# Simulation Modeling of Manufacturing Systems for the Serial Route and the Parallel One

Tadeusz Witkowski, Paweł Antczak, Arkadiusz Antczak

*Abstract*—In the paper we discuss the influence of the route flexibility degree, the open rate of operations and the production type coefficient on makespan. The  flexible job-open shop scheduling problem FJOSP (an extension of the classical job shop scheduling) is analyzed. For  the   analysis of the production process we used a hybrid heuristic of the GRASP (greedy randomized adaptive search procedure) with simulated annealing algorithm. Experiments with different levels of factors have been considered and compared. The GRASP+SA algorithm has been tested and illustrated with results for the serial route and the parallel one.

*Keywords*—Makespan, open shop, route flexibility, serial and parallel route,  simulation modeling, type of  production.

## I. INTRODUCTION

SIMULATION modeling [1]-[6] is a common paradigm for analyzing complex systems. The greatest overall benefit of using simulation in a manufacturing environment is that it allows a manager or engineer to obtain a system wide view of the effect of "local" changes to the manufacturing system.
Simulation is  well known as a powerful tool supporting the design, layout or redesign of production systems. Recently, many successful applications proved that it can also support the operation of manufacturing systems, especially in the area of scheduling and control.

There are a number of other well-known [7]-[13], general-purpose simulation packages, including e.g.  ], SIMUL8 [7,10],   and  SLX  [8],  GPSS/H  [9],  AnyLogic  [11], HyPerformix Workbench [12], Micro Saint [13].
Several simulation products such as Simulik, ACSL, and Dymola have been specifically designed for building continuous simulation models. In addition, Discrete Event simulation packages Arena [14] and Extend have continuous modeling capabilities.

We list [15]-[19] some of the application-oriented simulation packeges that are currently available which are used in manufacturing and supply chain: Simulation Dynamics, AutoMod, Enterprise Dynamics, QUEST [15], Flexsim [16],    PROMODEL [19], and Witness [18].
The broad goal of manufacturing operation management, such as a resource constrained scheduling problem, is to achieve a coordinated efficient behavior of manufacturing in servicing production demands, while responding to changes on shop-floors rapidly and in a cost effective manner. Shop floor scheduling,  such as resource constrained scheduling problems in general, is complex, NP-hard problem, thus is unfeasible to be solved computationally by the sole use of conventional operations research  approaches.

The development of decision-making methodologies is currently headed in the direction of simulation and search algorithm integration. This leads to a new approach, which successfully joins simulation and optimization. Hybrid techniques using domain specific heuristics are necessary to guide the search and to provide satisfactory solutions in due time [20]-[22] .

The paper is organized as follows. Section II formulates the problem. The diagram  of  simulated annealing and GRASP algorithm is given in section III. In section IV we describe the dependency of makespan on the flexibility of route, the open rate of operations and the production type coefficient for the serial route and the parallel one.

## II. PROBLEM FORMULATION

The *n x m* classical job shop scheduling problem (JSP) involves *n* jobs and m machines. Each job is to be processed on each machine in a predefined sequence. We consider of the flexible open - shop scheduling problem.

In an open shop scheduling problem (OSP), a set of *n* jobs $I_1, I_2,..., I_n$ has to be processed on a set of *m* machines $M_1$, $M_2,...,M_m$.  The processing of a job on machine is denoted as an operation, and the sequence in which the operations of a job are processed on the machines is immaterial.

Flexible job shop scheduling problem (FJSP) extends the JSP by allowing each operations to be processed on more than machine. With this extension, we are now confronted with two subtask: assignment of each operation to an appropriate machine and sequencing operations on each machine.

The FJOSP is formulated as follows. There is a set of jobs $Z = \{Z_i\}$, $i \in I$, where $I = \{1, 2, ..., n\}$ is an admissible set of parts, $U = \{u_k\}$, $k \in 1, m$, is a set of  machines. Each job $Z_i$ is a group of parts $\Pi_i$ of equal partial task $p_i$ of a certain range of production. Operations of  technological processing of the *i*-th part are denoted by $\{O_{ij}\}_{j=\xi}^{H_i}$. Then for $Z_i$ , we can write $Z_i =$ ($\Pi_i \{O_{ij}\}_{j=\xi}^{H_i}$), where  $O_{ij} = (G_{ij}, t_{ij\ (N)})$ is the  *j*- th operation of processing the *i*-th group of parts;   $\xi_i$ is the number of operation of the production  process at which one should start the processing the *i*- th group of parts; $H_i$ is the number of the last  operation  for  a given group; $G_{ij}$ is a group of interchangeable machines that is assigned to the operation $O_{ij}$; $G$ is a set of all groups of machines arose in the matrix  $\|\{ Z_i \}\|$; $t_{ij\ (N)}$ is an elementary duration of the operation $O_{ij}$ with one part $d_i$ that depends on the number of machine $N$ in the group (on the specified operations); $t'_{ij}$ is the duration of set up before the operation $O_{ij}$; $N_{gr}$ is the number of all groups of

machines. In this problem the sequence in which the operations of a job are processed on the machines is immaterial (with different open rates of operations). The most widely used objective is to find feasible schedules that minimize the completion time of the total production program, normally referred to as makespan (*Cmax*).

In this paper the dependency of makespan on the route flexibility degree, open rate of operations and production type coefficient for serial route have been described. The algorithm SA+GRASP [25]-[26] for the optimization of the production process has been used.

