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Simulation of Propagation of Cos-Gaussian Beam in Strongly Nonlocal Nonlinear Media Using Paraxial Group Transformation

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Abstract**—**In this paper, propagation of cos-Gaussian beam in strongly nonlocal nonlinear media has been stimulated by using paraxial group transformation. At first, cos-Gaussian beam, nonlocal nonlinear media, critical power, transfer matrix, and paraxial group transformation are introduced. Then, the propagation of the cos-Gaussian beam in strongly nonlocal nonlinear media is simulated. Results show that beam propagation has periodic structure during self-focusing effect in this case. However, this simple method can be used for investigation of propagation of kinds of beams in ABCD optical media.

Keywords**—**Paraxial group transformation, nonlocal nonlinear media, Cos-Gaussian beam, ABCD law.

I. Introduction

INVESTIGATION of propagation of optical beams in optical media is so important. Analytical solution is the common method of investigation of propagation of optical beams. However, this solution is an exact solution, but, it is complicated. Therefore, we can use approximate solution instead of exact solution. One of the approximate solution method is numerical solution. Furthermore, we can use Huygens-Fresnel integral to study how the beam diffracts in the medium. This integral is modified by Collins integral and rewritten with respect to ABCD matrix of the optical system [1]. Along with it, Bandres et al. presented the beam propagation method in the group of symmetries of the paraxial wave equation, called paraxial group, and obtained closed form expressions for the propagation of any paraxial beam through misaligned ABCD optical systems [2]. This method was used for propagation of Ince–Gaussian beams in strongly nonlocal nonlinear media successfully [3].

In this paper, at first, cos-Gaussian beam, strongly nonlocal nonlinear media, and paraxial group transformation method are introduced. Then, the propagation of the cos-Gaussian beam in strongly nonlocal nonlinear media is stimulated. Recently, there has been growing interest in the study of Hermite-sinusoidal-Gaussian (HSG) beams as a result of the works of Casperson and Tovar [4], [5] and Chen et al. [6]. As a special case of the HSG beams, the cos-Gaussian beams have many interesting applications such as optical telecom-munication and improved pump lasers with flat top beam shape for more efficient optical lasers and amplifiers [7]. This is because of its unique profile as a Gaussian beam that modulates with a cos function. Also, strongly nonlocal nonlinear media is a media in which refractive index of the point depends on the beam intensity of the other points. Physical mechanisms responsible for this type of nonlinear response includes various transport effects, such as heat conduction in materials with thermal nonlinearity [8], diffusion of molecules or atoms accompanying nonlinear light propagation in atomic vapors [9], drift and diffusion of photoexcited charges in photorefractive materials [10], and it appears as a result of many body interaction processes in the description of Bose-Einstein condensates [11]. The propagation of optical beams in nonlocal nonlinear media have attracted considerable interest in recent years. Therefore, we investigate propagation of cos-Gaussian beam in strongly nonlocal nonlinear media by using paraxial group transformation. Results show this method is simple and practical, and we can use it for simulation of complicated optical beams in ABCD optical media.

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II. Theory

A. Cos-Gaussian Beam

In this section, we consider the optical distribution of the field of the cos-Gaussian beam in free space which is expressed as [12]:

$$E(x,y)=\frac{i\pi w_0^2 A_0}{i\pi w_0^2+\lambda z}\exp\left[-\frac{\lambda z w_0^2(\beta_x^2+\beta_y^2)}{4i\pi w_0^2+4\lambda z}\right] \times\exp\left[-\frac{i\pi(x^2+y^2)}{i\pi w_0^2+\lambda z}\right]\times\cos\left(\frac{i\pi w_0^2\beta_x x}{i\pi w_0^2+\lambda z}\right)\cos\left(\frac{i\pi w_0^2\beta_y y}{i\pi w_0^2+\lambda z}\right) \quad (1)$$

where w_0 is the beam width of Gaussian beam, A_0 is the amplitude, β_x and β_y are the beam parameters associated with the cos part, and λ is the wavelength of beam.

