

Robust Iterative PID Controller Based on Linear Matrix Inequality for a Sample Power System

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Abstract—This paper provides the design steps of a robust Linear Matrix Inequality (LMI) based iterative multivariable PID controller whose duty is to drive a sample power system that comprises a synchronous generator connected to a large network via a step-up transformer and a transmission line. The generator is equipped with two control-loops, namely, the speed/power (governor) and voltage (exciter). Both loops are lumped in one where the error in the terminal voltage and output active power represent the controller inputs and the generator-exciter voltage and governor-valve position represent its outputs. Multivariable PID is considered here because of its wide use in the industry, simple structure and easy implementation. It is also preferred in plants of higher order that cannot be reduced to lower ones. To improve its robustness to variation in the controlled variables, H_∞ -norm of the system transfer function is used. To show the effectiveness of the controller, divers tests, namely, step/tracking in the controlled variables, and variation in plant parameters, are applied. A comparative study between the proposed controller and a robust H_∞ LMI-based output feedback is given by its robustness to disturbance rejection. From the simulation results, the iterative multivariable PID shows superiority.

Keywords—Linear matrix inequality, power system, robust iterative PID, robust output feedback control

I. INTRODUCTION

MODERN control theory is desirable to improve the a.c. turbogenerator system performance and overcome limitations in stability boundaries caused by the use of larger generator size with lower specific inertia, and also with longer transmission lines [1,2].

One of the difficulties faced when considering the application of optimal control theory relies in the measurement of all state variables when state feedback control is considered. This burden is reduced by using output feedback control instead [3,4].

PID controller is widely used in the industry owing to its simple structure [5,6], easy implementation and found to be adequate for most plants. However, it is not robust to disturbances in the controlled variables and system parameters change.

Many control problems and design specifications have constraints expressed as LMIs. This is especially true for Lyapunov-based analysis and design, optimal LQG control, H_∞ -control, covariance control, etc. [7,8]. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

Three factors make LMI techniques appealing:

- A variety of design specifications and constraints can be expressed as LMIs.
- Once formulated in terms of LMIs, a problem can be solved exactly by efficient convex optimization algorithms
- While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework.

Robust output-feedback control using H_∞ -norm is robust and widely preferred when the minimization of the effect of the disturbance on selected outputs is sought. However, due to its complexity in implementation and its order that is usually the same as the plant one, it is not highly desired. The design of such controllers is efficiently handled with Linear Matrix Inequalities (LMI) technique [8,9].

This paper presents the design steps of a robust iterative LMI-based multivariable PID (abbreviated: PID) whose effectiveness is evaluated through a comparison with a robust LMI-based H_∞ -controller (abbreviated: ROB) [3,10,11]. Both voltage and speed/power control-loops are lumped in one forming a Multi-Input Multi-Output (MIMO) controller. The controllers are used in conjunction with a synchronous generator connected to an infinite-bus through a step-up transformer and a transmission line [12]. Both controllers are tested using step/tracking in the control variables and variation in the plant parameters. The test results show superiority of the first (PID) over the second (ROB). Simulation is done using Matlab platform and LMI toolbox [13].

II. SYSTEM MODELING

Fig. 1 shows the block diagram of the controlled sample power system that comprises a steam turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The output real power P_t and terminal voltage V_t at the generator terminals are measured and fed to the controller. The outputs of the controller (system control inputs) are fed into the generator-exciter and governor-valve.

In the simulation studies described here, the nonlinear equations of the synchronous generator are represented by a third-order nonlinear model based on park's equations [1]. The steam turbine, governor valve and exciter are each represented by a first order-order model. The model equations are as follows. The data and symbols nomenclature are defined in the Appendix [14].

$$\begin{cases} \dot{x}_{CL} = A_{CL}x_{CL} + B_{CL}w \\ z = C_{CL}x_{CL} + D_{CL}w \end{cases} \quad (4)$$

be the corresponding closed-loop state-space equations with

$$x_{CL} = [x \quad \zeta]^T \quad z = e$$

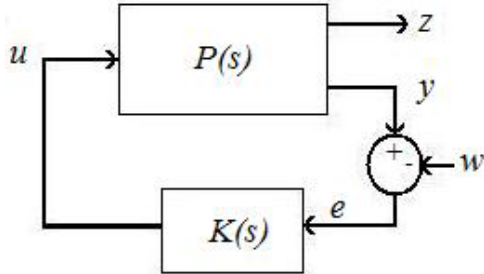


Fig. 2 Output feedback block diagram

The design objective for finding $K(s)$ is to minimize the H_∞ -norm, i.e., disturbance rejection or minimization of the effect of the worst-case disturbance on the output z , of the closed-loop transfer function $G(s)$ from w to z , i.e.,