### III. Diagram of the GRASP+SA Algorithm

In this paper, for the analysis of the manufacturing system we used the GRASP+SA hybrid heuristic (figure 1).
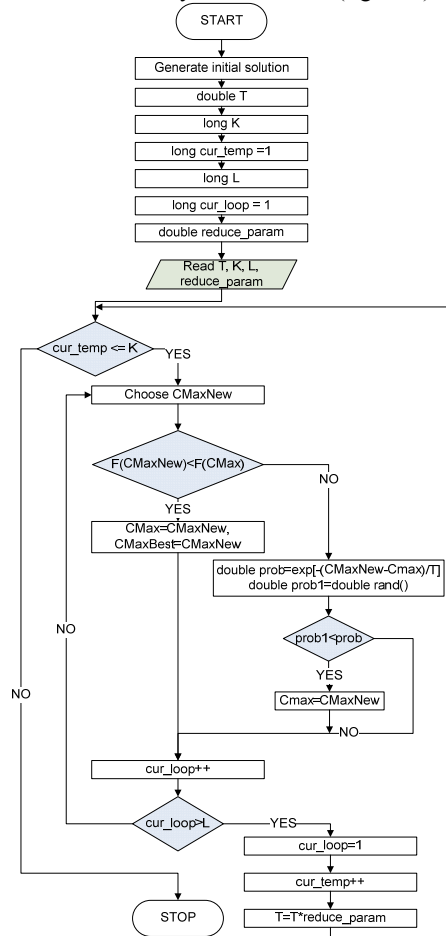


Fig. 1 Diagram of GRASP+Simulated Annealing algorithm

The research was carried out on a computer with an Intel Core2 2.4 GHz processor and 2047 MB of RAM for the following settings of the GRASP+SA algorithm: reducing temperature parameter $r$=0.95; number of temperature changes $K$ =100; number of iterations with steady temperature $L$ =100; initial temperature $T$ =100; parameter $\alpha$ =0 [26].

### IV. Computer Experiments

If a model has only one factor, the experimental design is simple: we just run the simulation at various values, or levels, of the factor, perhaps forming a confidence interval for the expected response at each of the factor levels.

In this experiment there are $k = 3$ factors and we want to get an initial estimate of how each factor affects the response [23]. We might also want to determine if the factors interact with one another, i.e., whether the effect of one factor on the response depends on the levels of the others. On way to measure the effect of a particular factor would be to fix the levels of the other $k$-1 factors at some set of values and make simulation runs at each of two lewels of the factor of interest to see how the response reacts to changes in this single factor. A much more ecomical strategy for determining the effects of factors on the response with which we can also measure interactions, called a $2^k$ factorial design, requires that we choose just two levels for each factors and then calls for simulation runs at each of the $2^k$ possible factor-level combinations, which are sometimes called design points. Because we are using only two levels for each factor, we assume that that the response is approximately linear over the range range of the factor.

In this work we examined open job problem defined in such a way that an operation (of the same type) of a job can be performed on any machine of changeable machines group (CMG) and the sequence in which the operations are processed on the machines is immaterial. The open rate of process was defined as the number of operations in a process can be performed with immaterial sequence. For example, let there be 10 operations for given part. If two operations can be performed on any machine in the CMG group and sequence in which these operations are processed is immaterial then the open rate is 2/10= 0.2 = 20 %.

The route flexibility defined in such a way that an operation of the same type can be performed on any machine of changeable machines group. The flexibility rate $E$ was defined as the number of machines in a group capable of performing a given operation. For example, let there be 10 changeable machines divided into 5 groups of two machines. If a given operation can be performed on any machine in the CMG group, then the flexibility rate is 2/10=0. In this work the following route flexibility rates were used $E$ : 0.2; 0.4; 0.6; 0.8 and 1.0.

We can use value of production type coefficient $K_{tp}$, which is described:

$$K_{tp} = \frac{\sum_{i=1}^{m} P_0 \times F \times K_0}{\sum_{i=1}^{n} N_i \times t_i} \qquad (1)$$

where: $\sum_{i=1}^{m} P_0$ - general number of production operations that were used at $i$ working position in a month, $i \in 1, m; F -$ a total of work time in a month; $N_o$ - an average coefficient

of a norm execution; $N_i$ monthly program of $i$ assortment; $t_i$ − labor of $i$ assortment; $n$ - a number of products assortment. With coefficient $K_{tp}$ (further marked with the symbol $k$) we can show relationships between a type of process, a form of process organization, and a kind of production system technical base [27]. In a serial route an entire batch of parts is processed on one machine and only when all of the products in the batch have been processed are they sent to the next machine. In a parallel route individual items of the batch are sent to the next machines as soon as they have been processed on the previous machine. For the solution of the FJOSP problem special software to realize the heuristic was created. Experiments were carried out for data presented in [25-26], for the number of operations - 160, and the number of machines - 26.

### A. Simulation modeling for the serial route

The three-factor experiments carried-out show the influence of the factors (E (or FD) flexibility degree, O (or OR) open rate, k coefficient (or BS batch size)) on the makespan $C_{max}$ value (Table I).