B. Strongly Nonlocal Nonlinear Media

We consider a model of nonlocal nonlinear media that the refractive index change, Δn, can be presented in general form as [1]:

$$\Delta n(I)=\int R(x'-x)I(x',z)dx' \quad (2)$$

where x and z denote transverse and propagation coordinate, respectively. The function $R(x)$ is the response function that is real, localized, and symmetric of the nonlocal medium, and $I(x,z)$ is a light intensity. With increasing width of $R(x)$, the light intensity in the vicinity of the point x also contributes to the index change at that point. Nonlocal criterion defines as $\gamma = w_m / w_0$ where w_m is the width of response function. If $\gamma > 1$, medium is a strongly nonlocal nonlinear medium. Critical power, is a power in which Gaussian spot size remains constant during the propagation.

$$P_{cr} = \frac{1}{\gamma^2 z_0^2} = \frac{4}{k^2 \gamma^2 w_0^2} \tag{3}$$

where k is wave number, and z_0 is Rayleigh distance.

Propagation of paraxial optical beams in strongly nonlocal nonlinear media describe by nonlocal nonlinear Schrödinger equation:

$$2ik\frac{\partial \varphi}{\partial z} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi - k^2\gamma^2 P_0 (x^2 + y^2)\varphi = 0 \tag{4}$$

With due attention to governing equation of the nonlocal nonlinear media, we can write ABCD transfer matrix of media in terms of critical power, and Rayleigh distance as:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} COS(\sqrt{\frac{P_0}{P_{cr}}}\frac{z}{z_0}) & -\frac{z_0}{\sqrt{\frac{P_0}{P_{cr}}}}\sin(\sqrt{\frac{P_0}{P_{cr}}}\frac{z}{z_0}) \\ \frac{1}{z_0}\sqrt{\frac{P_0}{P_{cr}}}\sin(\sqrt{\frac{P_0}{P_{cr}}}\frac{z}{z_0}) & \cos(\sqrt{\frac{P_0}{P_{cr}}}\frac{z}{z_0}) \end{pmatrix} \tag{5}$$

C. Paraxial Group Transformation

In paraxial group transformation method, we obtain the output field after propagation through an arbitrary axially symmetric *ABCD* system [2]:

$$E(r,z) = \frac{1}{A}\exp\left(\frac{ikCr^2}{2A}\right)E_{free}\left(\frac{r}{A}, \frac{B}{A}\right) \tag{6}$$

where $E_{free}(r,z)$ is the field in free space, and A, B, C, D are the elements of transfer matrix. Therefore, if a closed-form expression for the free-space propagation of a paraxial beam is available, $E_{free}(r,z)$, we can readily obtain the field distribution after propagation through an *ABCD* optical system without solving Collins diffraction integral.

III. Simulation

In this part, propagation of cos-Gaussian beam in strongly nonlocal nonlinear media has been simulated. For this purpose, we suppose $\beta_x = 4/w_0$, $\beta_y = 5/w_0$, $w_0 = 1$, and the optical wavelength is $\lambda = 800nm$. The cos-Gaussian beam profile has been simulated in Fig. 1.

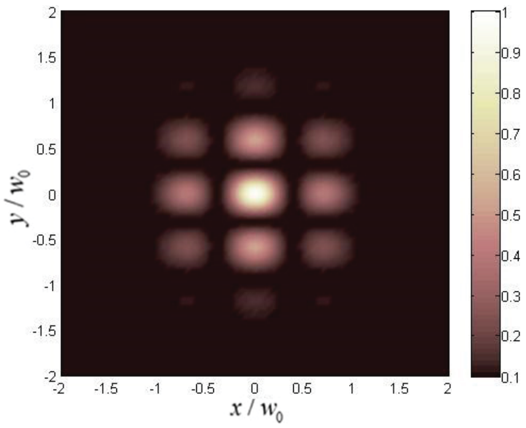


Fig. 1 Cos-Gaussian beam profile ($\beta_x = 4/w_0$, $\beta_y = 5/w_0$, $w_0 = 1$)

In Fig. 2 propagation of the cos-Gaussian beam in free space for different propagation distances is simulated. As it is seen, the beam diverges under propagation and loss the original shape as we expected.

