Semi-Analytic Solution and Hydrodynamics Behavior of Fluid Flow in Micro-Converging plates

A. Al-Shyyab and A. F. Khadrawi

Abstract—The hydrodynamics behavior of fluid flow in micro-converging plates is investigated analytically. Effects of Knudsen number (***Kn***) on the microchannel hydrodynamics behavior and the coefficient of friction are investigated. It is found that as ***Kn*** increases the slip in the hydrodynamic boundary condition increases. Also, the coefficient of friction decreases as ***Kn*** increases.

Keywords—Converging plates, hydrodynamic behavior, micro-plates, microchannel, slip velocity

I. INTRODUCTION

MICRO-electromechanical devices (MEMs) refer to devices and products whose characteristic length is less than 1 mm but larger than 1 micrometer. MEMs devices are usually part of integrated systems where microchannel flow and heat transfer are found. Micropumps, microturbines, micro heat exchangers, micronozzles, microvalves, micropipes and ducts, biological cells separation reactors and blood analyzers are just a few examples of micro channel flow and heat transfer. Ever continuous demand of miniaturizing devices and gadgets to increase power/weight ratio of products and tools in biological, medical, manufacturing processes, aerospace and many other industrial applications increased the interest of researchers to model and simulate such devices to better understand its physics and the governing laws as well as the fundamental differences of these physics and laws when regarding microscale devices rather than macroscale ones and the dominant parameters and factors imposing these differences. Experimental works have showed that conventional physics of fluid flow and heat transfer described successfully the macroscale devices and processes break down regarding microgeometry flow and heat transfer, where surface-to-volume ratio becomes drastically large and rarified gas flow with nonlinear pressure drop for gaseous flow and granular flows of liquids are observed [1], [2].

Continuum flow model hereafter refer to Navier-Stokes equations and conservation of mass (second-order differential equations). These equations are valid as long as the flow is near thermodynamic equilibrium, where conventional linear constitutive relations of shear stress-strain rate can be expressed in terms of first order differential gradients of flow velocity. The usual no-slip boundary condition becomes invalid as approaching microflows before continuum models break down. For gaseous flow, the continuum flow models as

well as the no-slip velocity at the flow boundaries fail as some characteristic length of the flow gradients (L) approaches the molecular mean free path λ. The ratio of this characteristic length to the mean free path is known as Knudsen number $Kn, (Kn = \lambda/L)$, it describes the degree of flow rarefication, hence, validity of the continuum model where rarefication effects are ignored. Therefore, for $Kn < 0.001$ continuum equation subject to no slip boundary conditions are accurate. For $0.001 < Kn < 0.1$ continuum model equations are still valid, yet subject to velocity-slip boundaries to account for fraction of diffuse (randomly reflect) molecular collisions with the solid boundaries which are quantified by tangential-momentum-accommodation-coefficient. Beyond this range of Kn the continuum model breaks down. For $0.1 < Kn < 10$, the flow is in transition towards rarefied gas flow, where $Kn > 10$ distinguishes rarefied free molecular flow [3].

Rarefication and compressibility are the most dominant factors affect microchannel flow, while effects due to non-parabolic velocity profile and acceleration have secondary effects and can be neglected [4]. Even the number of published microchannel flow research [5], [6] is large, and the mature kinetic theory of gases has helped researchers interpret many aspects of microchannel gas flow, many parts of the physical laws governing heat transfer and the fluid flow are still unknown or needs rigorous experimental investigations. In a review paper by Ho and Tai [7], discrepancies between macroscale Stokes flow theory and microchannel flow behavior were summarized. There has been an enormous interest in research works of micro-system over the last decade [8], [17]. It is widely accepted that microchannel gas flow rarefication introduced as a velocity slip and temperature jump is responsible for the deviations of the microgeometry flow behavior compared with the macroscale one [18]-[25].

The objective of this study is to analytically investigate the gaseous flow characteristics in converging micro plate flow problem. In particular, this study will focus on the effect of Knudsen number (Kn) on the velocity slip at the wall(s), flow coefficient of friction C_f and the spatial velocity components distribution.