TABLE I

$_{MAX}$ VALUES FOR THE THREE-FACTOR EXPERIMENT

| Experiment | Open rate level | Open rate value | Flexibility degree level | Flexibility degree value | Coefficient k level | Coefficient k value | Makespan value average | change | min | max |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | O1M | 0 | E1M | 0,2 | S1M | 32 | 28084,1 | - | 27738,8 | 28642,6 |
| 2 | O1M | 0 | E1M | 0,2 | S2S | 19 | 32021,4 | 14,02% | 31461,5 | 32466,2 |
| 3 | O1M | 0 | E1M | 0,2 | S3D | 7 | 53358,2 | 66,63% | 53358,2 | 53358,2 |
| 4 | O1M | 0 | E2S | 0,4 | S1M | 32 | 24709,1 | -53,69% | 24050,8 | 25058,6 |
| 5 | O1M | 0 | E2S | 0,4 | S2S | 19 | 28715,2 | 16,21% | 28284,2 | 29105,1 |
| 6 | O1M | 0 | E2S | 0,4 | S3D | 7 | 57350,3 | 99,72% | 54612,8 | 59514,9 |
| 7 | O1M | 0 | E3D | 1 | S1M | 32 | 22611,5 | -60,57% | 22511,3 | 22671,0 |
| 8 | O1M | 0 | E3D | 1 | S2S | 19 | 27276,7 | 20,63% | 26711,2 | 27910,9 |
| 9 | O1M | 0 | E3D | 1 | S3D | 7 | 53923,8 | 97,69% | 53358,2 | 55428,6 |
| 10 | O2S | 0-6 | E1M | 0,2 | S1M | 32 | 26413,2 | -51,02% | 26080,3 | 26709,3 |
| 11 | O2S | 0-6 | E1M | 0,2 | S2S | 19 | 28185,3 | 6,71% | 27591,3 | 28527,9 |
| 12 | O2S | 0-6 | E1M | 0,2 | S3D | 7 | 44505,2 | 57,90% | 44378,3 | 44781,4 |
| 13 | O2S | 0-6 | E2S | 0,4 | S1M | 32 | 23034,4 | -48,24% | 22866,2 | 23228,0 |
| 14 | O2S | 0-6 | E2S | 0,4 | S2S | 19 | 25820,0 | 12,09% | 25442,1 | 26056,2 |
| 15 | O2S | 0-6 | E2S | 0,4 | S3D | 7 | 44100,2 | 70,80% | 42924,0 | 44763,5 |
| 16 | O2S | 0-6 | E3D | 1 | S1M | 32 | 21170,8 | -51,99% | 20957,3 | 21381,3 |
| 17 | O2S | 0-6 | E3D | 1 | S2S | 19 | 24446,1 | 15,47% | 24059,4 | 24721,9 |
| 18 | O2S | 0-6 | E3D | 1 | S3D | 7 | 42790,3 | 75,04% | 41620,9 | 43482,1 |
| 19 | O3D | 0-15 | E1M | 0,2 | S1M | 32 | 18940,8 | -55,74% | 18940,8 | 18940,8 |
| 20 | O3D | 0-15 | E1M | 0,2 | S2S | 19 | 18940,8 | 0,00% | 18940,8 | 18940,8 |
| 21 | O3D | 0-15 | E1M | 0,2 | S3D | 7 | 19213,1 | 1,44% | 18940,8 | 19572,9 |
| 22 | O3D | 0-15 | E2S | 0,4 | S1M | 32 | 14100,7 | -26,61% | 14065,5 | 14129,9 |
| 23 | O3D | 0-15 | E2S | 0,4 | S2S | 19 | 13741,7 | -2,55% | 13525,9 | 13831,1 |
| 24 | O3D | 0-15 | E2S | 0,4 | S3D | 7 | 14890,1 | 8,36% | 14038,7 | 15612,4 |
| 25 | O3D | 0-15 | E3D | 1 | S1M | 32 | 14063,6 | -5,55% | 14054,4 | 14071,5 |
| 26 | O3D | 0-15 | E3D | 1 | S2S | 19 | 13458,6 | -4,30% | 13448,6 | 13472,3 |
| 27 | O3D | 0-15 | E3D | 1 | S3D | 7 | 13066,3 | -2,91% | 12921,8 | 13273,2 |

The experiments proved that the $C_{max}$ values do not always increase when the open rate increases, as was the case with the one-factor experiment (in the sequence O1', O2', O3'). For the open rate O1, the $C_{max}$ values are the lowest (regardless of the flexibility degree $E$ and the $k$ coefficient). However for the other two levels of the open rate, the $C_{max}$ values depend on the $E$ flexibility degree and/or the $k$ coefficient. The $C_{max}$ value is always lower for O2 than O3, but only when we take into account the same levels of the flexibility degree and of the $k$ coefficient. Therefore it can occur that with the same $k$ coefficient and different flexibility degrees, the $C_{max}$ values

will be lower for O3 than O2 (e.g. at the same k1 level the $C_{max}$ value for O3 and E3 is lower than the $C_{max}$ value for O2 and E1). Similarly, with the same flexibility degree the $C_{max}$ value for the O3 open rate can be lower than for O2, while the $k$ coefficient changes (figure 1).
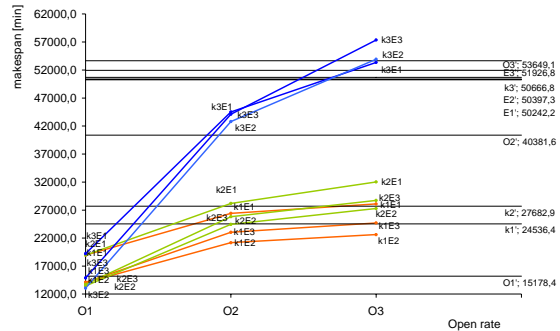


