

# Sensitivity computations of Time Relaxation Model with an application in Cavity Computation

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**Abstract**—We present a numerical study of the sensitivity of the so called time relaxation family of models of fluid motion with respect to the time relaxation parameter  $\chi$  on the two dimensional cavity problem. The goal of the study is to compute and compare the sensitivity of the model using finite difference method (FFD) and sensitivity equation method (SEM).

**Keywords**—sensitivity, time relaxation, deconvolution, Navier-Stokes equations.

## I. INTRODUCTION

**T**HE study of sensitivity has become an important tool in the understanding of fluid behavior. To obtain a meaningful solution for the Navier Stokes equations one needs to work with a fine mesh, which becomes expensive as well as time consuming. Fluid models are developed in order to avoid these obstacles. Even when a fluid flow model has performed well in practice, the reliability of the model is often not addressed [2]. The reliability can be affected if the model displays sensitivity to certain parameters. Sensitivity analysis eliminates the arising uncertainties and provides a reliable interval for the parameters to be chosen from. During the years, there have been investigations on the sensitivity topics, [1]–[3], [8], [13], [14], [16]. Two methods could be used in order to calculate the sensitivities: Forward Finite Difference method (FFD) or the Sensitivity Equation Method (SEM). SEM can be classified into two methods: Continuous Sensitivity Equation Method (CSEM) and Automatic Differentiation Method (ADM). The difference between ADM and CSEM is in the order of operations of discretization and differentiation. CSEM implements differentiation first and then implements discretization, whereas ADM implements discretization first and then uses differentiation, [10]. When using a flow solver code, finite difference quotient is easy to use. Nevertheless, it might not be a reliable way to compute sensitivities of the model, see [6]. When solving sensitivity for the flow using CSEM, when the flow is obtained, only a linear equation needs to be solved in order to compute the sensitivity [1]. CSEM has been extensively used to compute the sensitivities with respect to different regularization parameters, see [5]–[7], [9], [11]. This paper explores the sensitivity of a time relaxation type model with respect to a regularization parameter defined below.

The governing equations of fluid motion are the Navier Stokes equations, which are defined as follows:

$$\begin{aligned} \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \nu \Delta \mathbf{u} &= \mathbf{f}, \text{ in } \Omega \times [0, T] \\ \nabla \cdot \mathbf{u} &= 0, \text{ in } \Omega \times [0, T] \end{aligned}$$

Here  $\mathbf{u}$  and  $p$  represent velocity vector and pressure respectively,  $\nu$  represents the viscosity and  $\mathbf{f}$  represents the body

force. Time Relaxation model (TRM) was introduced by Stolz, Adams and Kleiser and was developed from regularized Chapman-Enskog expansion of conservation laws [21]. The model was computationally tested on compressible flows with shocks and on turbulent flows [17], [18], [21], i.e. on the aerodynamic noise [19]. A continuous finite element analysis for the model along with numerical results can be found in [21]. TRM consists of the Navier-Stokes equations with an addition of a stabilization term:

$$\begin{aligned} \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \nu \Delta \mathbf{u} \\ + \chi(\mathbf{u} - G_N \bar{\mathbf{u}}) &= \mathbf{f}, \text{ in } \Omega \times [0, T] \\ \nabla \cdot \mathbf{u} &= 0, \text{ on } \Omega \times [0, T] \end{aligned} \quad (1)$$

Here,  $\bar{\mathbf{u}}$  represents an averaged function of  $\mathbf{u}$  satisfying:

$$\begin{aligned} -\delta^2 \Delta \bar{\mathbf{u}} + \bar{\mathbf{u}} &= \mathbf{u}, \text{ in } \Omega \\ \bar{\mathbf{u}} &= 0, \text{ on } \partial \Omega \end{aligned} \quad (2)$$

