

# Series-Parallel Systems Reliability Optimization Using Genetic Algorithm and Statistical Analysis

Essa Abraham Abdulgader Saleem, Thien-My Dao

**Abstract**—The main objective of this paper is to optimize series-parallel system reliability using Genetic Algorithm (GA) and statistical analysis; considering system reliability constraints which involve the redundant numbers of selected components, total cost, and total weight. To perform this work, firstly the mathematical model which maximizes system reliability subject to maximum system cost and maximum system weight constraints is presented; secondly, a statistical analysis is used to optimize GA parameters, and thirdly GA is used to optimize series-parallel systems reliability. The objective is to determine the strategy choosing the redundancy level for each subsystem to maximize the overall system reliability subject to total cost and total weight constraints. Finally, the series-parallel system case study reliability optimization results are showed, and comparisons with the other previous results are presented to demonstrate the performance of our GA.

**Keywords**—Genetic algorithm, optimization, reliability, statistical analysis.

## I. INTRODUCTION

THE system reliability optimization has become a very important subject matter area in industry design and operation of large scale manufacturing systems. The main issue that will be dealt with it in this study is the optimizing reliability of a series-parallel system using GA via implementing solutions for the redundancy allocation problem. The problem is to select redundancy level for each subsystem, component, and the best redundancy strategy in order to maximize the system reliability under system-level constraints. This type of problems includes a determination of components with many selections and redundancy levels that create the maximum advantages, and are subject to the cost and weight constraints at the system level. These are extremely common problems confronted in the theoretical design of numerous engineering systems. It has become progressively necessary to develop adequate solutions to these issues since various mechanical and electrical systems are becoming more complex, even as developing plans take smaller and reliability requirements display very hard and fast. It is very important that the systems achieve their purpose under circumstances and operating conditions in a certain way. Nevertheless, the reliability level is a function of the investment amounts of a system. Consequently, using the optimization models is required to make an effective decision and perform analysis. The optimization of system reliability

(OSR) models has been advanced to resolve the problems in whatever reliability is involved as objective function or constraint. The problem in this research is to optimize a combinatorial engineering design problem by considering the system of reliability constraint, which involves a redundant number of selected components to maximize reliability system or minimize cost system under numerous resources of the constraints.

The series-parallel system considered (Fig. 1) has  $M$  number of subsystems in series, see Coit et al. [5] and Zhao et al. [18]. In turn, subsystem  $i$  contains  $N_i$  number of active (i.e., operating) units in parallel. If any one of the subsystems fails, the system fails. Each block in the diagram represents a unit. Reliability allocation is an essential step in system design. It allows determination of the reliability of vector of subsystems and components to obtain targeted overall reliability. For a system with identified cost, reliability, weight, volume, and other system parameters, the corresponding design problem becomes a combinatorial optimization problem, see Coit et al. [6] and Khorshidi et al. [8]. The best-identified reliability design problem of this type is denoted as the redundancy allocation problem.

Our goal in this paper is to present GA and statistical analysis approach, based on redundancy allocation problem to find the number of redundant components that either maximize reliability or minimize cost under numerous resources of the constraints. The redundancy allocation problem is fundamentally a nonlinear integer programming problem. Most of these problems cannot be answered by direct or indirect or mixed search methods because of separate search space. According to Chern [4], redundancy allocation problem with multiple constraints is somewhat frequently hard to find feasible solutions. This redundancy allocation problem is Non-Deterministic Polynomial-time hard (NP-hard) and it has been well discussed in Chambari et al. [3], Kuo and Prasad [9], Liang et al. [11], Sharifi et al. [14], and Tillman et al. [16].

The penalty function is used in constrained problems optimization, see Smith and Coit [15], Kuri-Morales and Gutiérrez-García [10], Yeniay [17]. Some researchers used statistical analysis to do this work for evolutionary algorithms, see François and Lavergne [7], Mills et al. [12], Castillo-Valdivieso et al. [2], Petrovski et al. [13], Bayabatli and Sabuncuoglu [1].

In the next section, we present our solving methodology using GA and statistical analysis.