Fig. 1 Relationship between the $C_{max}$ value and the open rate for various values of the flexibility degree and the $k$ coefficient (serial route)

For the experiment with one factor, the $C_{max}$ values increased, when the $k$ coefficient decreased (according to the sequence k1', k2', k3'). In case of the experiment with three factors the $C_{max}$ values did not increase when the $k$ coefficient changed from k1 to k2 to k3. The maximum $C_{max}$ value was obtained at the k3 coefficient level at O2 and O3 of the open rate. On the other hand with the same k3 coefficient – the $C_{max}$ values are the lowest when the open rate is O1 (figure 2).
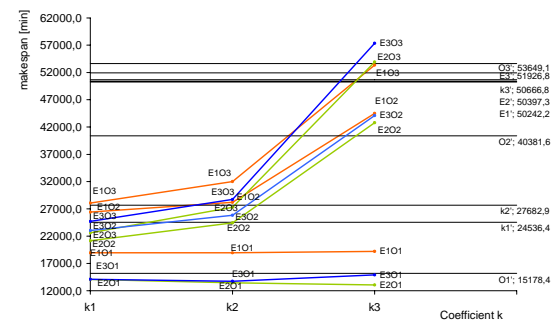


Fig. 2 Relationship between the $C_{max}$ value and k coefficient for various values of open rate and flexibility degree

For the experiment with one factor the $C_{max}$ values increased when the flexibility degree is as follows: E1', E2', E3'. In the case of the three-factor experiment the $C_{max}$ values are various for various flexibility degree levels. The average minimum $C_{max}$ value was obtained at the E2 flexibility degree - and the maximum $C_{max}$ values were obtained at the E3 flexibility degree (figure 3).

Fig. 3 Relationship between the $C_{max}$ value and the flexibility degree for various values of k coefficient and open rate
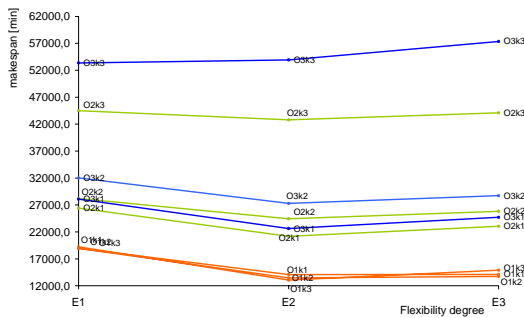
### B. Simulation modeling for the parallel route

The three-factor experiments showing the influence of the factors (flexibility degree $E$, open rate $O$, coefficient $k$) on the makespan $C_{max}$ value (Table II) were carried-out also for the parallel route.

TABLE II
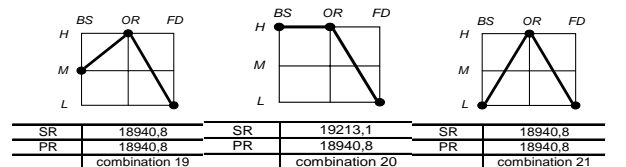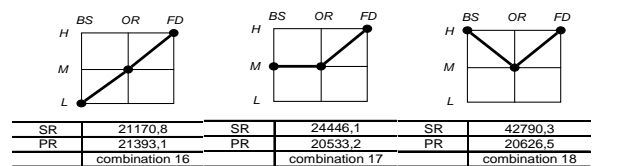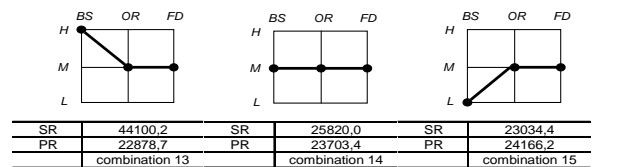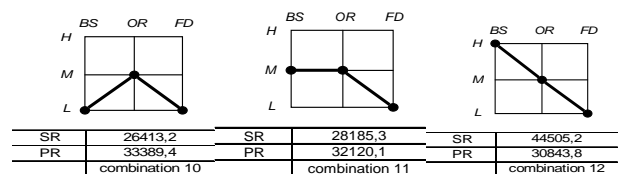$C_{MAX}$ VALUES FOR THE THREE-FACTOR EXPERIMENT
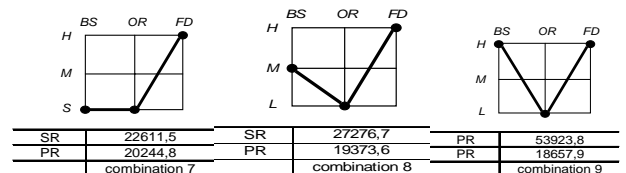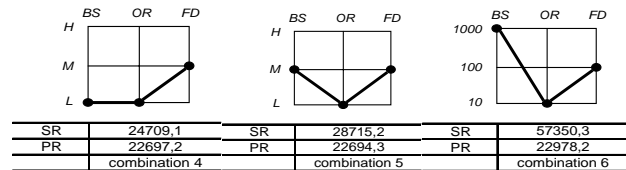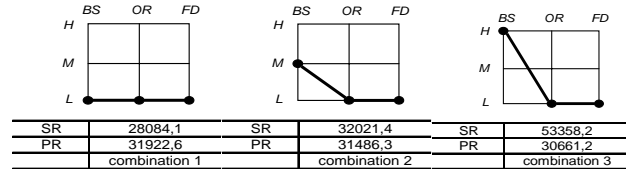( PARALLEL ROUTE)