$G_N$  represents the continuous van Cittert deconvolution operator and is defined as follows:

$$G_N \mathbf{u} := \sum_{n=0}^N (I - G)^n \mathbf{u}$$

For order of deconvolution  $N = 0$  and  $N = 1$ , and  $\mathbf{u} \in \mathbf{X}_h$  we have:

$$\begin{aligned} G_0 \mathbf{v} &= \mathbf{v}, \\ G_1 \mathbf{v} &= 2\mathbf{v} - \bar{\mathbf{v}}, \\ G_2 \mathbf{v} &= 3\mathbf{v} - 3\bar{\mathbf{v}} + \bar{\bar{\mathbf{v}}}, \\ G_3 \mathbf{v} &= 4\mathbf{v} - 6\bar{\mathbf{v}} + 4\bar{\bar{\mathbf{v}}} - \bar{\bar{\bar{\mathbf{v}}}}. \end{aligned}$$

Higher order of deconvolution increases accuracy, however it also requires significant computational time [15]. Herein, computations are carried out for the fundamental case, i.e. order of deconvolution  $N = 0$ . Here,  $\chi$  represents the time relaxation coefficient and has units 1/time. Since different values of  $\chi$  will cause different responses of the flow, it is natural to explore how the change of the flow will be affected by altering this parameter. In this paper we obtain sensitivity computations using both FFD and SEM. The sensitivity using FFD is obtained by the formula:

$$\frac{\mathbf{u}(\chi + \Delta\chi) - \mathbf{u}(\chi)}{\Delta\chi} \quad (3)$$

Sensitivity of the solution  $(\mathbf{u}, p)$  with respect to  $\chi$  for the SEM is obtained by differentiating (1) (with  $N = 0$ ) with respect

to  $\chi$ :

$$\begin{aligned} & \mathbf{s}_t + \mathbf{u} \cdot \nabla \mathbf{s} + \mathbf{s} \cdot \nabla \mathbf{u} + \nabla r - \nu \Delta \mathbf{s} \\ & + (\mathbf{u} - \bar{\mathbf{u}}) + \chi(\mathbf{s} - \mathbf{w}) = \mathbf{f}, \text{ in } \times [0, T] \\ & \nabla \cdot \mathbf{s} = 0, \text{ in } \Omega \times [0, T] \\ & s = 0, \text{ on } \partial\Omega \times [0, T] \end{aligned} \quad (4)$$

where  $s = \frac{\partial \mathbf{u}}{\partial \chi}$ ,  $r = \frac{\partial p}{\partial \chi}$  and  $\mathbf{w} = \frac{\partial \bar{\mathbf{u}}}{\partial \chi}$ . Here  $w$  satisfies the following filtering equation:

$$\begin{aligned} -\delta^2 \Delta \mathbf{w} + \mathbf{w} &= \mathbf{s}, \text{ in } \Omega \\ \mathbf{w} &= 0, \text{ on } \partial\Omega \end{aligned} \quad (5)$$

As we see in (4),  $\mathbf{u}$  appears in the sensitivity equation. Hence, in order to obtain the solution for (4) we need to couple (1) with (4).

## II. NUMERICAL SCHEME

This section presents the algorithm in order to numerically solve (1) and (4). The finite element spaces  $(X^h, Q^h)$  are defined respectively below:

$$\begin{aligned} X^h \subset X &= H_0^1(\Omega) := \{v \in H^1(\Omega) : v|_{\partial\Omega} = 0\}, \\ Q^h \subset Q &= L_0^2(\Omega) := \{q \in L^2(\Omega) \mid \int_{\Omega} q = 0\}. \end{aligned}$$

Also, bilinear  $a(\cdot, \cdot) : X \times X \rightarrow \mathbb{R}$  and trilinear  $b^*(\cdot, \cdot, \cdot) : X \times X \times X \rightarrow \mathbb{R}$  forms are defined respectively as follows:

$$\begin{aligned} a(\mathbf{u}, \mathbf{v}) &:= \nu(\nabla \mathbf{u}, \nabla \mathbf{v}), \\ b^*(\mathbf{u}, \mathbf{v}, \mathbf{w}) &:= \frac{1}{2}(\mathbf{u} \cdot \nabla \mathbf{v}, \mathbf{w}) - \frac{1}{2}(\mathbf{u} \cdot \nabla \mathbf{w}, \mathbf{v}) \end{aligned}$$

The Crank-Nicolson method, which is second order approximation in time, is used for the discretization of the time derivative.