II. MATHEMATICAL MODEL

Consider an incompressible fluid flow between two converging plates toward a sink as shown in Fig.1. The flow is assumed to be non-compressible Newtonian fluid. The governing equations of this flow are continuity, and momentum equations. The continuity equation is,

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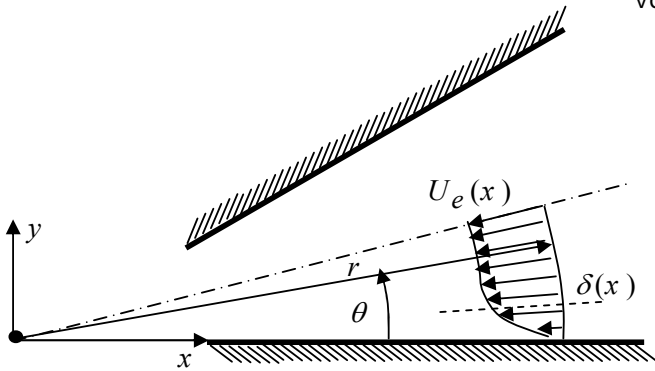
Fig. 1 Convergent channel flow, point sink flow

A. Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

And the momentum equations are,

B. x-momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

Applying this momentum component outside the boundary layer to get

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{(UL)^2}{x^3}, \tag{3}$$

Where the free stream velocity outside the boundary layer is

$$U_e(x) = -\frac{U\,L}{x}, \tag{4}$$

Here, U is the free stream velocity at some characteristic length L defined as distance along x-axis from the fluid sink.

C. y-momentum

$$\frac{\partial p}{\partial x} = 0, \tag{5}$$

By use of stream function $\psi(x,y) = -\sqrt{UL\nu}f(\eta)$, and similarity variable $\eta = y/x\sqrt{(UL/\nu)}$, the flow dimensionless velocity components that satisfy the continuity equation would be defined as $u/U_e = \partial\psi/\partial y = f'$ and $v/U_e = \eta\sqrt{\nu/UL}\,f'$, where "prime" denotes differentiation with respect to η. With these definitions the fluid governing equations are reduced to third order nonlinear ordinary differential equation. Multiply this equation by f' and simplify to get,

$$\left(\frac{f''^2}{2}\right)' - \left(\frac{f'^3}{3}\right)' + (f')' = 0 \tag{6}$$

Integrate the last equation (Eq. 2a) and use the flow conditions $f'(\eta \to \infty) \to 1$, and $f''(\eta \to \infty) \to 0$, to get,

$$\left(\frac{f''^2}{2}\right) - \left(\frac{f'^3}{3}\right) + f' = \frac{2}{3}, \tag{7}$$

The no-slip boundary conditions represent a macroscopic Newtonian fluid flow. It solution is well established. However, for micro-fluid flow the velocity component along the boundary ($y = 0$) can be assumed to be proportional to its gradient along the boundary normal to account for rarefication effects, where,

$$u(x,0) = \beta\,\lambda\,\frac{\partial u(x,0)}{\partial y}, \text{ or } f'(0) = \beta\,Kn\sqrt{\frac{UL}{\nu}}f''(0) \tag{8}$$

Where $Kn = \lambda/x$ is the Knudsen number and $\beta = (2 - \sigma_v)/\sigma_v$.

Substituting Eq. (8) into Eq. (7) to get,

$$\frac{1}{2}\frac{\nu}{UL}\left(\frac{f'(0)}{\beta\,Kn}\right)^2 - \frac{1}{3}\big(f'(0)\big)^3 + f'(0) = \frac{2}{3}, \tag{9}$$

The roots of the last equation satisfy the first order velocity slip boundary conditions. Let's denote these roots by $F_i, (i = 1, 2, 3)$.

The Integration of Eq. (2b) yields,

$$\eta = -\sqrt{2}\,tanh^{-1}\left(\sqrt{\frac{2+f'(\eta)}{3}}\right) + C, \tag{10}$$

Applying the slip boundary represented by the roots of Eq. (9), $f'(0) = F_i$, to get the values of the integration constant C_i as,

$$C_i = \sqrt{2}\,tanh^{-1}\left(\sqrt{\frac{2+F_i}{3}}\right), i = 1,2,3, \tag{11}$$

Rewrite Eq. (10) as,

$$f'(\eta) = 3\,tanh^2\left(\frac{\eta}{\sqrt{2}} + tanh^{-1}\left(\sqrt{\frac{2+F_i}{3}}\right)\right) - 2,\ i = 1,2,3 \tag{12}$$

Further integration of the last equation is not needed as both velocity components depend on the first derivative f'(η).