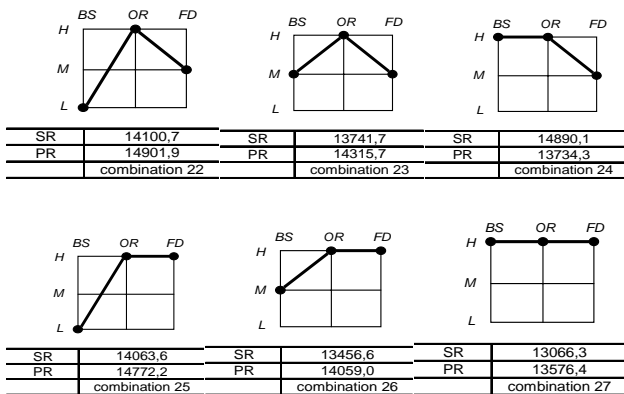| Experiment | Open rate level | Open rate value | Flexibility degree level | Flexibility degree value | Coefficient k level | Coefficient k value | Makespan value average | change | min | max |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | O1M | 0 | E1M | 0,2 | S1M | 32 | 31922,6 | - | 31922,6 | 31922,6 |
| 2 | O1M | 0 | E1M | 0,2 | S2S | 19 | 31486,3 | -1,37% | 31486,3 | 31486,3 |
| 3 | O1M | 0 | E1M | 0,2 | S3D | 7 | 30661,2 | -2,62% | 30661,2 | 30661,2 |
| 4 | O1M | 0 | E2S | 0,4 | S1M | 32 | 22697,2 | -25,97% | 22697,2 | 22697,2 |
| 5 | O1M | 0 | E2S | 0,4 | S2S | 19 | 22694,3 | -0,01% | 22694,3 | 22694,3 |
| 6 | O1M | 0 | E2S | 0,4 | S3D | 7 | 22958,2 | 1,16% | 22958,2 | 22958,2 |
| 7 | O1M | 0 | E3D | 1 | S1M | 32 | 20244,8 | -11,82% | 20244,8 | 20244,8 |
| 8 | O1M | 0 | E3D | 1 | S2S | 19 | 19373,6 | -4,30% | 19373,6 | 19373,6 |
| 9 | O1M | 0 | E3D | 1 | S3D | 7 | 18657,9 | -3,69% | 18657,9 | 18657,9 |
| 10 | O2S | 0-6 | E1M | 0,2 | S1M | 32 | 33389,4 | 78,96% | 33389,4 | 33389,4 |
| 11 | O2S | 0-6 | E1M | 0,2 | S2S | 19 | 32120,1 | -3,80% | 32120,1 | 32120,1 |
| 12 | O2S | 0-6 | E1M | 0,2 | S3D | 7 | 30843,8 | -3,97% | 30843,8 | 30843,8 |
| 13 | O2S | 0-6 | E2S | 0,4 | S1M | 32 | 24166,2 | -21,65% | 24166,2 | 24166,2 |
| 14 | O2S | 0-6 | E2S | 0,4 | S2S | 19 | 23703,4 | -1,92% | 23703,4 | 23703,4 |
| 15 | O2S | 0-6 | E2S | 0,4 | S3D | 7 | 22878,7 | -3,48% | 22878,7 | 22878,7 |
| 16 | O2S | 0-6 | E3D | 1 | S1M | 32 | 21393,1 | -6,49% | 21393,1 | 21393,1 |
| 17 | O2S | 0-6 | E3D | 1 | S2S | 19 | 20533,2 | -4,02% | 20533,2 | 20533,2 |
| 18 | O2S | 0-6 | E3D | 1 | S3D | 7 | 20626,5 | 0,45% | 20626,5 | 20626,5 |
| 19 | O3D | 0-15 | E1M | 0,2 | S1M | 32 | 18940,8 | -8,17% | 18940,8 | 18940,8 |
| 20 | O3D | 0-15 | E1M | 0,2 | S2S | 19 | 18940,8 | 0,00% | 18940,8 | 18940,8 |
| 21 | O3D | 0-15 | E1M | 0,2 | S3D | 7 | 18940,8 | 0,00% | 18940,8 | 18940,8 |
| 22 | O3D | 0-15 | E2S | 0,4 | S1M | 32 | 14901,9 | -21,32% | 14901,9 | 14901,9 |
| 23 | O3D | 0-15 | E2S | 0,4 | S2S | 19 | 14315,7 | -3,93% | 14315,7 | 14315,7 |
| 24 | O3D | 0-15 | E2S | 0,4 | S3D | 7 | 13734,3 | -4,06% | 13734,3 | 13734,3 |
| 25 | O3D | 0-15 | E3D | 1 | S1M | 32 | 14772,2 | 7,56% | 14772,2 | 14772,2 |
| 26 | O3D | 0-15 | E3D | 1 | S2S | 19 | 14059,0 | -4,83% | 14059,0 | 14059,0 |
| 27 | O3D | 0-15 | E3D | 1 | S3D | 7 | 13576,4 | -3,43% | 13576,4 | 13576,4 |

### C. Comparison of $C_{max}$ for the serial and parallel routes

Two types of routing were considered: a serial route (SR) and a parallel one (PR). In order to analyze the serial and parallel routes we will use the following symbols for the factors: BS – batch size (in relation to the $k$ coefficient); OR - open rate; FD - flexibility degree.

