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Robust Ellipse Detection by Fitting Randomly Selected Edge Patches

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***Abstract*—**In this paper, a method to detect multiple ellipses is presented. The technique is efficient and robust against incomplete ellipses due to partial occlusion, noise or missing edges and outliers. It is an iterative technique that finds and removes the best ellipse until no reasonable ellipse is found. At each run, the best ellipse is extracted from randomly selected edge patches, its fitness calculated and compared to a fitness threshold. RANSAC algorithm is applied as a sampling process together with the Direct Least Square fitting of ellipses (DLS) as the fitting algorithm. In our experiment, the method performs very well and is robust against noise and spurious edges on both synthetic and real-world image data.

***Keywords*—**Direct Least Square Fitting, Ellipse Detection, RANSAC

I. Introduction

THE ellipse detection is one of the key areas in shape analysis. It is a challenging problem to detect this geometric primitive in real-world images. Incomplete ellipse due to partial occlusion, noise or missing edges and outliers are typical in such images. Moreover, accurately detecting multiple intersecting ellipses is a complex task that requires a large computational resource. It is still a challenging task to solve all these problems.

A number of techniques have been proposed to detect or fit ellipses to edge data. They can be divided into two main groups: voting-based and searching-based techniques. The first group employs the idea of selecting model parameter based on the number of data supports. It is robust to outliers and noises, and can detect multiple ellipses simultaneously. However, the group suffers inaccuracy in the result and demands a huge computational resource. Hough transform (HT) and its variants such as Randomize Hough transform (RHT) [1], [2], the combinatorial Hough transform [3], the probabilistic Hough transform [4], the dynamic generalized Hough transform [5] are the main members of the group.

The searching-based category relies on finding model parameters that optimize its objective function. The objective function selected is normally based on how well the model fits to the data and how likely the model is an acceptable ellipse.

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The main benefits of the method are accuracy of the fitted model and efficiency; however, it only fits to one ellipse. There are also chances of getting local minima solutions, in other words chances that the most reasonable ellipse is undetected. To alleviate the problem, global search methods were proposed by some researchers. An example is the genetic algorithm for ellipse detection by Lutton and Martinez [6]. Techniques in this group normally introduce a scheme to make it capable of detecting multiple ellipses; however, this comes with an additional computing expense. For example, Yao [7] proposed a multiple populations Genetic Algorithm (GA) to detect multiple ellipses. Recently, to reduce computational complexity and local minima problems, some researchers turn to the idea of selecting good model candidates from sample of data instead of independently selecting a configuration to suit the data. Soetedjo and Yamada [8] introduced a method called "geometric fragmentation" to select a good set of boundary fractions for road sign detection; however, it is doubtful how the method could deal with partial occlusions. Kawaguchi and Nagata [9] selected good data set called "line-support regions" using gradient orientation in combination with GA. However, fault grouping and eliminating of line-support regions may result in undetected ellipses. Song and Wang [10] presented a Pseudo Random Sample Consensus (PRANSAC) method to detect ellipses. Only three sampling points are selected from one connected curve to find an ellipse parameter. The main weakness of the method is the ability to detect intersecting ellipses.

In this paper, a novel method to detect multiple ellipses is presented. It is based on randomly selecting good data set to fit an ellipse. RANSAC algorithm [11] is applied as a sampling process together with the Direct Least Square fitting of ellipses (DLS) [12] as the fitting algorithm. Nevertheless, our ellipse fitting framework is very close to PRANSAC [10], however, with an additional support for intersecting ellipses. Comparison results are in Experimental Results section.

II. Proposed method

An overview of our method is shown in Fig. 1. It is an iterative method that finds and removes the best ellipse until no reasonable ellipse is found. An edge map is supplied as the input data set for our algorithm. At each run, the best ellipse is extracted, its fitness calculated and compared to a fitness threshold. If the fitness value exceeds the threshold, data points close to the identified ellipse (within a prespecified distance) are removed. The process stops when the best fitness value of the last run falls below the threshold.

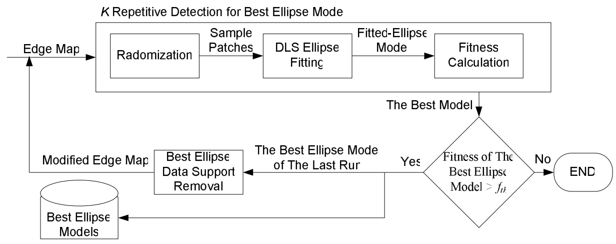


Fig. 1 System Flow Chart

At each run, the best ellipse is extracted by a *K*-repetition process of randomly selecting two patches, fitting them to find a model estimate and computing the model fitness. *K* is the expected number of iterations required to obtain the best model parameter as described in section A. The fitting procedure and the fitness calculation are explained in sections B and C respectively.

