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Robust Integrated Navigation of a Low Cost System

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Abstract**—**Robust nonlinear integrated navigation of GPS and low cost MEMS is a hot topic of research these days. A robust filter is required to cope up with the problem of unpredictable discontinuities and colored noises associated with low cost sensors. H∞ filter is previously used in Extended Kalman filter and Unscented Kalman filter frame. Unscented Kalman filter has a problem of Cholesky matrix factorization at each step which is a very unstable operation. To avoid this problem in this research H∞ filter is designed in Square root Unscented filter framework and found 50% more robust towards increased level of colored noises.

Keywords**—**H_∞ filter, MEMS, GPS, Nonlinear system, robust system, Square root unscented filter.

I. INTRODUCTION

THIS research designs a robust stable filter tolerant towards increased levels of non white Gaussian noises. The most popular approaches of nonlinear filtering i.e., Kalman filter (KF) and Unscented Kalman filter (UKF) have been utilized in H∞ filter framework in [1] using game theory approach. The results with UKFH∞ were found better than EKFH∞ as UKF is more robust with initial condition errors and gives 3rd to 4th order of accuracy with nonlinear system. On the other hand the UKF needs Cholesky matrix factorization of all covariance matrices which requires these to be positive definite at each time step. Sometimes this condition is violated when noises are non Gaussian and colored, to resolve this problem Square root unscented filter (SRUKF) was developed in [2] by utilizing three methods of linear algebra. This filter is so far the most stable form of UKF. In this paper H∞ filter is utilized in SRUKF frame to increase robustness margins [3] and called SRUKFH∞.

II. H∞UKF AND H∞SRUKF FILTERS

H∞UKF filters are described in [4]. Reference [4] utilized modified recursive Riccati equation to incorporate H∞ norm bound in standard UKF set up described in [5].The new state and covariance update equation in UKF set up can be given as (1)-(2).

$$P_k = P_{k-1} - [P_{X_kY_k} \quad P_{k-1}] \begin{bmatrix} P_{Y_k} + I & {P_{X_kY_k}}^T \\ P_{X_kY_k} & P_{k-1} - \gamma^2 I \end{bmatrix} [P_{X_kY_k} \quad P_{k-1}]^T \quad (1)$$

$$X_k = X_{k|k-1} + P_{X_kY_k}(R + P_{Y_k})^{-1}(Y_k - Y_{k|k-1}) \quad (2)$$

Here P_k and P_{k-1} is system covariance matrix. $P_{X_kY_k}$ is cross covariance matrix of measurement and state. γ is the designer selected value for bounded variance. This modification gives significant improvement in the presence of colored noise. For H∞ filter, as in [4] the condition in (3) must hold:

$$P_k^{-1} = P_{k-1}^{-1} + H_k^T R^{-1} H_k - \gamma_k^{-2} I > 0 \quad (3)$$

Here H_k is the observation mapping matrix and R is measurement covariance matrix, so we can choose γ for UKF H∞ Filter as in (4):

$$\gamma_k^2 = \alpha_{\max}[eig(P_{k-1}^{-1} + P_{k-1}^{-1} P_{X_kY_k} R^{-1} (P_{k-1}^{-1} P_{X_kY_k})^T)^{-1}] \quad (4)$$

Here γ should be greater than 1.To apply robust recursive Riccati equations in Square root unscented filter set up, this research has used (5)-(8).

Square root of $P_{X_kY_k}$ can be given as in (5):

$$S_{xyk} = S_{xk} H_k^T \quad (5)$$

And U can be calculated as in (6):

$$U = \left[S_{xyk} \quad S_{xk}\right] \begin{bmatrix} S_{yk} & {S_{xyk}}^T \\ S_{xyk} & -\gamma S_k + S_{xk} \end{bmatrix}^{-1} \left[S_{xyk} \quad S_{xk}\right]^T \quad (6)$$

Here S_k is the scaling matrix. The rest of the calculation of SRUKF remains same as described in [2].To ensure stability the factor γ can be given as in (7):

$$\gamma_k^2 = \alpha \max[eig((S_{xk} * {S_{xk}}^T)^{-1} + H_k^T R^{-1} H_k)^{-1})] \quad (7)$$

III. RESULTS AND DISCUSSION

A trajectory with bigger (low dynamic loops) and smaller (high dynamic loops) is generated with 6Dof software. The signals are corrupted using colored noises generated through (8):

$$\eta_{AR} = -1.5\eta_{AR}(t-1) - 0.75\eta_{AR}(t-2) - 0.125\eta_{AR}(t-3) + w + 0.5 \quad (8)$$

The level of noise is selected by multiplying signal to noise ratio of white noise w as 10,20,30,40,50.These values are added into values of accelerometer and gyro data at each time stept.The different parameter values can be given as:

$$x_k = [\delta r, \delta v, \delta q, \delta_{acc}, \delta_{gyr}]$$

$$P_k = diag[(4)^2,(4)^2,(4)^2,(4)^2,(4)^2,(4)^2,(0.5)^2,(0.5)^2,(0.5)^2,(0.5)^2 \\ ,(9.81e^{-3})^2,(9.81e^{-3})^2,(9.81e^{-3})^2,(2.4241e^{-5})^2, \\ (2.4241e^{-5})^2,(2.4241e^{-5})^2,.......]$$