$$G(s) = C_{CL}(s - A_{CL})^{-1}B_{CL} + D_{CL} \quad (5)$$

$$\|G(s)\|_{zw} < \gamma$$

using LMI technique [12,15-17]. This can be fulfilled if and only if there exists a symmetric matrix X such that the following LMIs are satisfied

$$\begin{pmatrix} A_{CL}X + XA_{CL}^T & B_{CL} & XC_{CL}^T \\ B_{CL}^T & -I & D_{CL}^T \\ C_{CL}X & D_{CL} & -\gamma^2 I \end{pmatrix} < 0 \quad (6)$$

$X > 0$

IV. ITERATIVE MULTIVARIABLE PID

Consider the linear time-invariant given by (1), i.e.,

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (7)$$

With the following PID controller [10]

$$u = F_1 y + F_2 \int_0^t y dt + F_3 \frac{dy}{dt} \quad (8)$$

Where

x	state variables
u	control inputs
y	outputs
A, B and C	matrices with appropriate dimensions
F_1, F_2, F_3	matrices to be designed.

Let $z_1 = x$ and $z_2 = \int_0^t y dt$. Denote $z = [z_1^T \ z_2^T]^T$. The

variable z can be viewed as the state vector of a new system whose dynamics is governed by

$$\dot{z}_1 = \dot{x} = Az_1 + Bu, \quad \dot{z}_2 = y = Cz_1 \quad (9)$$

i.e.

$$\dot{z} = \bar{A}z + \bar{B}u \quad (10)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Combining (7) and (9) yields

$$y = Cz_1 = [C \ 0]z, \quad \int_0^t y dt = z_2 = [0 \ I]z$$

$$\frac{dy}{dt} = C\dot{x} = CAx + CBu = [CA \ 0]z + CBu.$$

Let

$$\bar{C}_1 = [C \ 0], \quad \bar{C}_2 = [0 \ I], \quad \bar{C}_3 = [CA \ 0]$$

Then,

$$\bar{y}_i = \bar{C}_i z \quad (i=1-3)$$

Thus we have

$$u = F_1 \bar{y}_1 + F_2 \bar{y}_2 + F_3 \bar{y}_3 + F_3 CBu \quad (11)$$

Suppose $(I - F_3 CB)$ is invertible. Let

$$\bar{y} = [\bar{y}_1^T \ \bar{y}_2^T \ \bar{y}_3^T]^T$$

$$\bar{F} = [\bar{F}_1 \ \bar{F}_2 \ \bar{F}_3]$$

$$\bar{C} = [\bar{C}_1^T \ \bar{C}_2^T \ \bar{C}_3^T]^T$$

$$\bar{F}_1 = (I - F_3 CB)^{-1} F_1$$

$$\bar{F}_2 = (I - F_3 CB)^{-1} F_2$$

$$\bar{F}_3 = (I - F_3 CB)^{-1} F_3$$

The problem of PID controller design reduces to that of Static Output Feedback (SOF) [10,14,17] controller design for the following system:

$$\dot{z} = \bar{A}z + \bar{B}u$$

$$\bar{y} = \bar{C}z$$

$$u = \bar{F}\bar{y}$$

Once $\bar{F} = [\bar{F}_1 \ \bar{F}_2 \ \bar{F}_3]$ is found, the original PID gains can be recovered from

$$F_3 = \bar{F}_3(I + CB\bar{F}_3)^{-1}$$

$$F_2 = (I - F_3 CB)\bar{F}_2$$

$$F_1 = (I - F_3 CB)\bar{F}_1$$

V. ITERATIVE PID H ∞ (IPIDHI)

The design problem of PID controllers under H ∞ performance specification is handled by first considering the system (1) rewritten as [10-14] (Fig. 3):

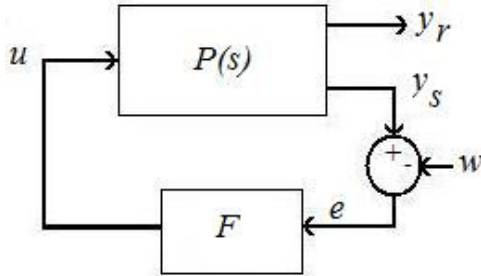


Fig. 3 Iterative PID block diagram

$$P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ y_s = C_s x \\ y_r = C_r x + Du \end{cases} \quad (13)$$

where

x state variable
 u system inputs (manipulated by the controller)
 w system inputs (here: reference of controlled variables, not manipulated by the controller)
 y_s vector of sensed or measured outputs
 y_r vector of regulated or controlled outputs

A, B_1, B_2, C_s , and C_r are matrices with appropriate dimensions.

