International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:11, No:9, 2017

Simulation of Piezoelectric Laminated Smart Structure under Strong Electric Field

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Abstract—Applying strong electric field on piezoelectric actuators, on one hand very significant electroelastic material nonlinear effects will occur, on the other hand piezo plates and shells may undergo large displacements and rotations. In order to give a precise prediction of piezolaminated smart structures under large electric field, this paper develops a finite element (FE) model accounting for both electroelastic material nonlinearity and geometric nonlinearity with large rotations based on the first order shear deformation (FSOD) hypothesis. The proposed FE model is applied to analyze a piezolaminated semicircular shell structure.

Keywords—Smart structures, piezolamintes, material nonlinearity, geometric nonlinearity, strong electric field.

I. Introduction

SMART structures are those integrated with smart materials like piezoelectric, shape memory alloys, etc., which have a controlled loop with feedback. They have great potential in the applications of vibration control, acoustic control and health monitoring. In order to obtain high actuation forces, strong electric driving field is one of the economical ways without any modification of structural construction. However, applying strong electric fields on piezo actuators, on one hand electroelastic material nonlinear phenomenon will be introduced to the system, on the other hand thin-walled structures will undergo large displacements and rotations. In such a case, numerical models should include both electroelastic material nonlinearity and geometric nonlinearity.

Regarding the modeling and simulation of piezoelectric smart structures, there are plenty of studies available in the literature developed linear numerical models, see e.g. [1], [2] among many others. Due to the simplified assumptions, linear models are only valid for structures subjected to weak electric fields and undergoing small displacements. The numerical models for piezoelectric structures under strong electric field must take into account the elctroelastic material nonlinearity. Nelson [3], Tiersten [4] first proposed nonlinear electroelastic equations for piezoelectric materials. Later, the irreversible piezoelectric nonlinearities were investigated experimentally by Li *et al.* [5], Masys *et al.* [6], and studied numerically by Landis [7], Ma *et al.* [8]. Furthermore, Wang *et al.* [9], Yao *et al.* [10] developed analytical models for cantilevered piezoelectric bimorph and unimorph beams under strong electric field. Kusculuoglu & Royston [11], Kapuria & Yasin developed numerical model for dynamic analysis and active vibration control of piezoelectric smart structures under strong electric fields.

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The above mentioned references considered electroelastic material nonlinearity with geometrically linear. Therefore, piezoelectric materials can be subjected to strong electric fields, with the structures undergoing small displacements and rotations. Concerning piezoelectric structures undergoing large displacements and rotations, but without taking into account the electroelastic material nonlinearity, a lot of papers can be found, e.g. von Kármán type nonlinear model [12], [13], moderate rotation nonlinear model [14], and large rotation nonlinear model [15], [16]. However, applying strong electric fields usually yields large displacements and rotations, which cannot be existing separately. In such a case, Yao *et al.* [17] developed a finite element model with von Kármán type nonlinearity and electroelastic material nonlinearity. Von Kármán nonlinearity is the simplest one including very week geometric nonlinearity, which is only valid for structures undergoing moderately large displacements and small rotations.

In order to simulate piezoelectric smart structures under strong electric fields with large displacements and rotations, the paper develops a numerical model with both geometric and material nonlinearities for piezoelectric integrated smart structures.

II. Mathematical Models

A. Electroelastic Material Nonlinearity

Considering electroelastic material nonlinearity, the constitutive equations of piezoelectric materials are expressed as [4]

$$\sigma_p = c_{pq}\varepsilon_q - e_{mp}E_m - \frac{1}{2}b_{mnp}E_mE_n\,, \tag{1}$$

$$D_m = e_{mq}\varepsilon_q + g_{mn}E_n + \frac{1}{2}h_{mkn}E_kE_n\,, \tag{2}$$

where p and q represent the numbers 1, 2, 3, 4, 5, and 6, while n, m, and k take the numbers 1, 2, 3. Assuming that the transverse normal strain is neglected for plates and shells and the electric field is applied only along the thickness direction, one yields $p, q = 1, 2, 4, 5$ or 6 and $n = m = k = 3$. Furthermore, σ_p and ε_q denote the components of strain and stress vector, E_m and D_m are the components of electric field and electric displacement vector, c_{pq} represent the components of elastic stiffness matrix.