With considering the equation of propagation of cos-Gaussian beam in free space, and transfer matrix of strongly nonlocal nonlinear media in paraxial group equation, propagation of cos-Gaussian beam in strongly nonlocal nonlinear media can be simulated as show in Fig. 3. As mentioned before, cos-Gaussian beam in free space has far field divergence, but, in this media, because of balancing the linear diffraction and nonlinear focusing, cos-Gaussian beam has self-focusing and it no experience divergence, so the beam envelope periodically during propagation and return to original beam shape after proper distance and it seems more stable than free space.

The effect of input power with respect to critical power investigates in Fig. 4. It can be seen from this figure for the same propagation distance z, for $P_0/P_{cr} = 1.2$ the beam is more focused than $P_0/P_{cr} = 1$, and $P_0/P_{cr} = 0.8$. The physical mining is when the input power respect to the critical power increases, the period of self-focusing decreases.

One can obtain the same result by using Collins integral or by applying the numerical method on wave equation, although the paraxial group method is easier.

IV. Conclusion

In this paper, we investigate the beam propagation in strongly nonlocal nonlinear media. The beam trajectory can be greatly affected by a strong nonlocal-nonlinearity. In this case, the propagation equation can be linearized, such as the beam propagation can be described by Collins formula based on ABCD matrix. Instead of it, we attempt to describe beam propagation using paraxial group theory.