The static output feedback H ∞ control problem is to find a controller of the form

$$u = F e \quad (14)$$

such that the ∞ -norm of the closed-loop transfer function from w to y_r is stable and limited as follows:

$$\|G_{wy_r}\|_{\infty} < \gamma \quad (15)$$

An iterative LMI algorithm is developed as follows [10,14]:

Step 0: Form the system state space realization: $(A, B_1, B_2, C_s, C_r, D)$ and select the performance index γ then compute $\bar{A}, \bar{B}_1, \bar{B}_2, \bar{C}_s, \bar{C}_r$ as defined in [10].

Step 1: Choose $Q_0 > 0$ and solve P for the Riccati equation:

$$\bar{A}^T P + P \bar{A} - P \bar{B}_2 \bar{B}_2^T P + Q_0 = 0, \quad P > 0$$

Set $i = 1$ and $X_i = P$

Step 2: Solve the following optimization problem for P_i, \bar{F} and α_i .

OP1: Minimize α_i subject to the following LMI constraints

$$\begin{bmatrix} \Sigma & P_i \bar{B}_1 & (\bar{C}_r + D \bar{F} \bar{C}_s)^T & (\bar{B}_2^T P_i + \bar{F} \bar{C}_s)^T \\ \bar{B}_1^T P_i & -\gamma & 0 & 0 \\ \bar{C}_r + D \bar{F} \bar{C}_s & 0 & -I & 0 \\ \bar{B}_2^T P_i + \bar{F} \bar{C}_s & 0 & 0 & -I \end{bmatrix} < 0 \quad (16)$$

$$P_i > 0$$

Where

$$\Sigma = \bar{A}^T P_i + P_i \bar{A} - X_i \bar{B}_2 \bar{B}_2^T P_i - P_i \bar{B}_2 \bar{B}_2^T X_i + X_i \bar{B}_2 \bar{B}_2^T X_i - \alpha_i P_i$$

Denote by α_i^* the minimized value of α_i .

Step 3: If $\alpha_i^* \leq 0$, the matrix pair (P_i, \bar{F}) solves the problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for P_i and \bar{F} .

OP2: Minimize $\text{trace}(P_i)$ subject to LMI constraints (16) with $\alpha_i = \alpha_i^*$. Denote by P_i^* the optimal P_i .

Step 5: If $\|X_i \bar{B} - P_i^* \bar{B}\| < \varepsilon$, where ε is a prescribed tolerance, go to Step 6; otherwise set $i := i + 1$, $X_i = P_i^*$, and go to Step 2.

Step 6: It cannot be decided by this algorithm whether the problem is solvable. Stop.

Now, the dynamics of the newly obtained closed-loop system have the form:

$$\dot{z} = \bar{A} z + \bar{B}_1 w + \bar{B}_2 u$$

$$\bar{y}_s = \bar{C}_s z$$

$$\bar{y}_r = \bar{C}_r z + D u \quad (17)$$

$$u = \bar{F} \bar{y}_s$$

Where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$$

$$\bar{C}_s = [C_s \quad 0] \quad \bar{C}_r = [C_r \quad 0]$$

Thus the feedback matrices $\bar{F} = (\bar{F}_1, \bar{F}_2, \bar{F}_3)$ can be obtained by applying the above Algorithm to system (17).

VI. SIMULATION RESULTS

To demonstrate the effectiveness of PID controller while driving the plant, several tests are carried out and the results are presented and compared with those of ROB. The simulation results are obtained using MATLAB package and LMI Toolbox.

