

# Solving the Economic Dispatch Problem by Using Differential Evolution

S. Khamsawang and S. Jiriwibhakorn

**Abstract**—This paper proposes an application of the differential evolution (DE) algorithm for solving the economic dispatch problem (ED). Furthermore, the regenerating population procedure added to the conventional DE in order to improve escaping the local minimum solution. To test performance of DE algorithm, three thermal generating units with valve-point loading effects is used for testing. Moreover, investigating the DE parameters is presented. The simulation results show that the DE algorithm, which had been adjusted parameters, is better convergent time than other optimization methods.

**Keywords**—Differential evolution, Economic dispatch problem, Valve-point loading effect, Optimization method.

## I. INTRODUCTION

RECENTLY, the electricity demand has grown rapidly, but energy resource decrease. Therefore, the energy usage must be economized. For that reason, economic dispatch (ED) problem is one of important problems in power system. The main objective of the ED problem is to minimize the fuel cost of generating units, satisfying various inequality and equality constraints. In classical ED problem, mathematical model of fuel cost function has been approximated as a single quadratic cost function [1].

In past decade, many researchers exert to improve optimization techniques for solving the ED problem [2], i.e. evolutionary programming (EP) applied to solve the ED problem with multiple fuel cost function [3,12]. Particle swarm optimization (PSO) is proposed to improve a solution quality and a new particle swarm optimization hybrid with local search is proposed in reference [4]. Tabu search algorithm (TSA) is introduced by many researchers, i.e., Lin et al. had presented an improved tabu search for solving the ED problem with multiple minima [5], Khamsawang et al. proposed the TSA for solving the ED problem consider valve-point loading effects [6]. Genetic algorithm (GA) had applied to solve the ED problem with many types of fuel cost function. Sheble et al. had introduced the GA to solve this problem with valve-point loading effects [8]. Wong et al. had presented the simulated annealing (SA) and the hybrid GA/SA [7,10].

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Al-Sumait et al. had reported the pattern search (PS) to solve the ED problem with multiple fuels cost function and valve-point effects [11].

This paper proposed the DE algorithm for solving the ED problem. The DE, one of popular optimization methods, was introduced by Storn and Price in 1995 [13]. This algorithm has high efficiency for solving continuous nonlinear optimization problems and multimodal environments [14-15]. The advantages of the DE are simple structure, a few control parameters and high reliable convergences. The DE is one type of modern optimization techniques, which based on a population searching mechanism like as GA [9], bee colony (BC) optimization [16] and PSO [17-18].

The paper is organized as follows: Section II formulates the ED problem. Section III describes detail of the DE algorithm. Section IV shows the test system and computational results. Lastly, conclusion is given in Section VI.

## II. FORMULATION OF ED PROBLEMS

The main objective of the ED problem is to determine minimum generation cost of the generating units, according to the operating constraints of the generators and the power system limits. The simplified fuel cost function of generators represent as quadratic functions, given in equation (1).

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (1)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are cost coefficients of generating unit  $i$ ,  $P_i$  is the real power output of generating unit  $i$ ,  $F_i(P_i)$  is the operating fuel cost of generating unit  $i$ .

Minimizing the fuel cost function (1) of all generating units in the power system is the objective of ED problem which represents as (2)

$$\text{Minimize } F_T = \sum_{i=1}^n F_i(P_i) \quad (2)$$

where  $F_T$  is total fuel cost,  $n$  is number of generating units.

To satisfy various constraints:

*Generating power output constraint:*

$$P_{i,\min} \leq P_i \leq P_{i,\max} \quad (3)$$

*Power balance constraint:*

$$\sum_{i=1}^n P_i = D + P_L \tag{4}$$

where  $D$  is total load demand,  $P_L$  is total transmission line loss,  $P_{i,\min}$ ,  $P_{i,\max}$  are minimum and maximum power output of unit  $i$ .

The effects of multi-valves steam turbine produce a ripple curve on quadratic fuel cost functions and represented as rectified sinusoidal function [1]. Considering the valve point loading effects, the quadratic cost function in (1) was modified as equation (5).

$$F_i(P_i) = a_i + b_i P_i + c_i P_{i+} + |e_{i1} \sin(f_{i1}(P_{i,\min} - P_i))| \tag{5}$$

where  $e_{ik}$  and  $f_{ik}$  are cost coefficients of generating unit  $i$ .