Given  $(\mathbf{X}_h, Q_h)$ , end-time  $T > 0$ , the time step is chosen  $\Delta t < T = M\Delta t$ , find the TRM solution  $(\mathbf{u}_h^{n+1}, p_h^{n+1}) \in (X_h, Q_h)$ , for  $n=0,1,2,\dots,M-1$  satisfying:

$$\begin{aligned} & \frac{1}{\Delta t}(\mathbf{u}_h^{n+1} - \mathbf{u}_h^n, \mathbf{v}_h) + \nu a(\mathbf{u}_h^{n+1/2}, \mathbf{v}_h) \\ & + b^*(\mathbf{u}_h^{n+1/2}, \mathbf{u}_h^{n+1/2}, \mathbf{v}_h) - (p_h^{n+1}, \nabla \cdot \mathbf{v}_h) \\ & + \chi(\mathbf{u}_h^{n+1/2} - \bar{\mathbf{u}}_h^{n+1/2}, \mathbf{v}_h) = (\mathbf{f}^{n+1/2}, \mathbf{v}_h), \\ & (\nabla \cdot \mathbf{u}_h^{n+1}, q_h) = 0, \quad \forall q_h \in Q_h \\ & \delta^2(\nabla \bar{\mathbf{u}}_h^{n+1}, \nabla \mathbf{v}_h) + (\bar{\mathbf{u}}_h^{n+1}, \mathbf{v}_h) = (\bar{\mathbf{u}}_h^{n+1}, \mathbf{v}_h), \\ & \quad \forall \mathbf{v}_h \in X_h \end{aligned}$$

and the sensitivity solution

$(\mathbf{s}_h^{n+1}, v_h^{n+1}) \in (X_h, Q_h)$ , for  $n=0,1,2,\dots,M-1$  satisfying:

$$\begin{aligned} & \frac{1}{\Delta t}(\mathbf{s}_h^{n+1} - \mathbf{s}_h^n, \mathbf{v}_h) + \nu a(\mathbf{s}_h^{n+1/2}, \mathbf{v}_h) \\ & + b^*(\mathbf{s}_h^{n+1/2}, \mathbf{u}_h^{n+1/2}, \mathbf{v}_h) + b^*(\mathbf{u}_h^{n+1/2}, \mathbf{s}_h^{n+1/2}, \mathbf{v}_h) \\ & - (r_h^{n+1}, \nabla \cdot \mathbf{v}_h) + (\mathbf{u}_h^{n+1/2} - \bar{\mathbf{u}}_h^{n+1/2}, \mathbf{v}_h) \\ & + \chi(\mathbf{s}_h^{n+1/2} - \mathbf{w}_h^{n+1/2}, \mathbf{v}_h) = (\mathbf{f}^{n+1/2}, \mathbf{v}_h), \\ & (\nabla \cdot \mathbf{s}_h^{n+1}, q_h) = 0, \quad \forall q_h \in Q_h \\ & \delta^2(\nabla \bar{\mathbf{w}}_h^{n+1}, \nabla \mathbf{v}_h) + (\bar{\mathbf{w}}_h^{n+1}, \mathbf{v}_h) = (\bar{\mathbf{s}}_h^{n+1}, \mathbf{v}_h), \\ & \quad \forall \mathbf{v}_h \in X_h \end{aligned}$$