A. Parameters of ROB controller:

Initial condition (operating point):

$$x_0 = [0.775 \quad 0 \quad 1.434 \quad -0.0016 \quad 0.8 \quad 0.8]$$

Plant $P(s)$:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -37.6 & -1.5 & -26 & 0 & 0 & 29.6 \\ -0.3 & 0 & -0.56 & 314 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 1.25 & -1.25 \end{bmatrix}$$

$$B_1 = 0_{6 \times 2} \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.1 \\ 18.89 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.27 & 0 & 0.88 & 0 & 0 & 0 \\ 0.03 & 0 & 0.53 & 0 & 0 & 0 \end{bmatrix}$$

Controller $K(s)$:

$$A_k = \begin{bmatrix} -999 & -335 & -226 & 700 & -362 & -1649 \\ -363 & -235 & -60 & 499 & 1176 & 542 \\ -426 & -75 & -86.7 & 312 & 14 & -550 \\ 518 & -86 & 81 & -379 & -128 & 516 \\ 164 & -441 & -24 & -93.8 & -217 & -114 \\ -871 & -1138 & -274 & 653 & -340 & -1801 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -8.11 & -2.46 \\ -3.75 & -1.58 \\ 7.15 & 5.23 \\ -45.7 & -30.3 \\ -96.1 & -61 \\ -176 & -235 \end{bmatrix}$$

$$C_k = \begin{bmatrix} 62.7 & 20.2 & 13.2 & -49.2 & -8.2 & 76.6 \\ -773 & 776 & -370 & -1672 & -12047 & -11561 \end{bmatrix}$$

$$D_k = 0$$

Desired H_∞ -norm: $\gamma=100$

Optimum H_∞ -norm: $\gamma_{\text{opt}} = 7.8603$

Closed-loop eigenvalues:

$$\lambda_{\text{CL}} = [-15370, -103 \pm 436i, -229, -10, -4.7, -2.2 \pm 2.7i, -1.3 \pm 2.8i, -0.63, -1]$$

B. Parameters of PID controller:

The controller gains are:

$$\bar{F}_1 = \begin{bmatrix} 0.6 & -9 \\ 29 & -146 \end{bmatrix}$$

$$\bar{F}_2 = \begin{bmatrix} -1.3 & -3.8 \\ 0.34 & -58 \end{bmatrix}$$

$$\bar{F}_3 = \begin{bmatrix} 0.8 & -1.8 \\ 15 & -31 \end{bmatrix}$$

In compact form:

$$\bar{F} = [\bar{F}_1 \quad \bar{F}_2 \quad \bar{F}_3]$$

Riccati starting matrix: $Q_0 = 10$

Desired dominant eigenvalue: $\alpha_{\text{opt}} = 0$

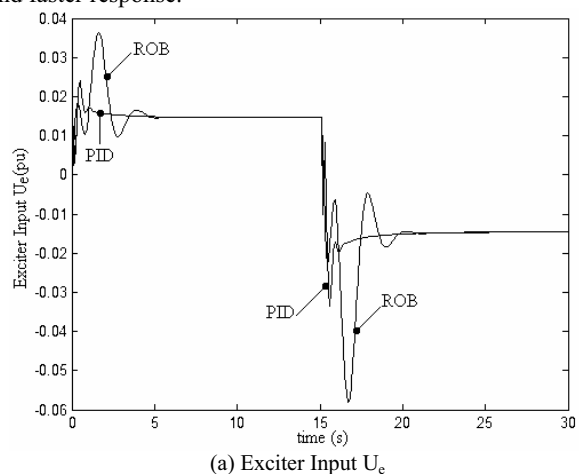
Obtained dominant eigenvalue: $\alpha_{\text{opt}} = -0.77$

