiation energy with conduction along *x*-direction

References

[1] G. O. Young, "Synthetic structure of industrial plastics (Book style with paper title and editor)," in *Plastics*, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.

[2] M. N. Borjini ,C. Mbow ,M. Daguenet ," Numerical analysis of combined radiation and unsteady natural convection within a horizontal annular space," *Int J Numer Methods Heat Fluid Flow*,vol.9,pp.742-764, 1999.

[3] C. Y. Han ,S. W. Baek ," The effects of radiation on natural convection in a rectangular enclosure divided by two partitions," *Numer Heat Transfer*, vol.37,pp.249–270,2000.

[4] G. D. Raithby , E. H. Chui," A finite-volume method for predicting a radiant heat transfer in enclosures with participating media," *J Heat Transfer* , vol.112, pp. 415–23,1990.

[5] J. C. Chai , S. V. Patankar ," Finite volume method for radiation heat transfer ," *Adv Numer Heat Transfer* ,vol.2, pp.110–35,2000.

[6] W. A. Fiveland ," Discrete-ordinates solution of the radiative transport equation for rectangular enclosures," *J Heat Transfer,* vol.106,pp.:699–706, 1984.

[7] W. A. Fiveland ," Discrete ordinate methods for radiative heat transfer in isotropically and anisotropically scattering media," *J Heat Transfer*, vol.12,pp.109-809,1987.

[8] W. A. Fiveland ," Three-dimensional radiative heat-transfer solutions by the discrete-ordinates method," *J Thermophys Heat Transfer*, vol.2,pp.309-316, 1988.

[9] J. S. Truelove ," Three-dimensional radiation in absorbing–emitting–scattering media using the discrete-ordinates approximation," *JQSRT,* vol.39,pp.27-31,1988.

[10] D. Balsara ,"Fast and accurate discrete ordinates methods for multidimensional radiative transfer," *JQSRT*,vol-7,pp.669-671, 2001.

[11] M.F. Modest ," *Radiative heat transfer*,". 2nd ed. New York: Academic Press; 2003.

[12] L.H. Liu ,L.M. Ruan,H.P. Tan ," On the discrete ordinates method for radiative heat transfer in anisotropically scattering media," *Int J Heat Mass Transfer*, vol.45,pp.3259-3262, 2002.

[13] R. Koch,W. Krebs, Wittig S, Viskanta R," Discrete ordinate quadrature schemes for multidimensional radiative transfer," *JQSRT* , vol.53,pp.353-372,1995.

[14] C.P. Thurgood , A. Pollard ,H. A. Becker," The TN quadrature set for the discrete ordinates method," *J Heat Transfer*, vol-117,pp.1068-1070, 1995.

[15] B. W. Li, Q. Yao,X. Y. Cuo, K. F. Cen ," A new discrete ordinates quadrature scheme for three-dimensional radiative heat transfer," *J Heat Transfer*, vol-120,pp.514-518, 1998.

[16] R. Koch,R. Becker," Evaluation of quadrature schemes for the discrete ordinates method," *JQSRT* , vol-84,pp.423-435,2004.

[17] H.S. Li, G. Flamant ,J.D. Lu ," A new discrete ordinate algorithm for computing radiative transfer in one-dimensional atmospheres," *JQSRT*, vol.83, pp.407–421, 2004.

[18] M.A. Ramankutty, A. L. Crosbie ," Modified discrete ordinates solution of radiative transfer in two-dimensional radiative rectangular enclosures," *JQSRT*, vol.57,pp.107-114, 1997.

[19] P. S. Cumber,"Improvements to the discrete transfer method of calculating radiative heat transfer," *Int. J Heat Mass Transfer*, vol.8 , pp.2251-2258, 1995.

[20] S. H. Kim, K. Y. Huh," A new angular discretization scheme of the finite volume method for 3D radiative heat transfer in absorbing, emitting and anisotropically scattering media," *Int. J Heat Mass Transfer* , vol-43, pp.1233-1242, 2000.

[21] J. C. Chai ," One-dimensional transient radiation heat transfer modeling using a finite-volume method," *Numer. Heat Transf.*, vol-44, pp.187-208, 2003.

[22] J. C. Chai, H. S. Lee , S. V. Patankar," Finite volume method for radiation heat transfer," *J. Thermophys. Heat Transf.*, vol-3, pp.419-425, 1994.