The coefficients appeared above can be obtained as

$$c_{11} = \frac{Y_1}{1-\nu_{12}\nu_{21}}\,, \qquad c_{12} = \frac{\nu_{12}Y_2}{1-\nu_{12}\nu_{21}}\,, \tag{3}$$
$$c_{22} = \frac{Y_2}{1-\nu_{12}\nu_{21}}\,, \qquad c_{44} = \kappa G_{23}\,, \tag{4}$$
$$c_{55} = \kappa G_{13}\,, \qquad c_{66} = G_{12}, \tag{5}$$

$$e_{31} = d_{31}c_{11} + d_{32}c_{12}\,, \tag{6}$$
$$e_{32} = d_{31}c_{21} + d_{32}c_{22}\,, \tag{7}$$
$$b_{331} = \beta_{331}c_{11} + \beta_{332}c_{12}\,, \tag{8}$$
$$b_{332} = \beta_{331}c_{21} + \beta_{332}c_{22}\,, \tag{9}$$
$$g_{33} = \epsilon_{33} - d_{31}e_{31} - d_{32}e_{32}\,, \tag{10}$$
$$h_{333} = \chi_{333} - d_{31}b_{331} - d_{32}b_{332}\,. \tag{11}$$

Here (Y_1, Y_2), (ν_{12}, ν_{21}), (G_{12}, G_{23}, G_{13}) and $\kappa = 5/6$ are respectively the Young's moduli, the Poisson's ratios, the shear moduli, and the shear correction factor; (d_{31}, d_{32}) and (ϵ_{33}) are the piezoelectric constants and the dielectric constant; (β_{331}, β_{332}) and (χ_{333}) denote the nonlinear electroelastic constants and nonlinear electroelastic susceptibility constants.

B. Geometric Nonlinearity

For plate and shell structures, using the first-order shear deformation hypothesis, one obtains the strain-displacement relations with fully geometric nonlinearity as (the details can be found in [18]-[20] [15], [16])

$$\varepsilon_{\alpha\beta} = \overset{0}{\varepsilon}_{\alpha\beta} + \Theta^3\,\overset{1}{\varepsilon}_{\alpha\beta} + (\Theta^3)^2\,\overset{2}{\varepsilon}_{\alpha\beta}, \tag{12}$$
$$\varepsilon_{\alpha3} = \overset{0}{\varepsilon}_{\alpha3}, \tag{13}$$

with the strain components

$$2\overset{0}{\varepsilon}_{\alpha\beta} = \overset{0}{\varphi}_{\alpha\beta} + \overset{0}{\varphi}_{\beta\alpha} + \underline{\overset{0}{\varphi}_{3\alpha}\overset{0}{\varphi}_{3\beta} + \overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta}}, \tag{14}$$
$$2\overset{1}{\varepsilon}_{\alpha\beta} = \overset{1}{\varphi}_{\alpha\beta} - b^{\lambda}_{\beta}\overset{0}{\varphi}_{\lambda\alpha} + \overset{1}{\varphi}_{\beta\alpha} - b^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta} \tag{15}$$
$$+ \underline{\overset{0}{\varphi}_{3\alpha}\overset{1}{\varphi}_{3\beta} + \overset{1}{\varphi}_{3\alpha}\overset{0}{\varphi}_{3\beta} + \overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{1}{\varphi}_{\delta\beta} + \overset{1}{\varphi}{}^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta}}, \tag{16}$$
$$2\overset{2}{\varepsilon}_{\alpha\beta} = -\,b^{\lambda}_{\beta}\overset{1}{\varphi}_{\lambda\alpha} - b^{\delta}_{\alpha}\overset{1}{\varphi}_{\delta\beta} + \underline{\overset{1}{\varphi}_{3\alpha}\overset{1}{\varphi}_{3\beta} + \overset{1}{\varphi}{}^{\delta}_{\alpha}\overset{1}{\varphi}_{\delta\beta}}, \tag{17}$$
$$2\overset{0}{\varepsilon}_{\alpha3} = \overset{1}{\upsilon}_{\alpha} + \overset{0}{\varphi}_{3\alpha} + \underline{\overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{1}{\upsilon}_{\delta} + \overset{0}{\varphi}_{3\alpha}\overset{1}{\upsilon}_{3}}. \tag{18}$$

Here $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\alpha3}$ represent respectively the in-plane strain components and the stress components. Dropping all terms with underlines yields linear plate/shell theory. Based on the linear theory, containing additionally the squares and products of transverse displacement gradients, one obtains von Kármán type nonlinear plate/shell theory. Including the fully geometrically nonlinear strain-displacement relations and considering finite shell director rotations, one leads to large rotation nonlinear theory.