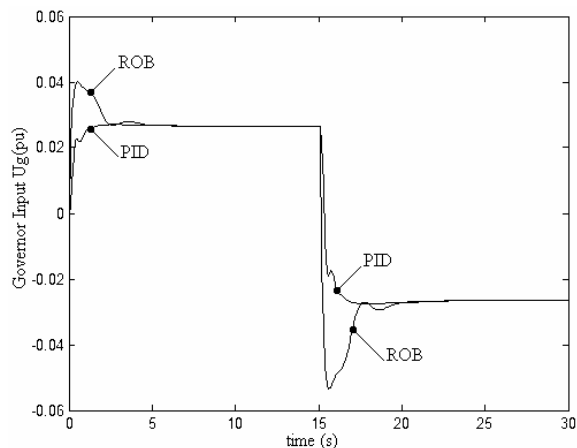
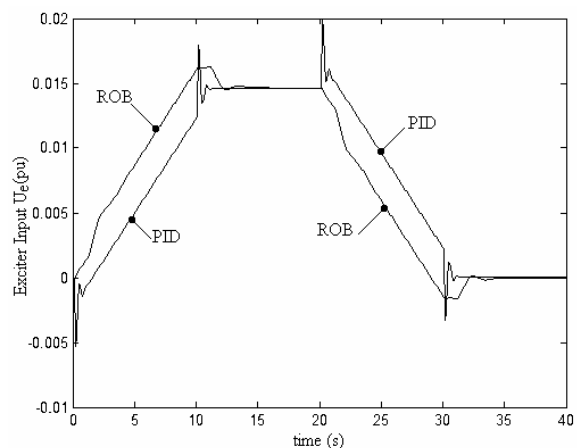
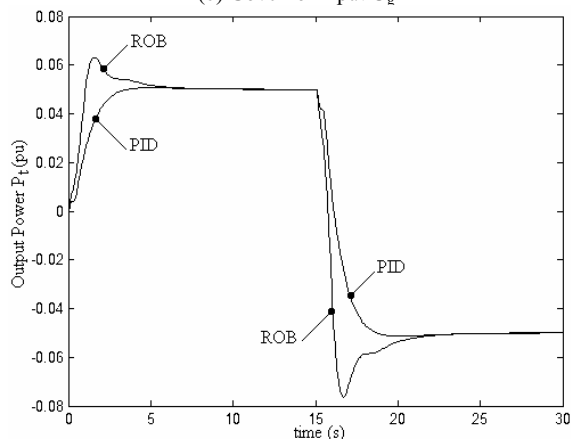
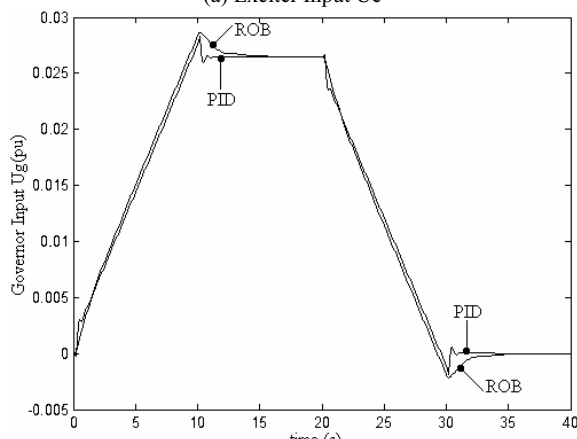
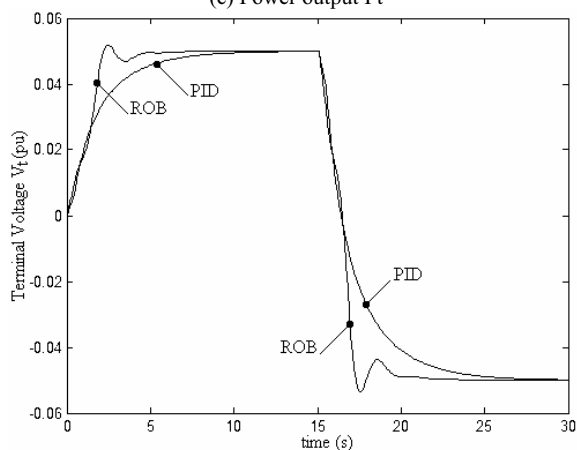
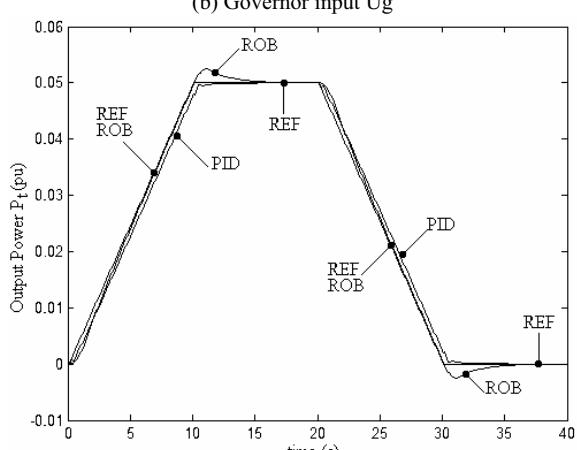
Closed-loop eigenvalues:

$$\lambda_{\text{CL}} = \begin{bmatrix} -111 \\ -3.26 \pm 10.3i \\ -9.2 \pm 4.4i \\ -7.4 \\ -0.85 \pm 0.31i \end{bmatrix}$$