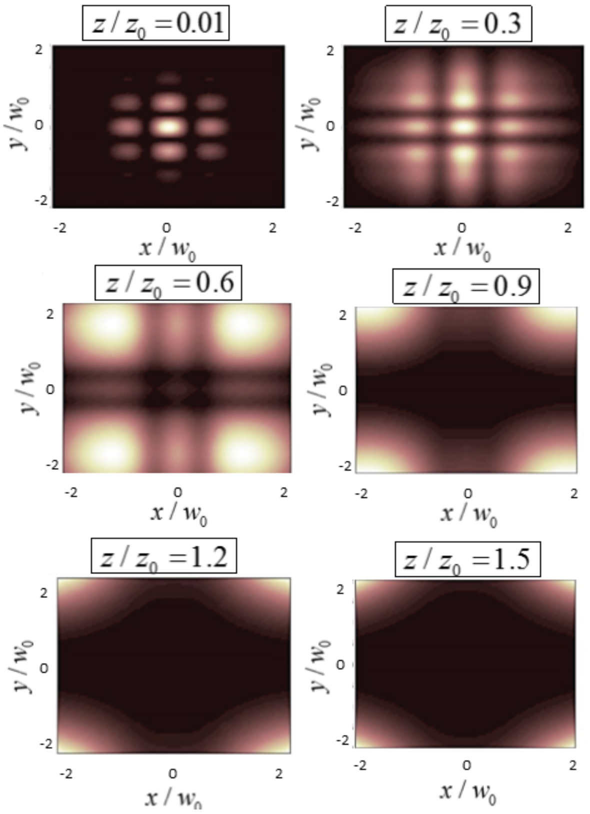


Fig. 2 Propagation of cos-Gaussian beam in free space

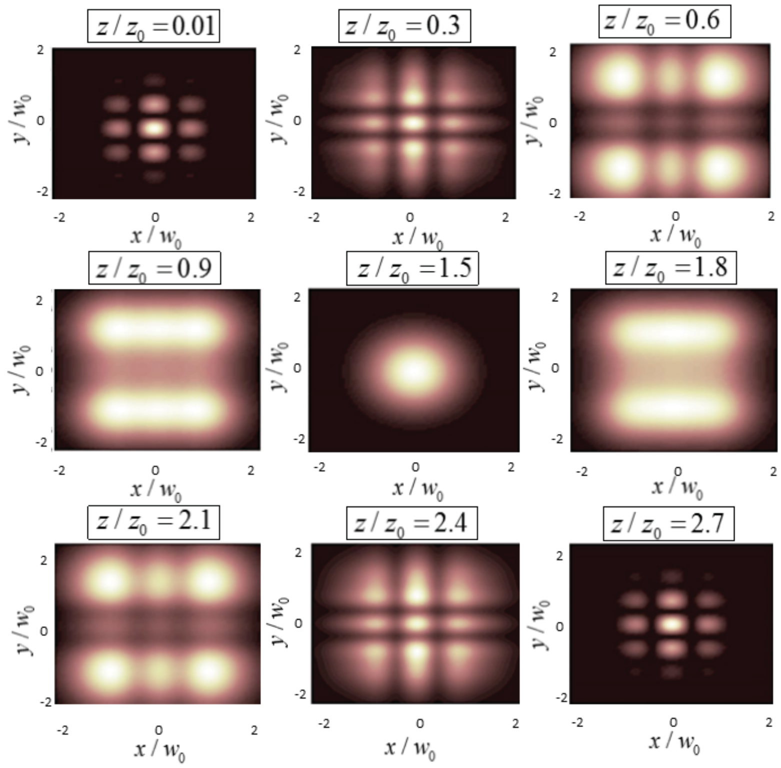
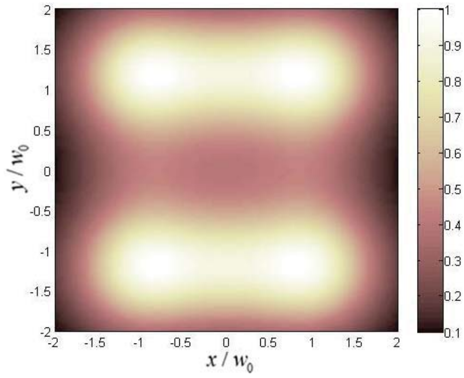
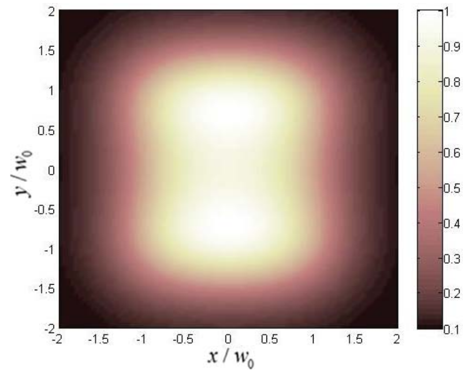


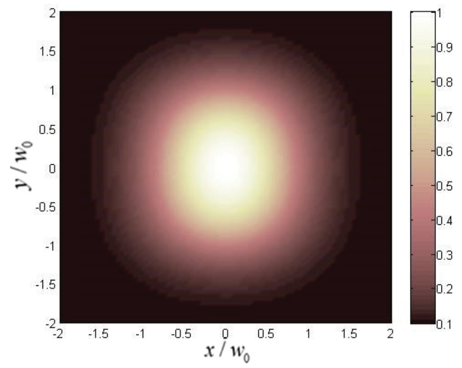
Fig. 3 Propagation of cos-Gaussian beam in strongly nonlocal nonlinear media with $P_0/P_{cr} = 1.3$ using paraxial group transformation method



(a)



(b)



(c)

Fig. 4 Propagation of cos-Gaussian beam in strongly nonlocal nonlinear media with $z/z_0 = 1.2$: (a) $P_0/P_{cr} = 0.8$ (b) $P_0/P_{cr} = 1$ (c) $P_0/P_{cr} = 1.2$

In this method, if we have equation of propagation of the beam and transfer matrix of the media, by having free space beam propagation, we can simulate the propagation of the beam in this media. So, by using this method, one can readily investigate propagation of complicated optical beams in ABCD optical media. For this case, propagation of the cos-Gaussian beam in strongly nonlocal nonlinear media is simulated. Results show the cos-Gaussian beam profile changes alternatively during propagation in this media and return to the initial shape after passing a periodic length. Also, by increasing the initial power, the period of self-focusing of the beam decreases as the self-focusing effect increases by increasing the input power with respect to critical power which is depended on nonlocality. As a result, we can control detergency of the beam by setting nonlocal nonlinearity of the medium. These results can be applied to many applications such as optical communication, optical limiting and laser amplifiers.

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