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$$Q_k = diag[1e^{-6}, 1e^{-6}, 1e^{-6}, 1e^{-13}, 1e^{-13}, 1e^{-13} ....]$$

$$R_k = diag[(2)^2, (2)^2, (2)^2, (2)^2, (2)^2, (2)^2]$$

$$\alpha_{\max} = 1.1342$$

$$S_k = diag[0.5, 0.5, 0.5, 0.7, 0.7, 1, 1, 0.5, 0.5, 1, 4, 1, 1, 10, 1, 1]$$

$$\alpha = 0.01, \beta = 2, \kappa = 3 - n$$

Here x_k is the state vector and all the rest are tuning parameter of filter as in [2] and [4]. The new filter SRUKFH∞ results are compared with UKFH∞ results with up to 5 times noise levels. Later is found more robust, whereas former diverged. Fig. 1 shows this result. Fig. 2 shows performance of two filters in the presence of GPS outage of two minutes at different interval. The system state vector is composed of 16 parameters, three positions in local frame, three velocities, four quaternion, and six random biases of accelerometer and gyro.

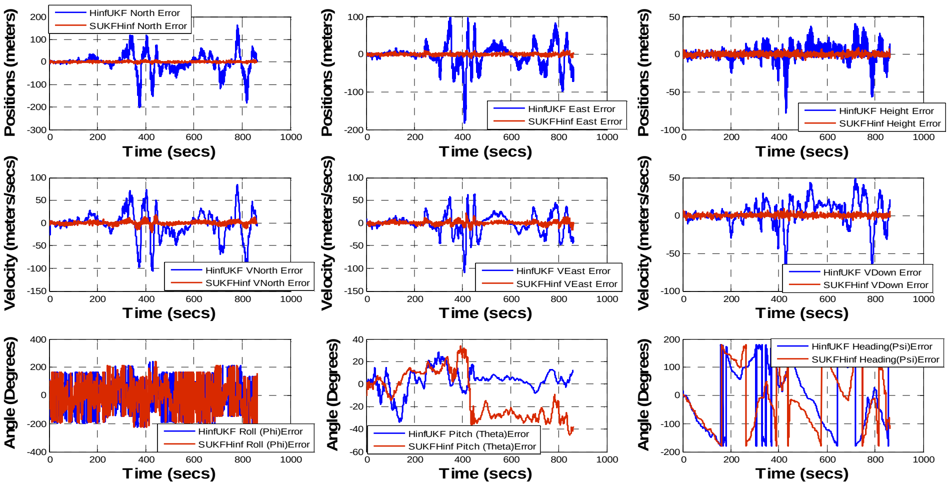

Fig. 1 UKFH∞/ SRUKFH∞ performance with 5x noises

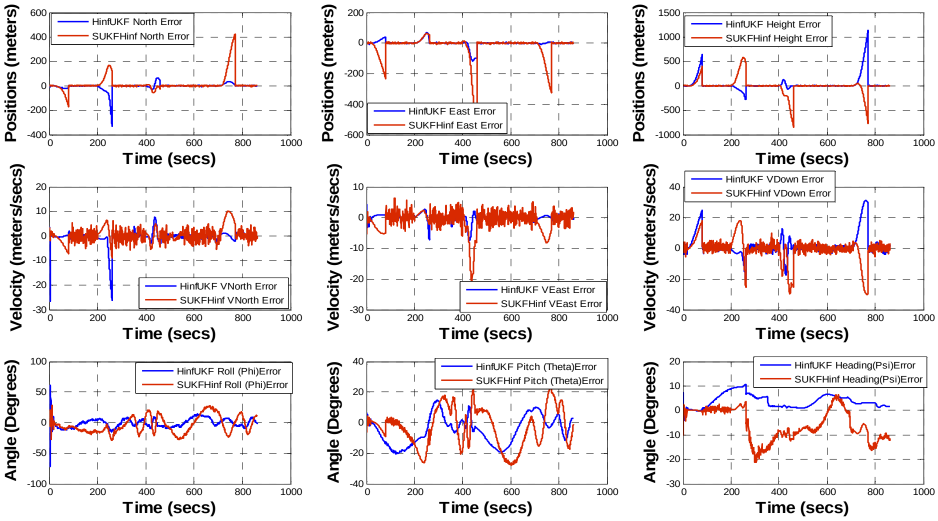

Fig. 2 UKFH∞/ SRUKFH∞ performance with GPS outage

A Monte Carlo analysis was made with increased level of noises with both UKFH∞ and SRUKFH∞ .The system simulated is composed of commercial grade MEMS IMU and a GPS receiver of moderated accuracy whose data is available at a rate of 1Hz with frequent outages. The accuracy of gyro and accelerometer is 5º/hr and 1mg respectively. The mechanization equation of this system can be given in [6].