Test 1: Step-response

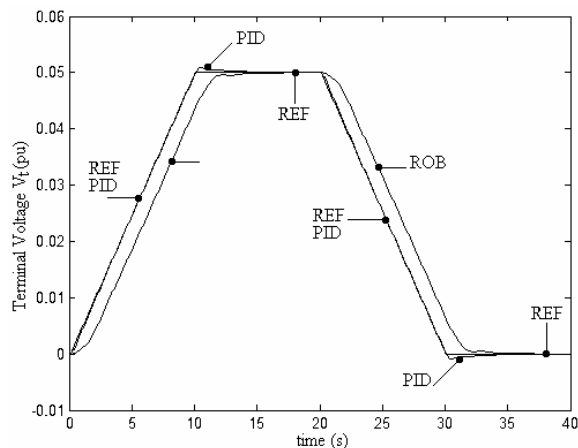
To test the effectiveness of the system equipped with (ROB, PID), one at a time, the system is subjected to an increase then a decrease by 5% in both Pref and Vref. The time responses of the exciter input voltage U_e , the governor valve position U_g , the output active power P_t , and the terminal voltage V_t , are shown in Fig. 4. Clearly, the PID shows better response for all mentioned variables. It shows good performance characterized by lower or no overshoot, less or no oscillations, and faster response.



(b) Governor input U_g (a) Exciter Input U_e (c) Power output P_t (b) Governor input U_g (d) Terminal voltage V_t (c) Power output P_t Fig. 4 Step-response following $P_{ref}=V_{ref}=5\%$

Test 2: Tracking-response

To test the effectiveness of the system to tracking the reference control values, the simulation period is divided into 4 regions where the reference values of the controlled variables vary as shown in Fig. 5(c) and the system responses of U_e , U_g , P_t and V_t are shown in Fig. 5(a) through Fig. 5(d). As in Test1, PID shows robustness and superiority with respect to ROB.

(d) Terminal voltage V_t Fig. 5 System responses with H_{∞} /PID due to reference tracking (test 2)

Test 3: Parameters Variation

To test the robustness to parameters change, a decrease by 5% in the inertia constant H and an increase by 50% in the field transient time constant τ_d are applied. Fig. 6 shows the system response following a step change by 5% then -5% in V_{ref} and T_{ref} with the system experiencing the described parameters change. It is clear that the system equipped with PID shows better performance than ROB from overshoot, oscillations and settling time point of view.

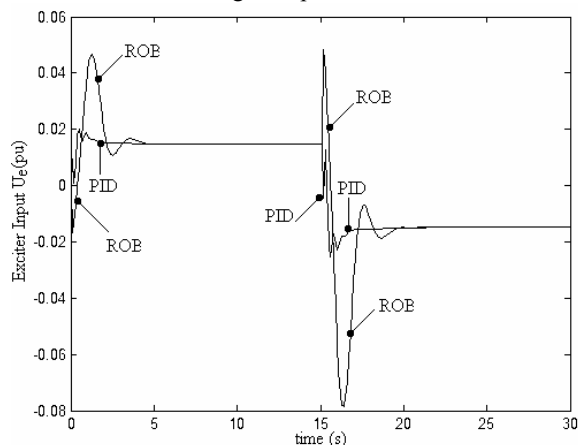
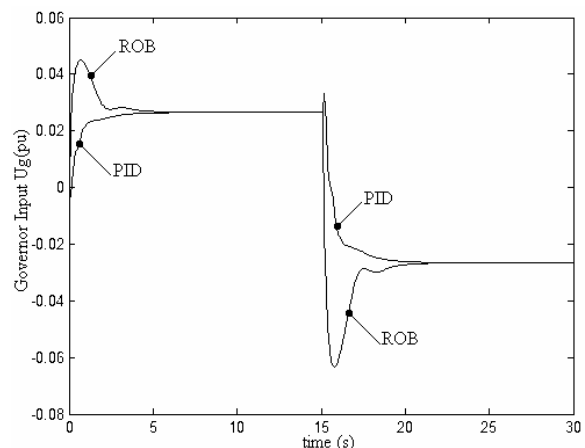
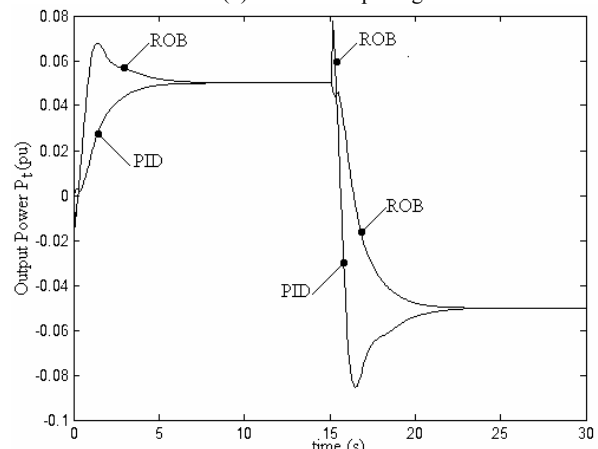
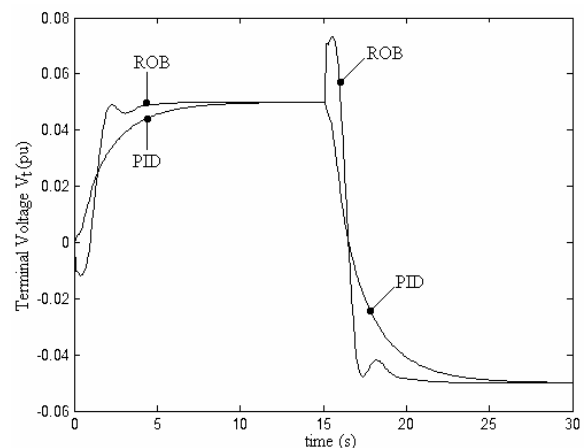
(a) Exciter Input U_e (b) Governor input U_g (c) Power output P_t (d) Terminal voltage V_t