The individual BS, OR and FD factors assume values of the linguistic variables from the set {L, M, H}. For the BS, OR and FD values we assume one of the following linguistic variables: L - low value, M – medium value, H - high value

(V-very high value in some combinations of factors). In this way 27 combinations with factors were created (figure 4). Figure 4 shows some of the results ($C_{max}$ value) for the test factors for the serial and parallel routes.



| | | |
|---|---|---|
| SR 28084,1 | SR 32021,4 | SR 53358,2 |
| PR 31922,6 | PR 31486,3 | PR 30661,2 |
| combination 1 | combination 2 | combination 3 |
| SR 24709,1 | SR 28715,2 | SR 57350,3 |
| PR 22697,2 | PR 22694,3 | PR 22978,2 |
| combination 4 | combination 5 | combination 6 |
| SR 22611,5 | SR 27276,7 | PR 53923,8 |
| PR 20244,8 | PR 19373,6 | PR 18657,9 |
| combination 7 | combination 8 | combination 9 |
| SR 26413,2 | SR 28185,3 | SR 44505,2 |
| PR 33389,4 | PR 32120,1 | PR 30843,8 |
| combination 10 | combination 11 | combination 12 |
| SR 44100,2 | SR 25820,0 | SR 23034,4 |
| PR 22878,7 | PR 23703,4 | PR 24166,2 |
| combination 13 | combination 14 | combination 15 |
| SR 21170,8 | SR 24446,1 | SR 42790,3 |
| PR 21393,1 | PR 20533,2 | PR 20626,5 |
| combination 16 | combination 17 | combination 18 |
| SR 18940,8 | SR 19213,1 | SR 18940,8 |
| PR 18940,8 | PR 18940,8 | PR 18940,8 |
| combination 19 | combination 20 | combination 21 |

| | | |
|---|---|---|
| SR | 14100,7 | SR 13741,7 | SR 14890,1 |
| PR | 14901,9 | PR 14315,7 | PR 13734,3 |
| combination 22 | combination 23 | combination 24 |

| | | |
|---|---|---|
| SR | 14063,6 | SR 13456,6 | SR 13066,3 |
| PR | 14772,2 | PR 14059,0 | PR 13576,4 |
| combination 25 | combination 26 | combination 27 |

Fig. 4 $C_{max}$ values for combinations of parameters (SR and PR route)

We can generally see that depending on the OR open rate we can single out 3 classes of quality results (with regard to the $C_{max}$ criterion) - very good (large open rate), average (medium open rate) and poor (small open rate). Moreover an increase of the FD value influences the $C_{max}$ more than the RS value, although there are a number of exceptions.

The best results of $C_{max}$ are always obtained at OR(H) regardless of the BS or FD values. E.g. at OR(H) value the best $C_{max}$ is achieved for combination BS(H)-OR(H)-FD(H) rather than for BS(M)-OR(H)-FD(H); at OR(H) value the best $C_{max}$ is achieved for combination BS(H)-OR(H)-FD(H) both for the serial route and the parallel route.

The poorest results of $C_{max}$ are generally obtained at OR(M) for the serial route and at OR(L) for the parallel route. E.g. at OR(L) for combinations BS(L)-OR(L)-FD(L), BS(M)-OR(L)-FD(L), BS(H)-OR(L)-FD(L), BS(L)-OR(L)-FD(M), BS(M)-OR(L)-FD(H), BS(L)-OR(L)-FD(H), BS(M)-OR(L)-FD(H), BS(H)-OR(L)-FD(H) better $C_{max}$ values are achieved for the parallel route than for the serial one; moreover for combination BS(H)-OR(L)-FD(M) the $C_{max}$ value is better than for BS(H)-OR(M)-FD(M).

At OR(M) for all combinations i.e. BS(L)-OR(M)-FD(L), BS(M)-OR(M)-FD(L), BS(H)-OR(M)-FD(L), BS(L)-OR(M)-FD(M), BS(M)-OR(M)-FD(M), BS(H)-OR(M)-FD(M), BS(L)-OR(M)-FD(H), BS(M)-OR(M)-FD(H), BS(H)-OR(M)-FD(H) better $C_{max}$ values are achieved for the serial route than for the parallel one.

Analyzing the influence of OR on the $C_{max}$ value we can observe the following behavior of the algorithm. At OR(L) an increase of BS from (L) to (M) and FD from (L) to (M) gives an insignificant increase of the average value of $C_{max}$ from ca. 28084.1 to 28715.2 i.e. by about 650 (min.) for the SR route and a decrease of the $C_{max}$ value from 31922.6 to 22694.3 i.e. by about 9000 (min.) for the PR route. An increase of BS from (L) to (M) at OR(L) and FD(L) gives a significant increase of the $C_{max}$ value from 28084.1 to 32021.4 i.e. by about 4000 (min.) for the SR route and an insignificant decrease of the $C_{max}$ value from 31922,6 to 31486,3 i.e. about 500 (min.) for the PR route.