The dimensionless coefficient of friction C_f can be written as,

$$C_f\big|_{y=0} = \frac{\mu\partial u/\partial y}{\rho\,U_e^2/2} = 2\sqrt{\frac{\nu}{U\,L}}f''(0), \tag{12}$$

III. RESULT AND DISCUSSIONS

The physical parameters of the problem are assigned typical values of light gases, $\nu = 352(10)^{-6}m^2s^{-1}, \alpha = 519(10)^{-6}m^2s^{-1}$, and $\sigma_v = \sigma_T = 0.8$. The free stream velocity $U = 25ms^{-1}$ at $L = (10)^{-6}m$. All figures include the conventional no-slip/jump boundary conditions $(0 < Kn < 0.001)$ for comparison sake.
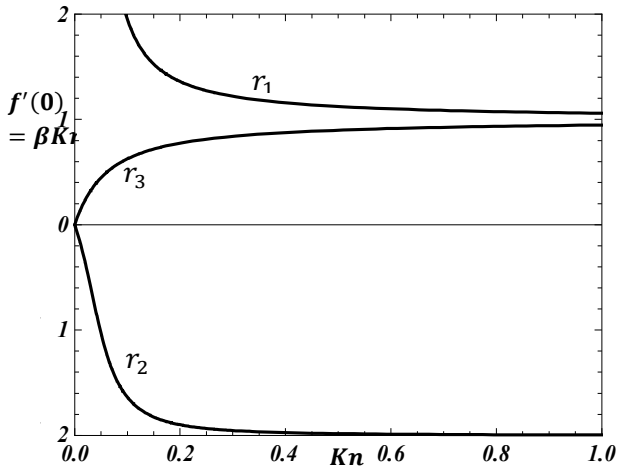
Fig. 2 three values of $f'(0)$ that satisfy the mathematical definition of the flow slip boundary conditions of Eq. (9)

The solution of the problem depends on the roots of Eq. (9). Fig. 2 shows the three roots r_1, r_2, and r_3, ($f'(0) \in [r_1, r_2, r_3]$), as a function of Knudsen number Kn. Root r_1 contradicts the flow physics, since it does imply the flow
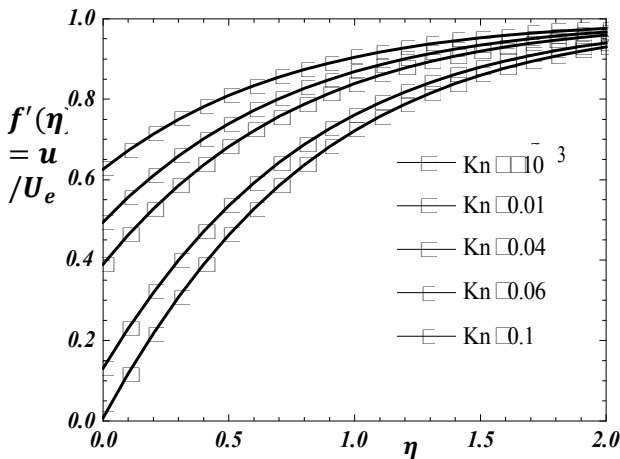

Fig. 3 variations of the dimensionless radial velocity $f'(\eta)$ as a function of Kn

velocity near the wall boundaries is faster than the velocity of the driving potential ($f'(0) > 1$), therefore it is discarded. Similarly, root r_2 is discarded since the direction of the rarefied flow velocity near the walls is opposite to the driving potential velocity direction ($f'(0) < 0$), which is not possible for flow between converging plates. The solution which corresponds to r_3, ($f'(0) = r_3$), is the only solution that satisfies the rarefied flow physics.
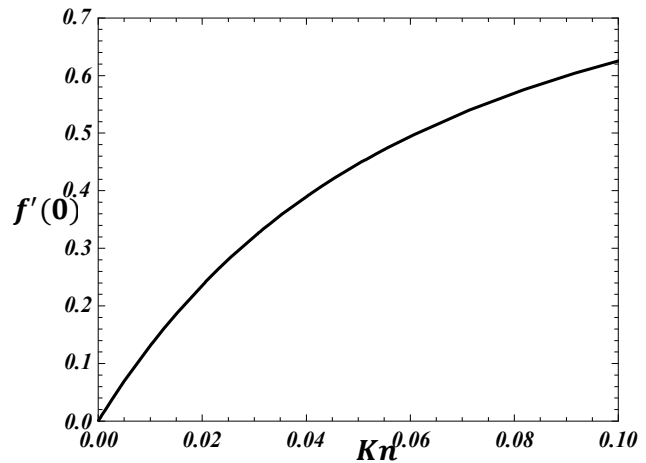