## III. SENSITIVITY COMPUTATIONS

$\chi$	Method	T=0.1	T=1
$\chi = 0.005$	FFD	0.0174288	3.71334
	SEM	0.0173	0.146137
$\chi = 0.05$	FFD	0.01738	4.69683
	SEM	0.0172	0.144227
$\chi = 0.5$	FFD	0.0169013	5.06646
	SEM	0.0169	0.127838
$\chi = 1$	FFD	0.016389	5.42801
	SEM	0.01.658	0.112962
$\chi = 10$	FFD	0.00987	0.0232246
	SEM	0.019	0.0384867

TABLE I

SENSITIVITY OF TRM ON THE FINE MESH  $h = 1/36$  AND  $Re = 10^4$

$\chi$	Method	T=0.1	T=1
$\chi = 0.005$	FFD	$1.73219 \cdot 10^{-5}$	$1.1232 \cdot 10^{-5}$
	SEM	$2.11071 \cdot 10^{-5}$	$1.12607 \cdot 10^{-5}$
$\chi = 0.05$	FFD	$1.73213 \cdot 10^{-5}$	$1.12315 \cdot 10^{-5}$
	SEM	$2.11134 \cdot 10^{-5}$	$1.12542 \cdot 10^{-5}$
$\chi = 0.5$	FFD	$1.73151 \cdot 10^{-5}$	$1.12257 \cdot 10^{-5}$
	SEM	$2.12031 \cdot 10^{-5}$	$1.1247 \cdot 10^{-5}$
$\chi = 1$	FFD	$1.73082 \cdot 10^{-5}$	$1.12194 \cdot 10^{-5}$
	SEM	$2.13586 \cdot 10^{-5}$	$1.13585 \cdot 10^{-5}$
$\chi = 10$	FFD	$1.71862 \cdot 10^{-5}$	$1.1106 \cdot 10^{-5}$
	SEM	$3.07014 \cdot 10^{-5}$	$2.41182 \cdot 10^{-5}$

TABLE II

SENSITIVITY OF TRM ON THE FINE MESH  $h = 1/36$  AND  $Re=1$

The experiment is done on the two-dimensional cavity problem with the flow domain  $\Omega = [0, 1] \times [0, 1]$ . The boundary and initial conditions for the problem are  $u(t, x, y) = (16x^2(1-x^2), 0)^t$  and  $u(0, x, y) = (3y^2 - y, 0)^t$  respectively. Since initial and boundary conditions have no dependence on  $\chi$ , they are set to zero for the sensitivity  $\mathbf{s}$ . In the computations for the cavity problem we use  $\nu = 0.0001$  and  $\nu = 1$ , implying  $Re = 10^4$  and  $Re = 1$  respectively. The time step is chosen to be  $\Delta t = 0.01$  and we use Taylor-Hood finite elements. To carry out the computations we use finite element method software Freefem++ [20]. The FFD computations are done with (3) and  $\Delta\chi = 0.01$ . We let  $s_{FFD}$  and  $s_{SEM}$  represent the sensitivity using FEM and SEM respectively. The following tables represent  $\|s_{FFD}(t)\|_{L^2(\Omega)}$  and  $\|s_{SEM}(t)\|_{L^2(\Omega)}$  for different parameter settings.

According to the results obtained from tables I-II and tables III-IV, the sensitivity is greatly decreased when a lower Reynolds number is used. Also, at the final time, the difference between FFD and SEM becomes more noticeable.

Tables V and VI computed the maximum sensitivity via SEM. It shows that this model is not strongly sensitive to the parameter  $\chi$  for this benchmark problem. Table VII shows the execution time for FFD and SEM for different values of  $\chi$ . The table suggests that SEM is computed significantly faster than the FFD and we believe that it is more accurate [6]. Figures 1 and 2 below show the similarity/difference of computations via SEM and FFD.