Similarly, an increase of BS from (M) to (H) and FD from (M) to (H) at OR(L) gives a very significant increase of the average $C_{max}$ value from ca. 28715.2 to 57350.3 for the SR route and a small decrease of the $C_{max}$ value from 22694.3 to 18657.9 (min.) i.e. by about 4000 (min.) for the PR route, etc.

For the SR serial route, e.g. when FD changes, the $C_{max}$ value increases in the following order: from BS(H)-OR(L)-FD(L) to BS(H)-OR(M)-FD(L) to BS(H)-OR(M)-FD(M) to BS(H)-OR(M)-FD(M) to BS(H)-OR(H)-FD(L) to BS(H)-OR(H)-FD(M) to BS(H)-OR(H)-FD(H), and from BS(H)-OR(L)-FD(L) to BS(H)-OR(M)-FD(L) to BS(H)-OR(M)-FD(M) to BS(H)-OR(M)-FD(H) to BS(H)-OR(H)-FD(L) to BS(H)-OR(H)-FD(M) to BS(H)-OR(H)-FD(H) for PR route.

For the PR parallel route e.g. when BS changes, the $C_{max}$ value decreases insignificantly in the following order: from BS(L)-OR(L)-FD(L) to BS(M)-OR(L)-FD(L) to BS(H)-OR(L)-FD(L) at OR(L) and FD(L); from BS(L)-OR(L)-FD(M) to BS(M)-OR(L)-FD(M) at OR(L) and FD(M) (for BS(H)-OR(L)-FD(L) the $C_{max}$ value increases insignificantly); from BS(L)-OR(L)-FD(H) to BS(M)-OR(L)-FD(H) to BS(H)-OR(L)-FD(H) at OR(L) and FD(H).

Table III shows the relative increase of the $C_{max}$ value defined by linguistic variables from the set {$S,A,B$}, and the decrease of the $C_{max}$ value defined by those from the set {$s,a,b$}. The comparison is made for each of the pairs assumed for the consideration of factor combinations, where $s,S$ - small, $a,A$ - average, and $b,B$ - big change value respectively.

TABLE III
COMPARISON OF COMBINATIONS FACTORS (SERIAL ROUTE)

Combinations of factors

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | s | b | S | s | b | s | s | b | s | S | a | a | S | S | S | S | a | S | S | S | S | A | A | A | A | A |
| 2 | | 0 | a | s | s | b | S | S | a | S | S | a | a | S | S | S | S | s | A | A | A | A | A | A | A | A | A |
| 3 | | | 0 | B | B | s | B | B | s | B | B | S | S | B | B | B | B | S | B | S | S | S | S | S | S | S | S |
| 4 | | | | 0 | S | b | S | s | b | S | s | s | a | a | s | S | S | S | a | S | S | S | S | S | S | S | S |
| 5 | | | | | 0 | b | s | s | b | S | S | S | a | a | S | S | S | S | a | S | S | A | A | A | A | A | A |
| 6 | | | | | | 0 | B | B | S | B | B | A | A | B | B | B | B | A | B | B | B | B | B | B | B | B | B |
| 7 | | | | | | | 0 | s | b | s | s | a | a | s | s | S | s | a | s | s | s | s | s | s | s | s | s |
| 8 | | | | | | | | 0 | b | S | s | a | a | S | S | S | S | a | S | S | S | A | A | A | A | A | A |
| 9 | | | | | | | | | 0 | B | B | S | B | B | B | B | B | A | B | B | B | B | B | B | B | B | B |
| 10 | | | | | | | | | | 0 | s | a | a | S | S | S | S | A | S | S | S | A | A | A | A | A | A |
| 11 | | | | | | | | | | | 0 | a | a | S | S | S | S | a | S | S | a | S | A | A | A | A | A |
| 12 | | | | | | | | | | | | 0 | S | A | A | A | S | B | B | B | B | B | B | B | B | B | B |
| 13 | | | | | | | | | | | | | 0 | A | A | A | A | s | B | B | B | B | B | B | B | B | B |
| 14 | | | | | | | | | | | | | | 0 | S | S | S | a | S | S | S | S | A | S | S | A | A |
| 15 | | | | | | | | | | | | | | | 0 | S | s | a | S | S | S | S | S | S | S | S | S |
| 16 | | | | | | | | | | | | | | | | 0 | s | a | S | S | S | S | S | S | S | S | S |
| 17 | | | | | | | | | | | | | | | | | 0 | a | S | S | S | S | S | S | S | S | S |
| 18 | | | | | | | | | | | | | | | | | | 0 | B | B | B | B | B | B | B | B | B |
| 19 | | | | | | | | | | | | | | | | | | | 0 | s | - | S | S | S | S | S | S |
| 20 | | | | | | | | | | | | | | | | | | | | 0 | S | S | S | S | S | S | S |
| 21 | | | | | | | | | | | | | | | | | | | | | 0 | S | S | S | S | S | S |
| 22 | | | | | | | | | | | | | | | | | | | | | | 0 | S | s | S | S | S |
| 23 | | | | | | | | | | | | | | | | | | | | | | | 0 | S | S | S | S |
| 24 | | | | | | | | | | | | | | | | | | | | | | | | 0 | S | S | S |
| 25 | | | | | | | | | | | | | | | | | | | | | | | | | 0 | S | S |
| 26 | | | | | | | | | | | | | | | | | | | | | | | | | | 0 | S |
| 27 | | | | | | | | | | | | | | | | | | | | | | | | | | | 0 |

For the evaluation of the manufacturing system with regard to the $C_{max}$ criterion we assume one of the following linguistic variables: V - very good, A - average, and P - poor.