## II. METHODOLOGY

From the study of the references [1], [2], [7], [10], [12],

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[13], [15], [17], we decided to use the design and statistical analysis of experiments to optimize two penalty factors in our reliability fitness function using GA. We used a full factorial design with three levels. This classification will allow us to determine the ranges of values of these two factors of penalty giving the best values of reliability using GA. We did 10 simulations for every point of our design of experiments and used the average of the ten reliability values found to improve the accuracy of our coming statistical analysis. The best configuration of each point corresponding to the biggest reliability value is given with the corresponding cost and weight values. It is known that GA is effective for solving

complex combinatorial optimization problems with large, and complex search spaces.

Assumptions:

- All the components  $r_{ij}$  have different value, and every branch has a different number of components in series-parallel.
- The failure rate of components in each subsystem is constant.
- Failure rate depends on the number of working elements.
- Components are not repairable; they are changeable only.
- Subsystems have internal linking cost.
- Failed components do not damage the system.

TABLE I  
INPUT DATA FOR RAP [18]

Gear pair	Stage											
	1			2			3			4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

TABLE II  
THE NOMENCLATURE AND NOTATION USED TO STATE THE MATHEMATICAL MODEL

$R_s$	system reliability
$C_s$	system cost
$W_s$	system weight
$C_{max}$	constraint of system cost
$W_{max}$	constraint of system weight
$s$	number of subsystems in the system
$i$	index of subsystem, $i \in (1, 2, \dots, s)$
$j$	index of component type
$k$	index of redundancy level
$m_i$	number of available component types in subsystem $i$
$P_i$	minimum number of components in parallel required for subsystem $i$ to function
$PN$	maximum number of components in parallel (user define)
$N_i$	a set of component types, $N_i = [1, 2, \dots, m_i]$
$x_{ki}$	a component type is assigned at the position $k$ of subsystem $i$ $x_{ki} \in (1, 2, \dots, m_i, m_{i+1})$
$x$	system configuration matrix
$n_i(x)$	total number of redundant components used in subsystem $i$
$n(x)$	$= [n_1, n_2, \dots, n_s]$
$r_{ij}$	reliability of the $j$ th component type for subsystem $i$
$c_{ij}$	cost of the $j$ th component type for subsystem $i$
$w_{ij}$	weight of $j$ th component type for subsystem $i$
$R_i(x)$	reliability of subsystem $i$
$C_i(x)$	total cost of subsystem $i$
$W_i(x)$	total weight of subsystem $i$

The input data for the reliability optimization of series-parallel systems problem are summarized in Table I. Zhao et al. [18] have provided this example problem. The system consists of four subsystems, and each subsystem has different design component type with same or different characteristics as reliability, cost, transmission ratio, material, dimension, weight, etc. The minimum gear pair  $p_i = 2$  and the maximum gear pair  $PN = 5$  will be used in the gearbox for all stages.

Each of the subsystems is represented by PN positions with each component listed according to their reliability index. The input data in Table I contain component reliability, weight, and cost. The objective is to maximize the system reliability with k-out-of-n subsystem connected in the series-parallel system under the given system requirement constraints.

TABLE III  
CERTAIN SYSTEM CONSTRAINTS VALUE USED

No.	System constraint	
	$C_{max}$	$W_{max}$
1	40	115
2	55	125
3	65	130
4	60	120
5	60	130
6	60	140
7	60	150
8	65	120
9	65	140
10	65	150
11	70	120
12	70	130
13	70	140
14	70	150
15	75	120
16	75	130
17	75	140
18	75	150

Fig. 1 presents a typical example of a series-parallel system configuration with k-out-of-n subsystem reliabilities. The system is separated into  $s$  subsystem indicated by the index  $i$  ( $i = 1, 2, \dots, s$ ).  $p_i$  number of effective components is required for the function at least in subsystem  $i$ . Each subsystem involves one or various components organized in parallel, and it

constitutes the lower bound of the redundancy level for subsystem  $i$ . The upper bound of the component redundancy level in subsystem  $i$  is  $PN$ . The system configuration can thus be described as a matrix of size  $PN \times s$ : The column index  $i$  ( $i = 1, 2, \dots, s$ ) represents subsystem  $i$ , and the row index  $k$  ( $k = 1, 2, \dots, PN$ ) of the matrix represents the position where a component will be used in the subsystem. Redundancy allocation problems (RAP) consist of defining the number of components of each type, so that the complete reliability system will be maximized by considering the given constraints such as cost and weight. The content of the case study is shown in Fig. 2. The problem used in this test to demonstrate the ability of this algorithm for solving RAP is a gearbox reliability optimization problem obtainable in Zhao et al. [18]. The authors in this reference presumed, in order to apply their method for all stages, that the minimum number of components is equal to 2, and the maximum number of components is equal to 5. The problem in the reference Zhao et al. [18] is to decide how many gear pairs and what kind of gear pair selected to be used in each stage to give maximum reliability of the gearbox with minimization of both system cost and weight. By assuming that all the gear pairs are active components in the stage, then the gearbox is analogous to a series-parallel system with  $k$ -out-of- $n$ :  $G$  subsystems.