In the FOSD hypothesis, 5 degrees of freedom at each node are usually considered. The fully geometrically nonlinear strain-displacement relations contain 6 parameters. In order to represent finite rotation of shell director, the 5 degrees of freedom are expressed by 6 parameters as (see [20])

$$\overset{0}{\upsilon}_1 = u, \tag{19}$$
$$\overset{0}{\upsilon}_2 = v, \tag{20}$$
$$\overset{0}{\upsilon}_3 = w, \tag{21}$$
$$\overset{1}{\upsilon}_1 = \sin(\varphi_1)\cos(\varphi_2), \tag{22}$$
$$\overset{1}{\upsilon}_2 = \sin(\varphi_2), \tag{23}$$
$$\overset{1}{\upsilon}_3 = \cos(\varphi_1)\cos(\varphi_2) - 1. \tag{24}$$

Here $\overset{0}{\upsilon}_i$ and $\overset{0}{\upsilon}_j$ are the 6 parameters in the strain-displacement relations. Moreover, u, v, w denote the translational displacements, φ_1 and φ_2 are the rotations. For linear theory and von Kármán type nonlinear theory, small or moderate rotations of the shell director are assumed, resulting in $\cos\varphi = 1$ and $\sin\varphi = \varphi$, which leads to $\overset{1}{\upsilon}_1 = \varphi_1$, $\overset{1}{\upsilon}_2 = \varphi_2$ and $\overset{1}{\upsilon}_3 = 0$.

C. Finite Element Model

Using the principle of virtual work yields nonlinear static equilibrium equations and sensor equations with consideration of both geometric and material nonlinearities as

$${}^{t}\boldsymbol{K}_{\mathrm{uu}}\Delta\boldsymbol{q} + {}^{t}\boldsymbol{K}_{\mathrm{u}\phi}\Delta\boldsymbol{\Phi}_{\mathrm{a}} = \boldsymbol{F}_{\mathrm{ue}} - {}^{t}\boldsymbol{F}_{\mathrm{ui}}, \tag{25}$$
$${}^{t}\boldsymbol{K}_{\phi\mathrm{u}}\Delta\boldsymbol{q} + {}^{t}\boldsymbol{K}_{\phi\phi}\Delta\boldsymbol{\Phi}_{\mathrm{s}} = \boldsymbol{G}_{\phi\mathrm{e}} - {}^{t}\boldsymbol{G}_{\phi\mathrm{i}}. \tag{26}$$

Here $\boldsymbol{K}_{\mathrm{uu}}$, $\boldsymbol{K}_{\mathrm{u}\phi}$, $\boldsymbol{K}_{\phi\mathrm{u}}$, $\boldsymbol{K}_{\phi\phi}$ represent the stiffness matrix, the piezoelectric coupled stiffness matrix, the coupled capacity matrix and the piezoelectric capacity matrix, respectively; $\boldsymbol{F}_{\mathrm{ue}}$, $\boldsymbol{F}_{\mathrm{ui}}$, $\boldsymbol{G}_{\phi\mathrm{e}}$ and $\boldsymbol{G}_{\phi\mathrm{i}}$ are, respectively, the external force vector, the in-balance force vector, the external charge vector and the in-balance charge vector; $\boldsymbol{\Phi}_{\mathrm{a}}$ and $\boldsymbol{\Phi}_{\mathrm{s}}$ denote, respectively, the actuation voltage vector and the sensor voltage vector.

For clear description, several abbreviations are introduced. The finite element model considers geometric and material linear effects based on the FOSD hypothesis is abbreviated as LIN5WE, in which WE denotes weak electric field. Employing the electroelastic material nonlinearity for strong driving electric field (SE), the FE models considering geometric linear and von Kármán nonlinear theory are respectively shorted as LIN5SE and RVK5SE. Furthermore, the model including large rotation geometric nonlinear theory with material nonlinear relations is denoted by LRT56SE.