$\chi$	Method	T=0.1	T=1
$\chi = 0.005$	FFD	0.023147	5.43861
	SEM	0.0231474	0.255481
$\chi = 0.5$	FFD	0.0223875	2.54603
	SEM	0.0223875	0.214977
$\chi = 1$	FFD	0.220898	0.81
	SEM	0.216504	0.1804
$\chi = 10$	FFD	0.0213324	0.01951
	SEM	0.0156569	0.043

TABLE III  
SENSITIVITY OF TRM ON A COARSE MESH  $h = 1/16$

$\chi$	Method	T=0.1	T=1
$\chi = 0.005$	FFD	$1.04284 \cdot 10^{-4}$	$9.80413 \cdot 10^{-5}$
	SEM	$1.01806 \cdot 10^{-4}$	$9.77097 \cdot 10^{-5}$
$\chi = 0.5$	FFD	$1.01786 \cdot 10^{-4}$	$9.80238 \cdot 10^{-5}$
	SEM	$1.04208 \cdot 10^{-4}$	$9.75814 \cdot 10^{-5}$
$\chi = 1$	FFD	$1.01584 \cdot 10^{-4}$	$9.78493 \cdot 10^{-5}$
	SEM	$1.0354 \cdot 10^{-4}$	$9.64125 \cdot 10^{-5}$
$\chi = 10$	FFD	$1.01362 \cdot 10^{-4}$	$9.7656 \cdot 10^{-5}$
	SEM	$1.03001 \cdot 10^{-4}$	$9.53558 \cdot 10^{-5}$

TABLE IV  
SENSITIVITY OF TRM ON A COARSE MESH  $h = 1/16$

$\chi$	T=0.1	T=1
0.005	0.000116971	0.000116971
0.05	0.000116965	0.000116965
0.5	0.000116906	0.000116906
1	0.000116841	0.000116841
10	0.000115714	0.000115714

TABLE V  
MAX. SENSITIVITY VIA SEM WITH  $h = 1/36$  AND  $Re = 10^4$

$\chi$	T=0.1	T=1
0.005	0.0173037	0.146137
0.05	0.0172701	0.144227
0.5	0.0169409	0.127838
1	0.0165878	0.0940275
10	0.0119636	0.0384867

TABLE VI  
MAX. SENSITIVITY VIA SEM WITH  $h = 1/36$  AND  $Re = 1$

$\chi$	0.005	0.05	0.5	1	10
FFD	353.94	354.92	431.89	357.02	366.72
SEM	177.84	205.61	187.05	179.41	179.23

TABLE VII  
COMPUTATIONAL TIME USING FFD AND SEM FOR WITH  $T = 0.1$ ,  
 $Re = 10^4$  AND  $h = 1/36$

#### IV. CONCLUSIONS AND FUTURE DIRECTIONS

Sensitivity of the time relaxation model to the time relaxation coefficient  $\chi$  on a two dimensional cavity problem was computed via SEM and FFD methods. The numerical comparison between the methods shows that the sensitivity is the highest about  $\chi = 1$ . Also, on a coarser mesh we can see that the sensitivity becomes larger than on the finer mesh, which agrees with the assumption that we need finer mesh for sensitivity computations [2]. Also, sensitivity is higher for larger  $Re$  based on our obtained results. Overall, based on



Fig. 1. Similarity of SEM (top) and FFD (bottom) at  $T = 0.1$  with  $\chi = 0.5$ ,  $h = 1/36$

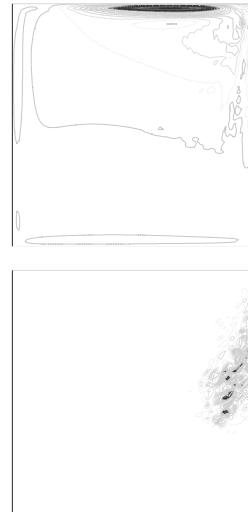


Fig. 2. Difference of SEM (top) and FFD (bottom) at  $T = 1$  with  $\chi = 0.5$ ,  $h = 1/36$

the maximum sensitivity results, the TRM does not seem to have strong sensitivity to the parameter  $\chi$  for this benchmark problem. The further studies will include stochastic finite element discretization, which should give more insights into the parameter sensitivity. Other benchmark problems will be investigated as well.

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