Fig. 4 dimensionless slip velocity $f'(0)$ as a function of Kn

The increase of Kn increases the ratio of specular reflections of molecules impinging the boundary walls, therefore retaining larger amount of their tangential momentum and consequently reduces the wall retarding viscous shear forces. Discrepancies among different velocity profiles correspond different Kn are shown in Fig. 3. The velocity profiles far from the boundaries approach the no-slip velocity profile. However, approaching the boundary walls,
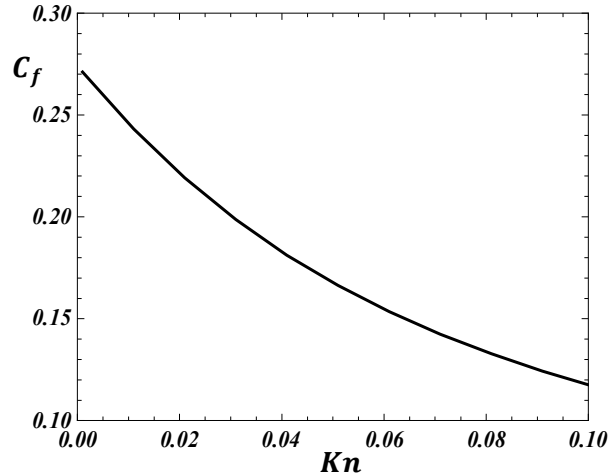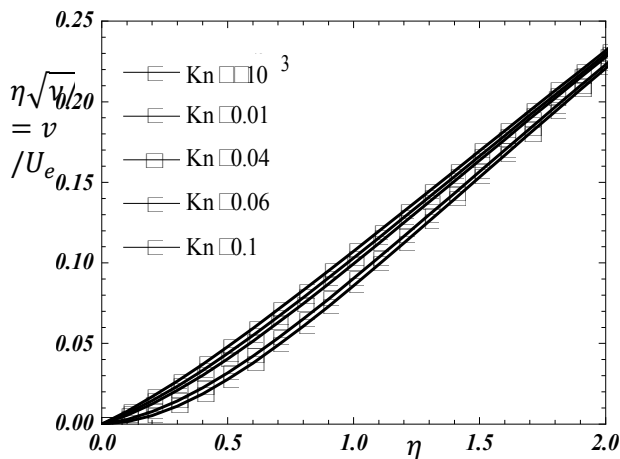

Fig. 5 variations of the coefficient of friction C_f as a function of Kn

the flow velocity increases and departs the no-slip velocity profile, this is because molecular rarefication measured by Kn increases. The velocity slip measured as difference between

Fig. 6 transverse velocity variations as a function of Kn

the wall and the flow tangential velocity ($f'(0)$) is shown in Fig. 4. The slip velocity importance stems from the fact that it is relevant to the surface coefficient of friction. Fig. 5 shows the dependence of the coefficient of friction C_f on Kn. C_f decreases as Kn increases because Kn is a measure of specular molecular collisions with the boundary. The effect of Kn on the transverse velocity component (v/U_e) is slight as shown in Fig. 6.

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IV. CONCLUSIONS

Effects of Knudsen number on the flow velocity components and the coefficient of friction are investigated. It is found that as Kn increases the slip in the boundary condition increases. Also, the coefficient of friction decreases as Kn increases. The effects of Kn on the transverse velocity component is slight.