III. DIFFERENTIAL EVOLUTIONARY (DE)

Price and Storn proposed a new evolutionary algorithm for global optimization which named it as differential evolution (DE). Easy method for implementation and has a few parameters for tuning made the algorithm quite popular very soon [14-15]. Like other evolutionary algorithms, the DE also starts with a population of NP D-dimensional search variable vectors, represented as

$$X_i^G = [x_{r1}^G, x_{r2}^G, \dots, x_{rD}^G] \tag{6}$$

For each variable, there may be a certain range within upper and lower limits. Changing each population member  $X_i^G$ , a mutant vector  $V^{G+1}$  is created as

$$V^{G+1} = [x_{r1}^G + F \times (x_{r2}^G - x_{r3}^G), x_{r2}^G + F \times (x_{r3}^G - x_{r4}^G), \dots, x_{rD}^G + F \times (x_{rD-2}^G - x_{rD-1}^G)] \tag{7}$$

The integers  $r1, r2, \dots, rD$  are chosen randomly from the interval  $[0, NP-1]$  and different from the running index  $i$ .  $F$  is a real and constant factor which controls the amplification of the differential variation which called scaling factor or amplification factor. The process of two dimensional examples is illustrated in Fig. 1.

To increase the potential diversity of the population, crossover scheme comes to play, the following vector is adopted:

$$U_i^G = [u_{i,1}^G, u_{i,2}^G, \dots, u_{i,D}^G] \tag{8}$$

with

$$u_{i,1}^G = \begin{cases} v_{i,1}^G & \text{if } rand() \leq CR \\ x_{i,1}^G & \text{otherwise.} \end{cases} \tag{9}$$

where  $CR$  is crossover rate in the range  $[0, 1]$ . In this way, each trial vector  $X_i^G$ , an offspring vector  $U_i^G$  is created. This idea is illustrated in Fig. 2, for  $D=7$ ,  $n=2$  and  $L=3$ . In order to decide the new vector  $U_i^G$  shall become a population member of the next generation, the selection process is evolved, at selection process can be expressed as

$$X_i^{G+1} = \begin{cases} U_i^G & \text{if } f(U_i^G) \leq f(X_i^G) \\ X_i^G & \text{otherwise.} \end{cases} \tag{10}$$

where  $f()$  is the objective function to be minimized. Thus, if the new trial vector ( $X_i^{G+1}$ ) yields the better objective function value than  $X_i^G$ ,  $X_i^{G+1}$  replaces its target in the next generation; otherwise the target vector ( $X_i^G$ ) is retained.

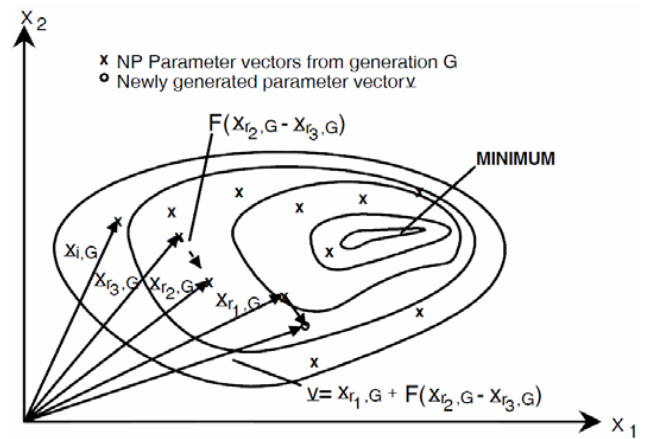
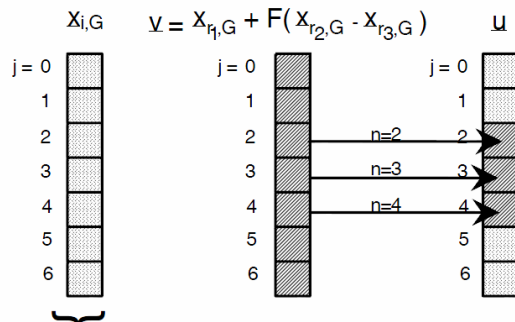


Fig.1 Two dimensional example for generates mutant vector.



Parameter vector containing the parameters  $x_j, j=0,1, \dots, D-1$

Fig. 2 Illustration of the crossover processes.

The above process of the mutation, the crossover operation and the selection are repeated generation until some stopping criteria are met. The original DE has suffered from local minimum solutions. In order to avoid these problems, the re-

generate population technique is proposed for enhancing the searching process and employed if a solution of current generation had higher than previous generation. The equations for generating the new populations express as follow

$$X_{i,\min} = x_{best}^{G-K} \text{ and } X_{i,\max} = x_{best}^{G-J} \quad (11)$$

$$X_i^{G+1} = X_{i,\min} + (X_i^U - X_i^L) \times rand + \min(X_{i,\max}, X_i^U) \times rand / X_{i,\min} \quad (12)$$

where  $X_i^U$  and  $X_i^L$  are upper limit and lower limit of decision variables (real power output),  $x_{best}^{G-K}$  and  $x_{best}^{G-J}$  are the best populations in  $K^{th}$  generation and  $J^{th}$  generation, respectively.