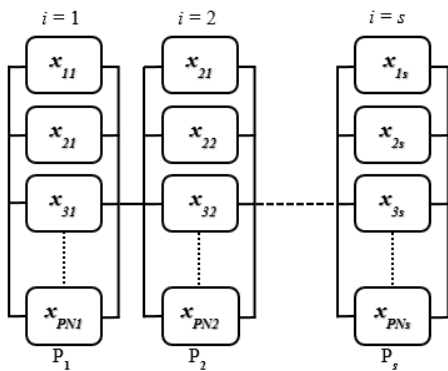


Fig. 1 Series-parallel system

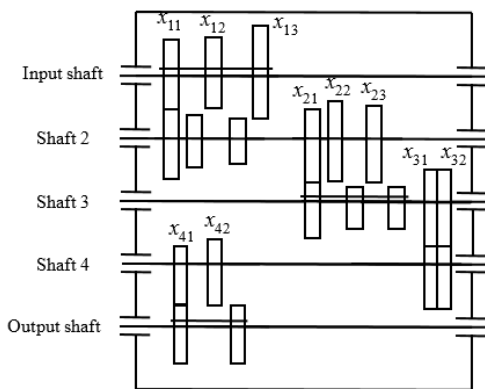


Fig. 2 Series-parallel system case study

The studied problem is modeled by Zhao et al. [18], and the mathematical model formulated as

$$\text{Maximize } R_s(x) = \prod_{i=1}^s [1 - \prod_{k=1}^{PN} 1 - r_{ix_{ki}}] \quad (1)$$

Subject to

$$C_s(x) = \sum_{i=1}^s C_i(x) = \sum_{i=1}^s \sum_{k=1}^{PN} C_{ix_{ki}} \leq C_{max} \quad (2)$$

$$W_s(x) = \sum_{i=1}^s W_i(x) = \sum_{i=1}^s \sum_{k=1}^{PN} W_{ix_{ki}} \leq W_{max} \quad (3)$$

$$P_i \leq n_i \leq PN \text{ and } \forall i, i = 1, 2, \dots, s \quad (4)$$

A technique based on GA to optimize series-parallel systems reliability is developed (Fig. 3) in order to find out the best compromise (optimal) solution of the problem. The different steps of the developed technique are given in the chart Fig. 3.

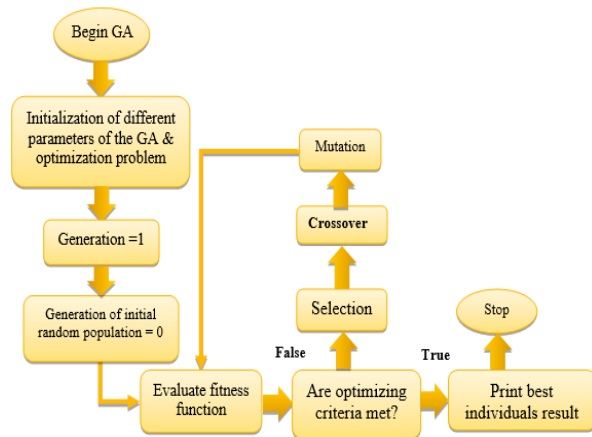


Fig. 3 Flow chart of the proposed GA for optimizing system reliability

We used the fitness function  $f(i)$  to do the reliability optimization of the series-parallel systems using GA in the following form:

$$f(i) = \text{rel}(i); \quad \text{if } c(i) > C_{max};$$

$$\text{then: } f(i) = f(i) - (CPen * (c(i) - C_{max})); \quad \text{if } w(i) > W_{max};$$

$$\text{then: } f(i) = f(i) - (WPen * (w(i) - W_{max}));$$

where  $\text{rel}(i)$  is the reliability,  $c(i)$  is the cost,  $w(i)$  is the weight,  $C_{max}$  is the maximum cost, and  $W_{max}$  is the maximum weight.  $CPen$  is the cost penalty factor, and  $WPen$  is the weight penalty factor. The range of the values in Table IV for cost penalty factor and weight penalty factor was found using trial and error. The dynamic penalty function was defined increasing the penalty for infeasible solutions as the search progresses. The GA implementation is doing with this experimental procedure for determining the initial population size considering the following GA parameters:

- The population size, which determines the size of the population at each generation is 1000, and our maximum number of iteration is 10000.
- We used 20 integers to code our chromosomes (maximum

- of five gear pair and four stages).
- The value 6 from the configuration means that this position is empty.
- We used four points of crossover generated randomly corresponding to our four subsystems to improve our GA search.
- We could obtain a better result by increase the population size to enable the GA to search for more points.
- Nevertheless, when the population size is large, the GA will take a long time to compute each generation
- Finally, it is very important to note that we set the population size to be at least the value of number of variables, so the individuals in each population span the space being searched.

III. RESULTS AND DISCUSSION

A numerical application has been demonstrated with the data obtained from test problem 1 of Zhao et al. (2007) [18], and the obtained results are presented.

We used  $C_{max} = 65$ , and the  $W_{max} = 130$  for our GA fitness function cost penalty (CPen), and weight penalty (WPen) statistical analysis. We used a 3-level full factorial design.

Our statistical analysis in Figs. 4 and 5 shows that all the residuals are zero, which means that our prediction is very good.

TABLE IV  
GA RESULTS OF THE DESIGN OF EXPERIMENTS POINTS IS USED FOR APPLYING STATISTICAL ANALYSIS DATA

No.	CPen	WPen	Average Reliability
1	0.1	0.1	0.9961
2	0.1	0.55	0.9965
3	0.1	1	0.9956
4	0.55	0.1	0.9967
5	0.55	0.55	0.9961
6	0.55	1	0.9967
7	1	0.1	0.9961
8	1	0.55	0.9964
9	1	1	0.9960

The contour plot in Fig. 6 displays the three-dimensional relationship in two dimensions. This plot is on the x-axis and y-axis scales factors by the predictor and the response values represented by contours. The contour plot can be used to investigate the possible relation between the three variables. We have an average reliability, cost penalty, and weight penalty. This plot shows how cost penalty on the x-axis and the weight penalty in y-axis affect the quality result. The darker indicates to the higher quality of the average reliability.

The response surface (Fig. 7) is obtained using the statistical analysis software STATISTICA and it generates the optimal designs. These numbers of the statistical analysis obtained are to choose the best GA for the selection of the optimal designs. The techniques for experimental model design objective are to optimize the response of the output variable (average reliability) which is influenced by cost penalty factor, and weight penalty factor. The response can be represented graphically in the contour plots that help us to

visualize the shape of the response. The darker regions indicate higher quality. The response surface plot for the cost penalty strength where the axis x is the redder color, the weight penalty for the axis y is the less red color, and the axis z is the average reliability.

ANOVA: Var.:Average Reliability; R-sqr=1. (Spreadsheet1.sta)						
2 3-level factors, 1 Blocks, 9 Runs						
DV: Average Reliability						
Factor		SS	df	MS	F	p
(1)CPen	L+Q	0.000000	2	0.000000		
(2)WPen	L+Q	0.000000	2	0.000000		
1*2		0.000001	4	0.000000		
Error		0.000000	0			
Total SS		0.000001	8			

Fig. 4 Model ANOVA result

Observed, Predicted, and Residual Values (Spreadsheet1.sta)			
2 3-level factors, 1 Blocks, 9 Runs			
DV: Average Reliability			
Case or Run	Observed	Predictd	Resids
1	0.996127	0.996127	0.000000
2	0.996507	0.996507	0.000000
3	0.995593	0.995593	-0.000000
4	0.996716	0.996716	0.000000
5	0.996087	0.996087	-0.000000
6	0.996748	0.996748	0.000000
7	0.996125	0.996125	-0.000000
8	0.996392	0.996392	0.000000
9	0.996016	0.996016	-0.000000

