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Simulated Annealing Application for Structural Optimization

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***Abstract*—**Several methods are available for weight and shape optimization of structures, among which Evolutionary Structural Optimization (ESO) is one of the most widely used methods. In ESO, however, the optimization criterion is completely case-dependent. Moreover, only the improving solutions are accepted during the search. In this paper a Simulated Annealing (SA) algorithm is used for structural optimization problem. This algorithm differs from other random search methods by accepting non-improving solutions. The implementation of SA algorithm is done through reducing the number of finite element analyses (function evaluations). Computational results show that SA can efficiently and effectively solve such optimization problems within short search time.

***Keywords*—**Simulated annealing, Structural optimization, Compliance, C.V. product.

I. Introduction

WEIGHT optimization is one of the most important issues in designing different structures such as machine elements, towers and frames. Design engineer working in the field of research and development has to often design completely new structures. The loading and support conditions of a particular design problem are usually known in advance, but the designer should decide on the final shape and geometry of structure. Weight is known to be one of the main criteria in structural design. For this reason, weight reduction is often set as the main objective of the design task. Due to financial and safety considerations, optimum design of structures has received much attention in the literature.

Among different optimization methods, Evolutionary Structural Optimization (ESO) is the most widely used techniques. ESO is based on a simple idea that an optimal structure (with maximum stiffness but minimum weight) can be achieved by gradually removing inefficient materials from design domain. This method was first proposed by Xie and Steven [1]. The basic concept of ESO is that by systematically removing inefficient materials, the structure evolves towards a lighter one. It is usually employed for sizing, shape and layout problems under stress considerations [1], [2].

A major drawback with ESO is that the optimization results are likely to be local optimums, as the removal is one-sided and thus some useful elements are probably deleted in the early evolutional generations. To overcome this problem, ESO has been extended to bidirectional evolutionary structural optimization (BESO) which allows materials to be added to the structure at the same time as the inefficient ones are removed [3].

However, the results from ESO/BESO are likely to be local optimums other than the global optimum desired [4-6]. One obvious deficiency of ESO/BESO is that these methods may result in a non-convergent solution [4]. In other words, the solution may be worse and worse in terms of the objective function, e.g. compliance, if the ESO/BESO procedure continues without stop. Apparently, the solution using this type of ESO/BESO procedure is problematic when a broken member with no or low strain energy happens to be a part of the final topology [5]. In addition, for BESO method, the new elements are introduced only around existing elements with a high value of the criterion function. Such an element introduction may be unsuccessful in rectifying incorrect element rejections. It could also be unsuccessful in connecting large areas/volumes where much material has been removed [6].

In this paper, a Simulated Annealing (SA) approach for structural optimization is presented. The objective is to find a design with minimum compliance with respect to a desired volume. The optimization is done in such a way that design specifications such as structure performance and overall geometry are maintained.

II. Simulated Annealing

Simulated annealing (SA), first proposed by Kirkpatrik *et al.* [7], is a method suitable for solving optimization problems of large scales. This algorithm, among few other heuristics, is suitable for complicated problems where global optimum is hidden among many local optima. The idea of the method is an analogy with the way molten metals cool and anneal. For slowly cooled process, system is able to find the minimum energy state. So slow cooling is essential for ensuring that a low energy state is achieved.

A standard SA procedure begins by generating an initial solution at random. At initial stages, a small random change is made in the current solution. Then the objective function value of new solution is calculated and compared with that of current solution. A move is made to the new solution if it has better value or if the probability function implemented in SA

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has a higher value than a randomly generated number. Otherwise a new solution generated and evaluated.

The probability of accepting a new solution is given as follows:

$$p=\begin{cases}1 & if\ \Delta<0\\ e^{-\Delta/t} & if\ \Delta\geq 0\end{cases} \quad (1)$$

The calculation of this probability relies on a temperature parameter, *T*, which is referred to as temperature, since it plays a similar role as the temperature in the physical annealing process. To avoid getting trapped at a local minimum point, the rate of reduction should be slow. In our problem the following method to reduce the temperature has been used:

$$T_{i+1}=cT_i \qquad i=0,1,... \quad and\ 0.9\leq c<1 \quad (2)$$

Thus, at the start of SA most worsening moves may be accepted, but at the end only improving ones are likely to be allowed. This can help the procedure jump out of a local minimum. The algorithm may be terminated after a certain volume fraction for the structure has been reached or after a pre-specified run time.

Additional concepts about SA and some of its applications can easily be found in literature (e.g., [8], [9]).

III. Problem Formulation

The SA formulation for structural optimization is based on the continuous problem of minimizing the total compliance in volume product (*C.V* Product) [2]. The general model for *C.V* optimization is as follows:

$$\begin{cases}\min C.V\\ Subject\ to \quad V/V_0\leq Volfract.\end{cases} \quad (3)$$

In the above model, *V* is the volume of structure at each iteration during optimization process, V_0 is initial volume and *volfrac* is the minimum volume ratio with respect to initial volume. In Finite Element (FE) analysis, the total compliance of structure is given by:

$$C(x)=\sum_{i=1}^{N}x_i^P\{u_i\}^T[K]\{u_i\} \quad (4)$$

In equation (4) x_i is the vector of design variable that defines the density of *i*-th element, *N* is the total number of elements, *P* is the penalization power (typically *P=3*), $\{u_i\}$ is the displacement vector for *i*-th element and [*K*] is the global stiffness matrix.