#### IV. SIMULATION RESULTS AND COMPARISON

This section proposes parameters tuning of the DE algorithm to solve the ED problem. Three thermal generating units considering the valve-point loading effects in fuel cost function are tested and shown in Table I [10-12].

TABLE I  
DATA OF THE TEST SYSTEM

$P_i$ (MW)	$P_{min}$	$P_{max}$	$a$	$b$	$c$	$e$	$f$
P1	100	600	561	7.92	0.00156	300	0.0315
P2	100	400	310	7.85	0.00194	200	0.042
P3	50	200	78	7.97	0.00482	150	0.063

TABLE II  
THE CR EMPIRICAL TESTS OF THE ORIGINAL DE

F=1	Generation cost (\$/hr)			Mean CPU	Std
CR	Min.	Mean.	Max.	time (s)	
0.1	8234.0770	8240.8359	8754.9641	1.40	44.0051
0.2	8234.0748	8243.1188	8760.1338	1.16	49.9236
0.3	8234.0740	8240.6756	9098.7896	0.92	48.2049
0.4	8234.0744	8237.3922	8924.1688	0.58	36.2734
0.5	8234.0745	8236.2191	8760.1338	0.41	29.4397
0.6	8234.0727	8235.6221	8935.2665	0.36	27.1168
0.7	8234.0731	8235.2255	8652.7811	0.34	15.6763
0.8	8234.0758	8235.5657	8702.7068	0.34	20.7676
<b>0.9</b>	<b>8234.0740</b>	<b>8237.6246</b>	<b>8343.9362</b>	<b>0.77</b>	<b>7.1427</b>

The DE, PSO, TSA, BC and GA methods are implemented in MatLab language and executed on an Intel Core 2 Duo 3.0 GHz personal computer with a 4.0 GB of RAM.

Table II shows the results of the original DE are obtained by tuning crossover rates (CR) between 0.1 and 0.9 while scaling factor (F) fixed. The results show that, CR will be selected at 0.9 for F=1. Table III shows some selected results of the scaling factors tuning. This procedure is varying the scaling factors between 0.1 and 2 while the crossover rate is set at the best CR obtained from Table II.

TABLE III  
EMPIRICAL TESTS OF F OF THE ORIGINAL DE

CR=0.9	Generation cost (\$/hr)			Mean CPU	Std
F	Min.	Mean.	Max.	time (s)	
0.8	8234.0752	8255.5943	8929.5178	0.68	86.4479
0.9	8234.0734	8245.2871	8929.5178	0.58	62.8190
<b>1</b>	<b>8234.0728</b>	<b>8234.7221</b>	<b>8343.9362</b>	<b>0.32</b>	<b>6.2303</b>
1.1	8234.0732	8241.1001	8988.8452	0.60	52.4493
1.2	8234.0742	8239.6225	8935.2665	0.80	37.7832
2	8234.0739	8240.3533	9220.8928	1.22	50.5432

TABLE IV  
EMPIRICAL TEST OF CR OF THE PROPOSED DE

F=1	Generation cost (\$/hr)			Mean CPU	Std
CR	Min.	Mean.	Max.	time (s)	
0.5	8234.0728	8234.1774	8234.8361	1.94	0.0930
0.6	8234.0731	8234.1223	8234.3136	1.10	0.0243
0.7	8234.0722	8234.1167	8234.1993	0.57	0.0161
0.8	8234.0732	8234.1169	8234.1400	0.37	0.0163
<b>0.9</b>	<b>8234.07360</b>	<b>8234.11737</b>	<b>8234.13999</b>	<b>0.31</b>	<b>0.0158</b>

TABLE V  
EMPIRICAL TESTS OF F OF THE PROPOSED DE

CR=0.9	Generation cost (\$/hr)			Mean CPU	Std
F	Min.	Mean.	Max.	time (s)	
0.8	8234.0731	8234.1245	8241.1752	0.58	0.2237
0.9	8234.0746	8234.1177	8234.1400	0.51	0.0164
<b>1.0</b>	<b>8234.0728</b>	<b>8234.1174</b>	<b>8234.1399</b>	<b>0.30</b>	<b>0.0158</b>
1.1	8234.0725	8234.1218	8234.8713	1.01	0.0367
1.2	8234.0737	8234.1676	8241.1796	1.45	0.4018

Table IV shows the obtained results from the empirical tests for determining the best CR of the proposed DE while F is fixed. The results are obtained from tuning F while CR fixed show in Table V. The results of Table I-V prove that the proposed DE algorithm has reached to the minimum solution, the lower computational time and the lower standard deviation of solutions than the original DE algorithm. The best simulation result is obtained from the proposed method (DE) compared with the GA, PSO, TSA, BC and optimization methods from the literatures are shown in Table VI.