Fig. 5 Display observed, predicted, and residual values

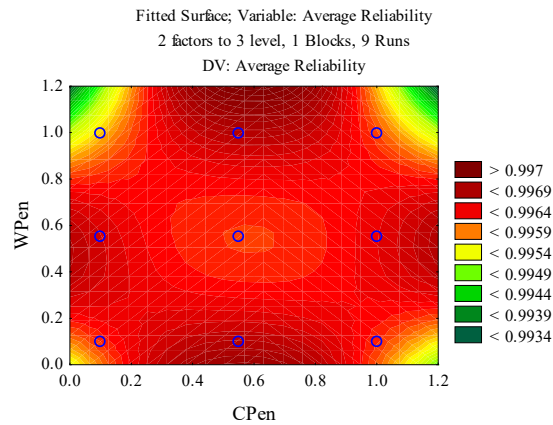


Fig. 6 Contour plot of average reliability versus WPen, CPen

The response curves (Fig. 6) and response surface (Fig. 7) show that the best parameters are around the cost penalty  $CPen = 0.55$  and the weight penalty factor  $WPen = 0.1$ .

We show here one of the ten results running our GA on the center of our design of experiments ( $CPen = 0.55$ ,  $WPen = 0.55$ ): the configuration is 6 3 6 3 3 5 5 6 3 6 5 6 3 4 5 6 2 2 2 6, the reliability = 0.997743, the cost = 62, the weight = 130, and the fitness = 0.997743. The reliability, cost, weight, and fitness graphics of this result are showed respectively on Figs. 8-11. It can be observed that from the plots which show that GA has already achieved the maximum score at the iteration of 10000.

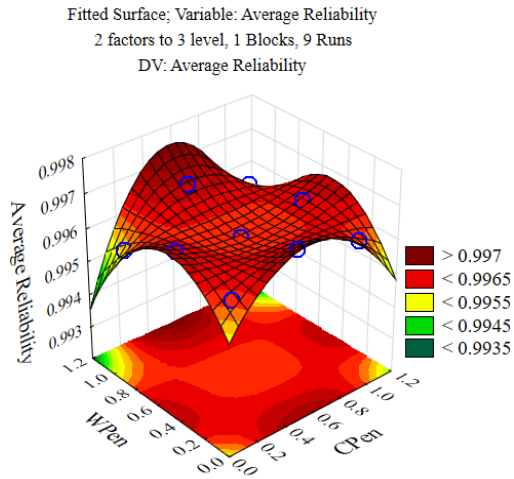


Table V reports all the allocated components for each subsystem that we have in our system. For example, at admissible constraint weight = 115 and constraint cost = 40, the best configuration of the 10 simulations that we obtained is = [3, 3, 6, 6, 6, 6, 3, 3, 3, 6, 5, 6, 5, 5, 6, 6, 2, 2, 6, 6], which means that, from the 20 positions, result is illustrated as:

- The first subsystem has two components of type 3.
- The second subsystem has three components of type 3.
- The third subsystem has three components of type 5.
- The fourth subsystem has two components of type 2.

The result obtained in Table V was just change by the values of the constraints. The cost penalty = 0.55 is constant, and the weight penalty = 0.1 is constant. These results are obtained using the maximum possible improvement with the best feasible solution, which improves the system reliability, cost, and weight.