Table IV shows some data representing the membership grade in fuzzy sets of very good, average, and poor (quality of results) corresponding to some combinations of factors. The fuzzy sets unions can be considered as representing rules such as [24]:

IF $E$ THEN $H$

where $E$ is the analyzed combination of factors and $H$ is the fuzzy set union. E.g. for serial route

IF COMBINATION BS(L)-OR(L)-FD(L)

THEN RESULT ($C_{max}$) (0.3 /V + 0.7/A + 0.0/P)

and for parallel route

IF COMBINATION BS(L)-OR(L)-FD(L)

THEN RESULT($C_{max}$) (0.0 /V + 0.2/A + 0.8/P)

where the expression in parentheses is the fuzzy set union quality of the result.

TABLE IV
MEMBERSHIP GRADES IN FUZZY SETS OF COMBINATIONS FACTORS (SERIAL AND PARALLEL ROUTES)

| Quality of results / Combinations | Very good (V) | | Average (A) | | Poor (P) | |
|---|---|---|---|---|---|---|
| | SR | PR | SR | PR | SR | PR |
| BS(L)-OR(L)-FD(L) | 0.3 | 0 | 0.7 | 0.2 | 0 | 0.8 |
| BS(M)-OR(L)-FD(L) | 0.1 | 0 | 0.9 | 0.2 | 0 | 0.8 |
| BS(H)-OR(L)-FD(L) | 0.9 | 0 | 0.1 | 0.2 | 0 | 0.8 |
| BS(L)-OR(L)-FD(M) | 0.6 | 0.2 | 0.4 | 0.8 | 0 | 0 |
| BS(M)-OR(L)-FD(M) | 0 | 0.3 | 1 | 0.7 | 0 | 0 |
| BS(H)-OR(L)-FD(M) | 0 | 0.2 | 0 | 0.8 | 1 | 0 |
| BS(L)-OR(L)-FD(H) | 0.6 | 0.4 | 0.4 | 0.6 | 0 | 0 |
| BS(M)-OR(L)-FD(H) | 0.3 | 0.4 | 0.7 | 0.6 | 0 | 0 |
| BS(H)-OR(L)-FD(H) | 0 | 0.5 | 0.2 | 0.5 | 0.8 | 0 |
| BS(L)-OR(M)-FD(L) | 0.6 | 0 | 0.4 | 0.1 | 0 | 0.9 |
| BS(M)-OR(M)-FD(L) | 0.3 | 0 | 0.7 | 0.2 | 0 | 0. |
| BS(H)-OR(M)-FD(L) | 0 | 0 | 0.6 | 0.4 | 0.4 | 0.6 |
| BS(L)-OR(M)-FD(M) | 0.6 | 0 | 0.4 | 0.9 | 0 | 0.1 |
| BS(M)-OR(M)-FD(M) | 0.4 | 0 | 0.6 | 1 | 0 | 0 |
| BS(H)-OR(M)-FD(M) | 0 | 0.1 | 0.6 | 0.9 | 0.4 | 0 |
| BS(L)-OR(M)-FD(H) | 0.7 | 0.2 | 0.3 | 0.8 | 0 | 0 |
| BS(M)-OR(M)-FD(H) | 0,6 | 0.3 | 0.4 | 0.7 | 0 | 0 |
| BS(M)-OR(L)-FD(H) | 0 | 0.3 | 0.7 | 0.7 | 0.3 | 0 |
| BS(L)-OR(H)-FD(L) | 0.8 | 0.4 | 0.2 | 0.6 | 0 | 0 |
| BS(M)-OR(H)-FD(L) | 0.8 | 0.4 | 0.2 | 0.6 | 0 | 0 |
| BS(H)-OR(H)-FD(L) | 0.8 | 0.4 | 0.2 | 0.6 | 0 | 0 |
| BS(L)-OR(H)-FD(M) | 0.9 | 0.9 | 0.1 | 0.1 | 0 | 0 |
| BS(M)-OR(H)-FD(M) | 0.9 | 0.9 | 0.1 | 0.1 | 0 | 0 |
| BS(H)-OR(H)-FD(H) | 0.8 | 0.9 | 0.2 | 0.1 | 0 | 0 |
| BS(L)-OR(H)-FD(H) | 0.9 | 0.9 | 0.1 | 0.1 | 0 | 0 |
| BS(M)-OR(H)-FD(H) | 0.9 | 0.9 | 0.1 | 0.1 | 0 | 0 |
| BS(H)-OR(H)-FD(H) | 1 | 0 | 1 | 0 | 0 | 0 |

## IV. CONCLUSION

We proposed the use of GRASP+SA hybrid metaheuristic in the analysis of the manufacturing systems (for flexible job-open scheduling problem) for the serial route and the parallel one. The influence of route flexibility degree, open rate of operations and production type coefficient on makespan is discussed. Experiments with one and three factors have been tested and different levels of factors have been considered and compared.

As a future perspective, it will be interesting to compare the proposed approach with the other methods and also to study the problem on a large set of benchmark data and real problems.

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