REFERENCES

[1] Gad-el-Hak, Mohamed, *MEMs Introduction and Fundamentals,* 2ed ed. Taylor and Frances Group,LLC, 2006, ch. 4.
[2] G. Karniadakis, M. Beskok*, Micro flows fundamentals and simulation*. New York: Springer 2002.
[3] Y. Zohar, Heat convection in micro ducts. Kluwer 2003.
[4] Y. Zohar, W. Lee, S. Lee, L. Jiang, and P. Tong, "Subsonic gas flow in a straight and uniform microchannel,"*J. Fluid Mech*. 472, pp. 125-151, 2002.
[5] C. Ho, Y. Tai, Micro-electro-mechanical systems (MEMS) and fluid flows. Ann. Rev. Fluid Mech. 30, 579612, 1998.
[6] M. A. Al-Nimr, and A. F. Khadrawi, Thermal behavior of a stagnant gas convicted in a horizontal microchannel as described by the dual-phase-lag heat conduction model, Int. J. Thermophysics, Vol. 25, pp. 1953, 2004.
[7] A. F. Khadrawi, A. Othman and M. A. Al-Nimr, Transient free convection fluid flow in a vertical microchannel as described by the hyperbolic heat conduction model, Int. J. Thermophysics, Vol. 26, pp.905, 2005.
[8] O. M. Haddad, M. M. Abuzaid and M. A. Al-Nimr, Entropy generation due to laminar incompressible forced convection flow through parallel-plates microchannel, Entropy, Vol. 6(5), pp. 413-426, 2004.
[9] O. M. Haddad, M. A. Al-Nimr, and M. M. Abuzaid, The effect of frequency of fluctuating driving force on basic slip micro-flows, Acta Mechanica , Vol. 179, pp. 249-259, 2005.
[10] O. M. Haddad, M. M. Abuzaid, and M. A. Al-Nimr, Developing free convection gas flow in a vertical open-ended micro-channel filled with porous media, Numerical Heat Transfer, Part A, Vol. 48 (2005).
[11] O. M., Haddad, M. A. Al-Nimr, and Y. Taamneh, Hydrodynamic and thermal behavior of gas flow in micro channels filled with porous media, J. Porous Media, Vol. 9(5), pp. 403-414, 2006.
[12] O. M., Haddad, M. A. Al-Nimr, and M. M. Abuzaid, Effect of periodically oscillating driving force on basic microflows in porous media, Journal Porous Media, Vol. 9(7), pp. 695-707, 2006.
[13] O.M. Haddad, M. A. Al-Nimr, and J. Sh. Al-Omary, Forced convection of gaseous slip-flow in porous micro-channels under local thermal non-equilibrium conditions, Transport in Porous Media, Vol. 67(3), pp. 453-471, 2007.
[14] O.M. Haddad, M. A. Al-Nimr, and M. Sari, Forced convection gaseous slip flow in circular porous micro-channels, Transport in Porous Media, Vol. 70(2), pp. 167-179 , 2007.
[15] J. Al-Jarrah, A. F. Khadrawi, and M. A. Al-Nimr, Film condensation on a vertical micro-channel, Int. Communication in Heat and Mass Transfer, Vol. 35(9), pp. 1172-1176, 2008.
[16] P. Wu, W. A. Little, Measurement of friction factors for the flow of gases in very fine channels used for microminiature Joule Thompson refrigerators. Cryogenics 23, 273277, 1983.
[17] S. B. Choi, R. F. Barron, R. O. Warrington, Fluid flow and heat transfer in microtubes. Micromech. Sensors Actuators Sys. 32, 123134 1991.
[18] J. C. Harley, Y. Huang, H. Bau, J. N. Zemel, Gas flows in micro-channels. J. Fluid Mech. 284, 257274, 1995.
[19] S. F. Choquette, M. Faghri, E. J. Kenyon, B. Sunden, Compressible fluid flow in micron-sized channels. Nat. Heat Transfer Conf. 5, 2532 1996.
[20] Z. Y. Guo, X. B. Wu, Compressibility effects on the gas flow and heat transfer in a microtube. Int. J. Heat Mass Transf. 40, 32513254 ,1997.
[21] S. Kiwan, and M. A. Al-Nimr, "Flow and Heat Transfer over a Stretched Micro-Surface", ASME J Heat Transfer, Vol. 131(6), 2009.
[22] M. A., Al-Nimr, A. M., Maqapleh, A. F. Khadrawi, and Ammourah S. A.: "Fully developed thermal behaviors for parallel flow microchannel heat exchanger", International Communications in Heat and Mass Transfer Vol. 36, pp 385–390, 2009.
[23] K. Bataineh, and M. A. Al-Nimr, 2-D "Navier–Stokes Simulations of Microscale Viscous Pump with Slip Flow", ASME Journal of Fluids Engineering, Vol. 131, Issue 5, 2009.
[24] G. P. Duncan, G. P. Peterson, Review of microscale heat transfer. Appl. Mech. Rev. 47(9), 397428, 1994.
[25] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces(Translation Journals style)," *IEEE Transl. J. Magn.Jpn.*, vol. 2, Aug. 1987, pp. 740–741 [*Dig. 9th Annu. Conf. Magnetics* Japan, p. 301,1982.

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