Fig. 7 3D Surface plot of average reliability versus CPen and WPen

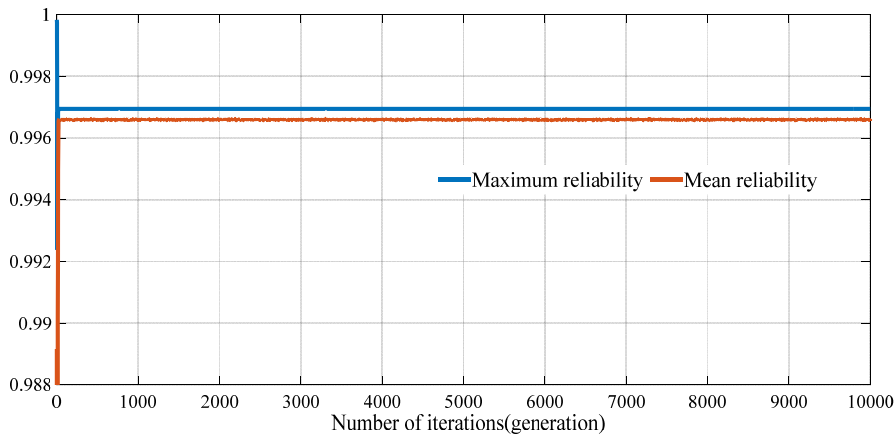


Fig. 8 Maximum and mean reliability

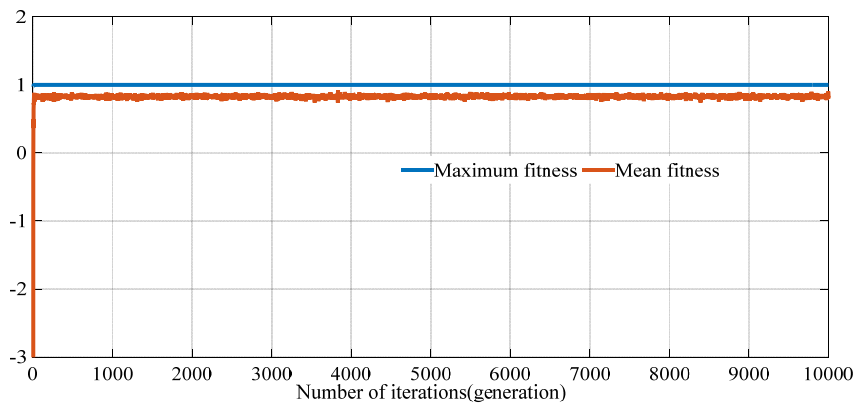


Fig. 9 Maximum and mean fitness

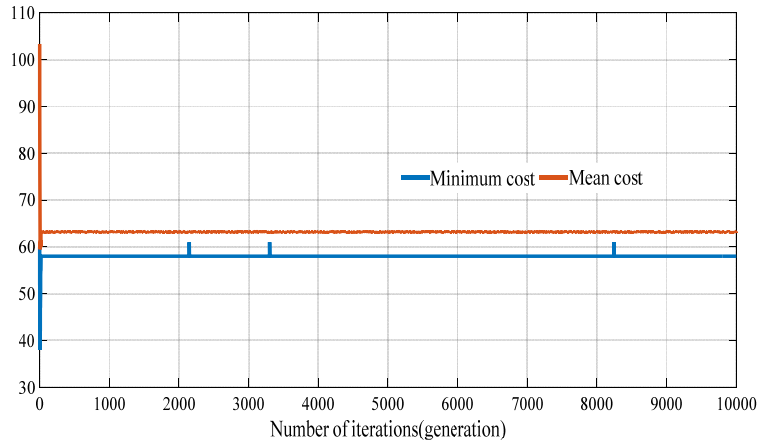


Fig. 10 Minimum and mean cost

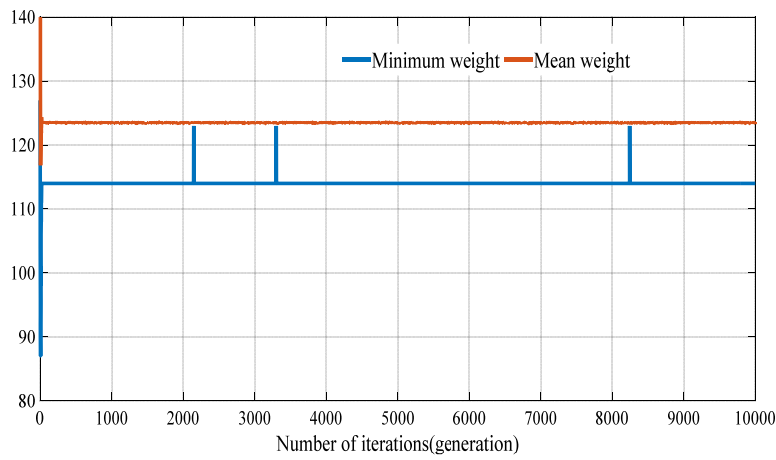


Fig. 11 Minimum and mean weight

TABLE V  
THE RESULTS OBTAINED BY GA

No.	C <sub>Max</sub>	W <sub>Max</sub>	Our obtaining result using GA																		Reliability	Cost	Weight	Fitness		
			Best Configuration of 10 simulations																							
1	40	115	3	3	6	6	6	6	3	3	3	6	5	6	5	5	6	6	2	2	6	6	0.9836	35	113	0.9836
2	55	125	1	6	3	3	6	5	6	2	6	3	4	5	5	6	6	2	6	2	6	2	0.9954	54	125	0.9954
3	65	130	3	6	6	3	3	5	6	5	3	1	6	6	4	4	5	2	2	6	2	6	0.9977	64	129	0.9977
4	60	120	3	3	6	6	3	3	5	6	5	6	4	4	5	6	6	6	2	1	2	6	0.9959	60	120	0.9959
5	60	130	3	3	6	3	6	5	3	6	5	6	5	5	4	5	6	2	6	6	2	2	0.9979	58	130	0.9979
6	60	140	3	1	3	6	1	2	6	6	5	5	6	5	5	5	5	6	2	2	6	2	0.9979	59	139	0.9979
7	60	150	3	6	3	3	6	5	3	5	3	6	4	5	5	5	6	6	6	2	2	2	0.9987	60	144	0.9987
8	65	120	3	3	6	3	6	5	5	5	6	6	4	6	5	4	6	6	2	2	2	6	0.9973	62	113	0.9973
9	65	140	3	3	3	1	6	5	5	6	6	5	5	4	5	6	5	6	2	2	6	2	0.9985	64	134	0.9985
10	65	150	3	3	6	6	3	6	3	3	5	5	6	5	5	5	4	2	6	2	6	2	0.9987	60	144	0.9987
11	70	120	6	3	3	3	6	6	5	6	2	5	6	4	4	6	4	2	2	2	6	6	0.9976	67	118	0.9976
12	70	130	3	3	3	6	6	1	5	5	6	5	4	6	4	5	6	6	2	2	6	2	0.9978	67	122	0.9978
13	70	140	3	1	6	6	3	3	6	5	5	5	6	4	5	3	4	2	6	2	6	2	0.9985	69	140	0.9985
14	70	150	3	3	1	6	6	3	5	5	5	6	5	6	5	4	5	2	2	1	6	2	0.9988	67	149	0.9988
15	75	120	6	3	3	3	6	5	6	5	6	5	4	4	6	6	4	2	6	6	2	4	0.9974	70	117	0.9974