Fig. 6 System responses with parameters change (test 3)

VII. CONCLUSION

Two controllers, a robust H_∞ -LMI based output feedback (ROB) and a H_∞ -LMI based iterative-LMI multivariable PID (PID), were designed for a sample power system comprising a steam turbine driving a synchronous generator connected to an infinite bus via a step-up transformer and a transmission line. The goal was to use the first (ROB) as a reference in evaluating the robustness of the second (PID) over the first (ROB).

From the simulation results, it is clear that PID shows better performance. Moreover, it is of a lower order, simple structure, and easier implementation.

As an extension, the performance of the PID via H_2 -norm optimization and pole placement, the extension to a multimachine power system, and the inclusion of the nonlinear features inherent in the system, will be considered in the future.

APPENDIX

A. List of symbols

V_d, V_q	<i>stator voltage in d-axis and q-axis circuit</i>
V_t	terminal voltage
ψ_{fd}	field flux linkage
x_{ad}	stator-rotor mutual reactance
x_{fd}	self reactance of field winding
V_{fd}	field voltage
r_{fd}	field resistance
e	busbar voltage resistance
U_e	input to exciter
δ	rotor angle, rad
T_e/T_m	electrical / mechanical torque
P_s	steam power
H	inertia constant
ω	angular frequency of rotor
ω_0	angular frequency of infinite busbar
K_d	mechanical damping torque coefficient
T_d	damping torque coefficient due to damper windings
P_t, P_b	real power output at terminals and busbar
τ_e	exciter time constant
τ_g	governor valve time constant
τ_b	turbine time constant
U_g	input to governor
G_v	governor valve position
K_v	valve constant

B. System Parameters

MVA	37.5
MW	30
p.f.	0.8 lagging
kV	11.8
r/min	3000
x_d	2 pu
x_q	1.86 pu
x_{ad}	1.86 pu
x_{fd}	2 pu

R_{fd}	0.00107 pu
H	5.3 MWs/MVA
T_d	0.05 s
x_t	0.345 pu
x_l	0.125 pu
e	1 pu
τ_e	0.1 s
τ_g	0.1 s
τ_b	0.5 s
K_v	1.889
K_e	0.01
K_1	1.2564
K_2	-0.9218
K_3	-0.5609
K_4	0.4224
K_5	0.7983
K_6	0.5905
K_7	0.3650
K_8	-39.559
K_9	-27.427
K_{10}	-0.2955
K_{11}	1.268
K_{12}	0.8791
V_d	0.5586
V_q	1.1076
V_t	1.2405
K_{13}	0.0287
K_{14}	0.52726

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