The variable *x* can be zero or one. Thus if an element exists in the model its density is 1 and otherwise it is set to zero; i.e., $x\in\{0,1\}$. In this paper, to avoid singularity we use *x* = 0.0001 instead *x* = 0.

IV. Example Problem and FE Analysis

Fig. 1 shows a typical topology optimization problem in which the compliance is to be minimized for various volume fractions [5]. There is a fix support at the left and a roller support at the top. Young's modulus is taken as unity and Poisson's ratio as zero. The element thickness may only be unity or zero. The load condition consists of a horizontal load of intensity 2.0 and a vertical load of intensity 1.0.

The objective is to minimize the product of compliance by weight in such a way that the overall shape of structure is maintained and the structure can still endure the applied load without failure.

During the optimization process, in each iteration, the value *C(x)* has to be calculated using FE analysis. For this purpose and to speed up search, we have created the FE analysis subroutine using MATLAB code and implanted it in the SA algorithm. In this way, the FE analysis is done much faster. In FE analysis a model with 100 four-node quadrilateral elements has been used.

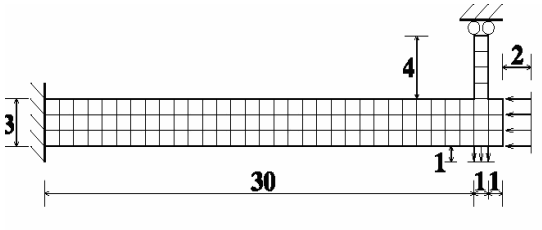

Fig. 1 test example, initial design with FE mesh

V. Implementation of SA Algorithm

As shown in Fig. 1, to begin the optimization procedure, the initial design domain is considered as a rectangular consists of 7 elements in vertical direction and 32 elements in horizontal direction. Then, the densities of 100 elements that constituted the original model (shape) are set to unity; i.e. $x_i=1$. These elements are shown in black in Fig. 2. The densities of remaining elements are set to zero (white color).

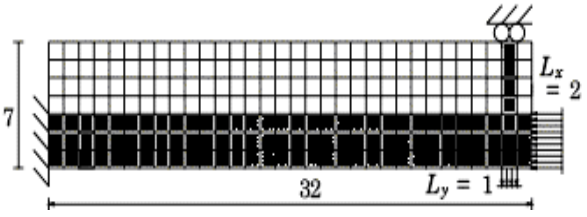

Fig. 2 Initial and original design domains

The search starts with the original design. In each iteration, SA randomly finds an element in original domain and changes its density from 1 to 0.0001 (near zero). Then FE program is executed to calculate compliance for the new shape. This compliance is then returned to main program to determine the objective function value. The new and current objective function values are then compared. A move to new solution (shape) is made under two conditions: 1) the value of new objective function is better than the current one and 2) the probability of accepting a new solution, given by equation 1, is greater than a uniform random number between (0,1].

Otherwise if none of these conditions is satisfied, the density of that selected element is returned to its previous value namely 1 and a new move is generated. The search is terminated after a pre-defined volume fraction is obtained or after a specified run time, whichever comes first.

For simulated annealing algorithm, we have set the initial temperature 5 times of compliance of initial design $(T_0 = 5 \times C_0)$. The temperature control function was set to $T_i = 0.9\ (T_{i-1})$, with *i* denoting number of iterations.

In designing the optimization procedure, important functional constraints have been taken into consideration. For instance, the algorithm was not allowed to remove those elements that create vertical tie and elements that carry out the load. Also to avoid discontinuity in structure, the removal of any element that resulted in discontinuity was prohibited to the algorithm.

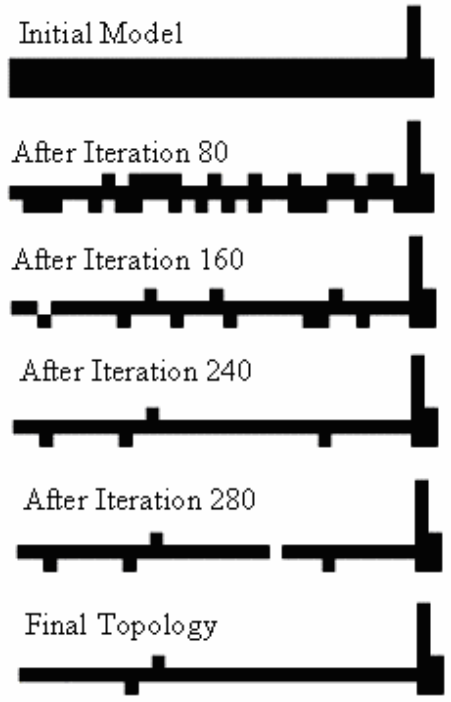

Fig. 3 Evolution of the solution during the search

For a sample run, the evolution of structure design, during different stages of the search, is illustrated in Fig. 3.