TABLE VI  
COMPARISON THE BEST RESULTS

Methods	Output power (MW)			Total (MW)	Total cost (\$/hr)
	P1	P2	P3		
SA[10]	300.3077	399.9016	149.7907	850.0	8,234.150
PS[11]	300.2663	149.7331	399.9996	850.0	8,234.050
GA	300.2655	400.0000	149.7345	850.0	8,234.073
PSO	300.2758	399.9911	149.7331	850.0	8,234.076
TSA	300.3066	399.9234	149.7700	850.0	8,234.131
BC	300.2847	399.9805	149.7348	850.0	8,234.083
DE	<b>300.2680</b>	<b>399.9985</b>	<b>149.7336</b>	<b>850.0</b>	<b>8,234.073</b>

TABLE VII  
COMPARISON THE RESULTS OF DE WITH OTHER METHODS

Methods	Generation cost (\$/hr)			Mean CPU	Std
	Min.	Mean	Max.	time (s)	
SA[10]	8234.15	-	-	-	-
PS[11]	8234.05	8352.41	8453.00	0.81	-
MFEB[12]	8234.08	8234.71	8241.80	8.00	-
GAF[12]	8234.07	-	-	24.65	-
GAB[12]	8234.08	-	-	35.8	-
IFEP[12]	8234.07	8234.16	8234.54	6.78	-
CEP[12]	8234.07	8235.97	8241.83	20.46	-
FEP[12]	8234.07	8234.24	8241.78	4.45	-
GA	8234.073	8237.545	8366.391	1.05	14.57
PSO	8234.076	8239.511	8343.936	1.65	7.05
TSA	8234.073	8283.399	8471.548	8.62	71.30
BC	8234.083	8237.149	8254.948	2.02	5.90
DE	<b>8234.073</b>	<b>8234.117</b>	<b>8234.140</b>	<b>0.30</b>	<b>0.0158</b>

The minimum generation cost of proposed method has closely to other methods while the average cost and the maximum cost are the better, shown in Table VII.

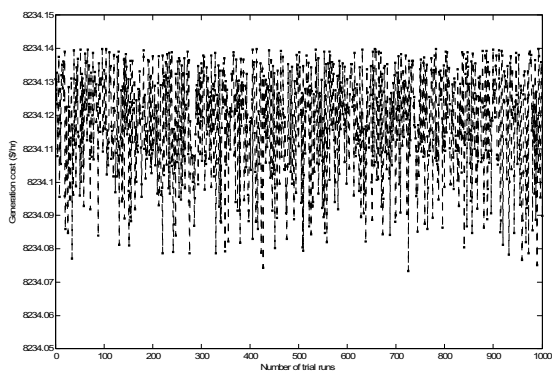


Fig. 3 Generation cost of 1000 differences running.

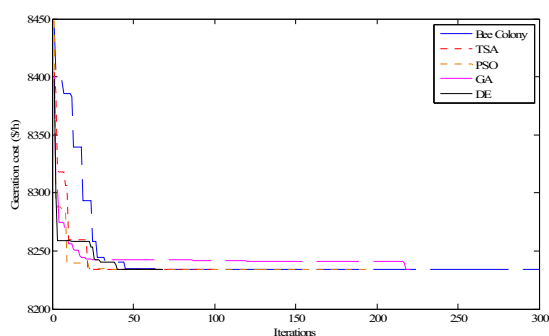


Fig. 4 Convergence of DE compared with BC, TSA, PSO and GA.

Fig.3 shows the generation cost profiles obtained from 1000 difference trials of the proposed DE. Fig.4 illustrates the convergence characteristic of the proposed DE compared with BC, TSA, PSO and GA.

## V. CONCLUSION

This paper reported the tuning parameters of the two DE algorithms, i.e. the original DE and the proposed DE. The proposed method based on the original DE with the re-generated population technique and tuning parameters. The effectiveness is tested with the three thermal generating units considering valve-point loading effects. The numerical solutions of the proposed DE are investigated with 1000 difference trials and compared with several optimization methods. Evidently, the proposed approach can improve the performance of the original DE and yield the best solution qualities than GA, PSO, TSA, BC and other methods form literatures.

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