III. Numerical Examples

In this section, a semicircular cylindrical shell is considered as the numerical example, where two piezoelectric layers are bonded at the inner and outer surfaces, as shown in Fig. 1. The host structure is made of graphite/epoxy (T300/976) stacked as 4 substrate layers $[45°/-45°]_{\mathrm{s}}$ with equal thickness of 0.254 mm. The piezoelectric layers made of 3203HD has the thickness of 0.254 mm. The material properties of 3203HD and T300/976 are given in Table I.

The semicircular cylindrical shell is subjected to a driving voltage of 254 V on both piezoelectric layers. Calculating the

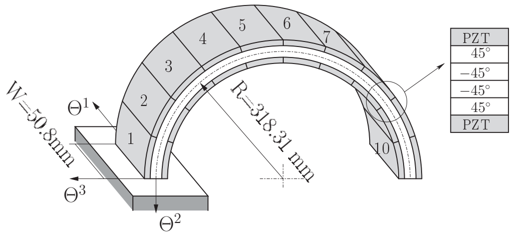

Fig. 1 Clamped piezolaminated semicircular cylindrical shell

TABLE I
MATERIAL PROPERTIES

Properties	3203HD[a,b]	T300/976
Y_1 (GPa)	60.24	150
Y_2 (GPa)	60.24	9
ν_{12}	0.253	0.3
ν_{23}	0.494	0.3
G_{12} (GPa)	20.04	7.1
G_{23} (GPa)	19.084	2.5
G_{13} (GPa)	19.084	7.1
d_{31} ($\times 10^{-12}$ m/V)	-320	/
d_{32} ($\times 10^{-12}$ m/V)	-320	/
ϵ_{33}/ϵ_0 [c]	3800	/
β_{331} ($\times 10^{-18}$ m^{2}V^{-2})	-520	/
β_{332} ($\times 10^{-18}$ m^{2}V^{-2})	-520	/

[a] CTS Corporation [21]
[b] Kapuria & Yasin [22]
[c] Electrical permittivity of air, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

hoop and radial displacements, one obtains the deformed shape of the central line by using different models, as presented in Fig. 2.

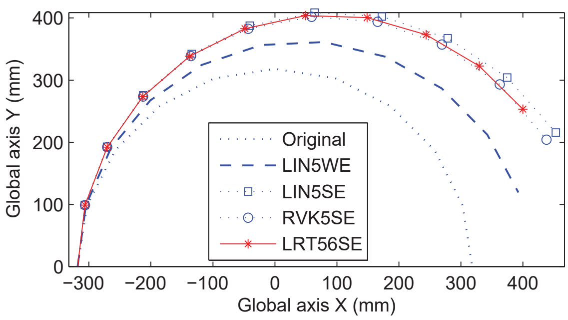

Fig. 2 Central line deformation of the semicircular shell

The numerical results are respectively using linear theory (LIN5WE), geometrically linear with material nonlinear (LIN5SE), von Kármán type nonlinear with material nonlinear (RVK5SE), and large rotation geometrically nonlinear with material nonlinear (LRT56SE). The results show that the linear prediction is far different from the nonlinear predictions. LIN5SE and RVK5SE curves are quite similar, but has relatively large discrepancy compared with LRT56SE result. This is because von Kármán type nonlinear theory is the simplest one, which includes very week geometric nonlinearity. However, the nonlinear model LRT56SE considers fully geometric nonlinearity with large rotation, as well as the electroelastic material nonlinearity.

IV. CONCLUSION

To simulate piezoelectric smart structures under strong electric fields with large displacements and rotations, the paper has developed a nonlinear FE model for piezolaminated smart structures based on the FOSD hypothesis, in which the electroelastic material nonlinearity and large rotation geometric nonlinearity are taken into account.

The proposed model is applied to analyze a piezolaminated semicircular shell structure. The results show that large discrepancy occurs between the linear and nonlinear predictions. Moreover, in the analysis of the semicircular shell, the RVK5SE curve is very close to LIN5SE, but both of them are different from LRT56SE curve, which indicates that von Kármán type nonlinearity is too weak to predict behavior precisely.

ACKNOWLEDGMENT

The work is supported by the National Natural Science Foundation of China (Grant No. 11602193), the State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology - China (Grant No. GZ15212), and the State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and astronautics - China (Grant No. MCMS-0517G01).

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