Since SA is a probabilistic search algorithm, different runs may result in dissimilar configurations. Some of these solutions may not be optimum, especially if the search is terminated prematurely. This fact is well demonstrated in Fig. 4 which shows the final structures for different runs with various numbers of iterations.

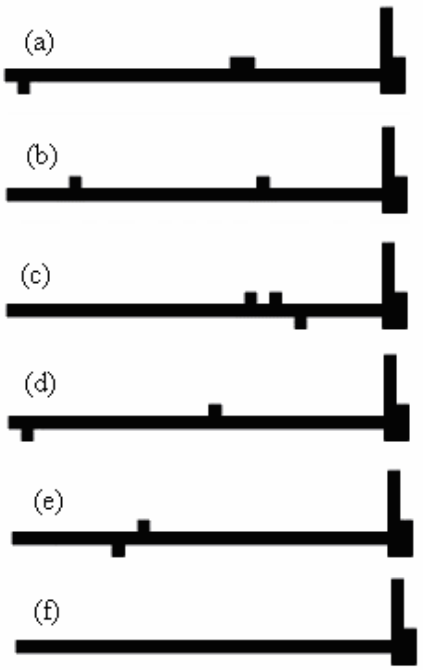

Fig. 4 Final results for different runs after: (a) 262 iterations, (b) 272 iterations, (c) 302 iterations, (d) 303 iterations, (e) 290 iterations, (f) 405 iterations (Best result)

It should be noted that all of the final designs of Fig. 4 satisfy our volume fraction constraint. Nevertheless, Fig. 4-f is the optimal solution for the example problem [10]. The ESO method also results in the same structural configuration.

Fig. 5 shows the changes in volume fractions during SA search. As can be seen, at the final stages of the search, the volume fraction reaches its optimal value with small variations.

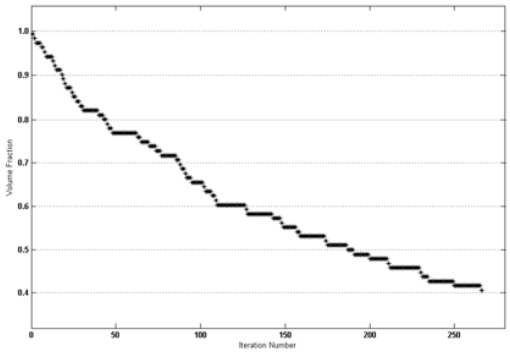

Fig. 5 Volume variations during optimization process

Similarly, Fig. 6 shows the changes in compliance values during the optimization process. Although the value compliance has increased in the final solution, the overall value of the objective function, with respect to volume fraction, has been minimized.

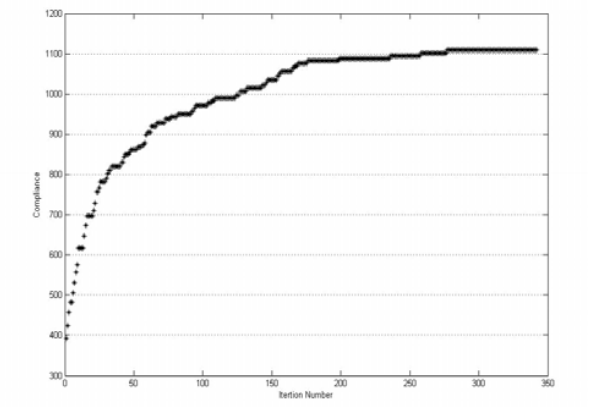

Fig. 6 Compliance variation during SA process

This fact is demonstrated in Fig. 7 that shows the convergence curve of *C.V* product (objective function value). This final structure design has much lower weight than the original one; while satisfies the design constraints and specifications.

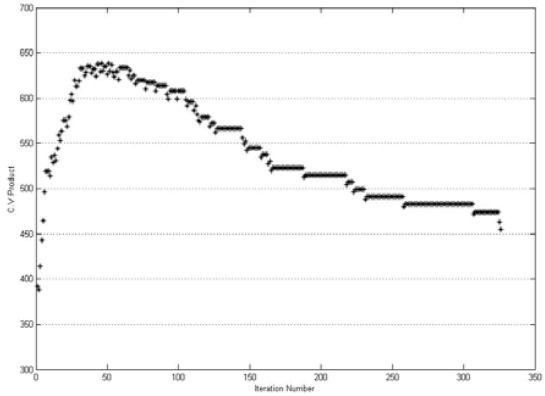

Fig. 7 *C.V* product (Objective Function) Variation during SA process

VI. Conclusion

In this research, as an alternative to ESO method, a simulated annealing (SA) algorithm has successfully been used to optimize the structure design problem. Based on the example shown, the proposed approach is quite capable of solving structural optimization problems within short search times. Computational results have also revealed that the SA solution procedure can results in optimal solutions in most cases. Given its flexibility and ease of adaptation, SA can be viewed as a valuable and attractive tool for structural optimization. An extension of this research may be developing hybrid heuristic algorithms and more robust solutions evaluation criteria in order to tackle such optimization problems more efficiently.

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