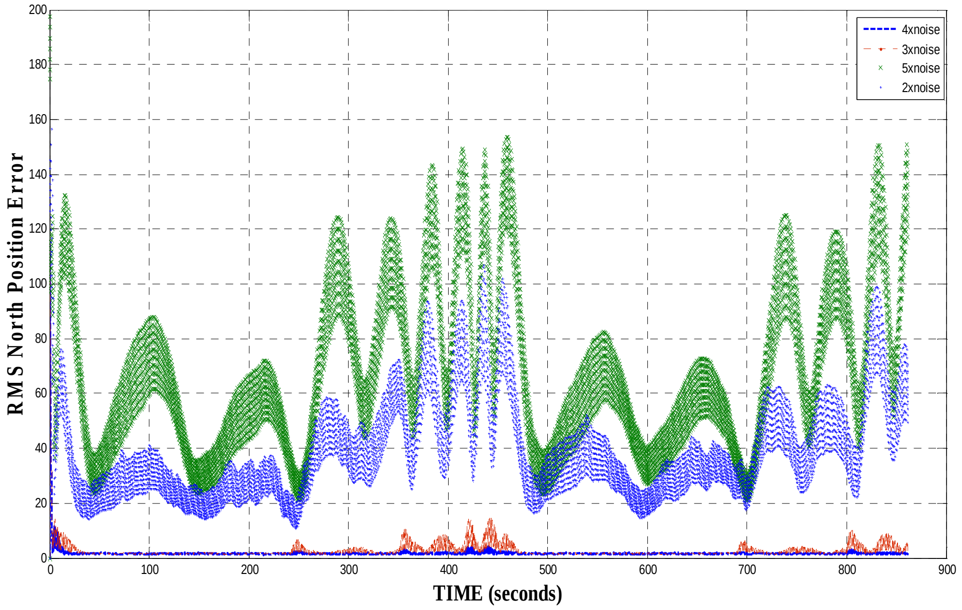

Fig. 3 UKFH∞ performance with 100 runs of Monte Carlo simulations in presence of different level of noises

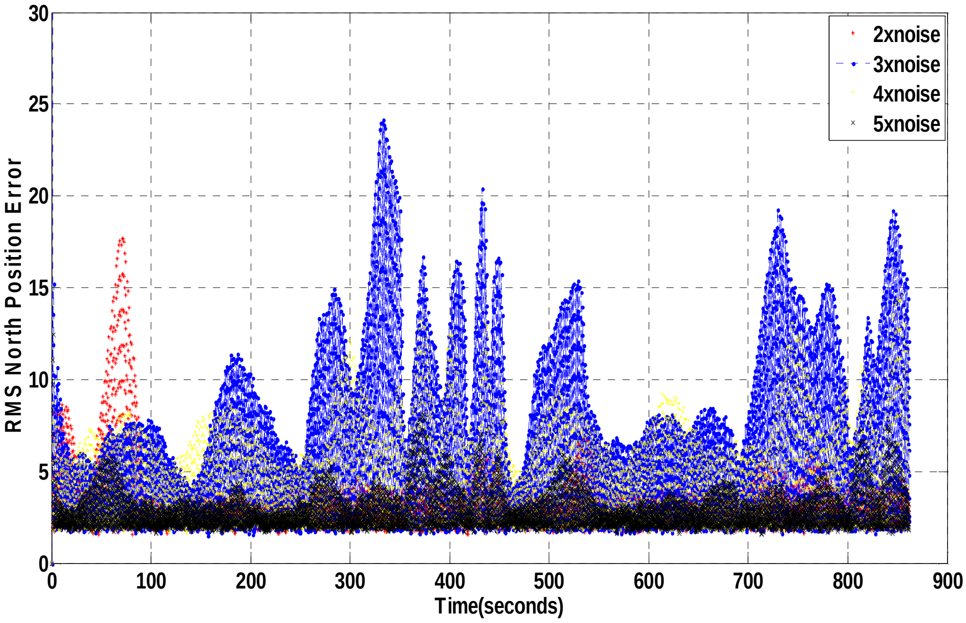

Fig. 4 SRUKFH∞ performance with 100 runs of Monte Carlo simulations in presence of different level of noises

Fig. 3 and Fig.4 show the results.. The result clearly shows that SRUKFH∞ are better and more stable towards increased level of noises and computation time is also relatively less than former approach. Table I summarizes these results.

TABLE I
COMPARISON OF H∞SRUKF AND H∞UKF

Noise level	H∞SRUKF		H∞UKF	
	CPU Time	STD	CPU Time	STD
0.01	~29sec	3.59	~31sec	3.532
0.03	~29sec	4.941	~31sec	3.913
0.09	~29sec	3.934	~31sec	17.61
0.3	~29sec	3.387	~31sec	29.71

IV. CONCLUSION

This study showed that as SRUKFH∞ is the most stable form of UKF filters, with lower level of noises it performed equally well as UKFH∞ but as the noise levels increased its performance remain consistent and the former filter started diverging and some simulations stopped due to violation of condition of positive definitiveness of covariance matrices. On the other hand in GPS outages UKFH∞ was better than new filter. Further work can be done to improve the computation complexity by using any switching technique.

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