16	75	130	3	3	6	3	6	5	5	3	6	5	4	4	6	4	6	6	6	2	2	2	0.9984	68	128	0.9984
17	75	140	3	6	6	3	3	5	5	5	3	6	4	6	5	3	4	6	2	2	6	2	0.9988	71	138	0.9988
18	75	150	3	3	6	3	6	5	5	3	6	1	4	5	6	5	4	2	2	2	6	1	0.9991	73	150	0.9991

TABLE VI  
THE COMPARISON OF ZHAO ET AL. [18] ANT COLONY SYSTEM (ACS) RESULT AND OUR GA RESULT

No.	$C_{Max}$	$W_{Max}$	Our GA result			Zhao, J. H., Liu, Z., & Dao, M. T. results using ACS-RAP		
			Reliability	Cost	Weight	Reliability	Cost	Weight
1	40	115	0.9836	35	113	0.9861	40	114
2	55	125	0.9954	54	125	0.9973	55	124
3	65	130	0.9977	64	129	0.9977	58	130
4	60	120	0.9959	60	120	0.9968	59	120
5	60	130	0.9979	58	130	0.9977	58	130
6	60	140	0.9979	59	139	0.9985	60	140
7	60	150	0.9987	60	144	0.9987	60	149
8	65	120	0.9973	62	113	0.9968	59	120
9	65	140	0.9985	64	134	0.9988	65	140
10	65	150	0.9987	60	144	0.9990	64	150
11	70	120	0.9976	67	118	0.9968	59	120
12	70	130	0.9978	67	122	0.9988	66	130
13	70	140	0.9985	69	140	0.9990	65	140
14	70	150	0.9988	67	149	0.9992	70	149
15	75	120	0.9974	70	117	0.9968	59	120
16	75	130	0.9984	68	128	0.9988	66	130
17	75	140	0.9988	71	138	0.9992	71	140
18	75	150	0.9991	73	150	0.9995	70	150

#### IV. CONCLUSION

We determined the best combination and the redundancy level for a case study of the series-parallel system reliability optimization problem and improved our GA implementation using statistical analysis. We used STATISTICA software to do our statistical analysis experimental which gave us to choose the best penalty factor values that improved our GA parameters. The best configuration of 10 simulations obtained gave us the best reliability as one can see in Table V.

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