A. Randomization

The algorithm for sampling edge points in our work is the RANSAC [11]. RANSAC is an algorithm for robust fitting of models in the presence of data outliers. In our method, *K* randomizations taken in each run, are obtained from (1). is the probability of at least one random sample is free from outliers, *err* is the proportion of outliers, and *s* is the minimum points for model fitting requirement.

$$K = \frac{\log(1- \ )}{\log(1-(1-err)^s)} \tag{1}$$

At each iteration, two points from the input data are randomly selected. The distance between the points must exceed 2*r* to proceed to the next step; otherwise, the points are reselected. Then all connected edge points to each of the selected points, which have their distances to their corresponding selected point within radius r, are chosen for the ellipse fitting explained in the next section.

B. irect Least quare itting of Ellipse

The *irect Least quare itting of Ellipse* (DLS) was first proposed by Andrew et al. [12]. Halif and Flusser [13] further elaborated the DLS mathematic for more stability. It is an efficient method to obtain an accurate fitting result. The mathematic form used in DLS is $(x)=ax^2+bxy+cy^2+dx+ey+f = 0$ which is a linear form of general conic equation. It can be hyperbola, parabola, circle or ellipse, depending on parameter conditions. The ellipse constrain is $b^2 \ 4ac < 0$. Fitzgibbon et al. [12] developed a least squares fitting algorithm of ellipse by applying an equal constraint $4ac\text{-}b^2=1$ as (2). **D** (n x 6) is the design matrix, n is a number of sampling points in each sampling data set, is the parameter vector of conic equation, and **C** (*constraint matrix*) is the integrated-ellipse-constraint matrix (6x6).

$$\min_{a} \sum_{i=1}^{n} \ (x_i, y_i)^2 = \min \|\mathbf{D} \ \|^2 \text{ subject to } \quad \mathbf{C} \ = 1 \tag{2}$$

To be specific,

$$\mathbf{D} = \begin{pmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^2 & x_i y_i & y_i^2 & x_i & y_i & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_n y_n & y_n^2 & x_n & y_n & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a & b & c & d & e & f \end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The solution can be determined by solving an eigenproblem to find the solutions (λ) of

$$\|\mathbf{D} \ \|^2 = \ ^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{D} \tag{4}$$

Eigenvector $_k$, corresponding to the minimal positive eigenvalue λ_k is the solution of (2).

C. itness Calculation

The ellipse fitting at each iteration is verified by calculating a fitness value. This value can be regarded as the ratio of non-occluded edge length to the ellipse perimeter. The fitness is computed from (5). *N* is the number of data points with shortest (perpendicular) distances to the ellipse perimeter of less than *d*.

$$fitness = \frac{N}{\text{perimeter of ellipse}} \tag{5}$$

III. Experimental Results

In this section, we present performance of our method and PRANSAC in terms of speed and accuracy of the solution on synthesis data and real-world images. The numbers of sampled points (*s*) for PRANSAC is 3, while ours is 5 (f=1). In addition, the other parameters of PRANSAC and ours are =0.99 and *err*=0.50. Therefore, the numbers of iterations of PRANSAC and ours are 35 and 146, consequently.

A. roposed Method versus RAN AC

Frameworks of proposed method and PRANSAC are shown in Table I.

TABLE I
PRANSAC AND PROPOSED METHOD FRAMEWORKS

PRANSAC	Proposed Method
1. Edge Detection	1. Edge Detection
2. Component Labeling	2. Ellipse Fitting (Randomizations 5-points, 146 iterations)
3. Parallel Thinning	• DLS fitting
4. Ellipse Fitting (Randomizations 3-points, 35 iterations)	3. Fitness Calculation
• Three Tangent Direction Calculation	4. Data Support Removing
• Center of Ellipse Approximation	
• Least Square Fitting	
5. Fitness Calculation	

Processing times of PRANSAC and our technique which are tested by images in Fig. 5 are shown in Table II. From experimental results, in case of a small image size, PRANSAC is faster. In other cases, our approach is faster than PRANSAC. Although, the number of iterations in Ellipse Fitting step of PRANSAC is lower (35 iterations), pre-processing of PRANSAC (Component Labeling and Parallel Thinning) takes the computational time more than post-processing of our method (Data Support Removing). In other words, the computational times of Component Labeling and Parallel Thinning depend on complexity of texture and the image size consequently, while the computational time of our proposed technique is constant. Moreover, the main drawback of PRANSAC is the incapability to resolve intersecting ellipses.

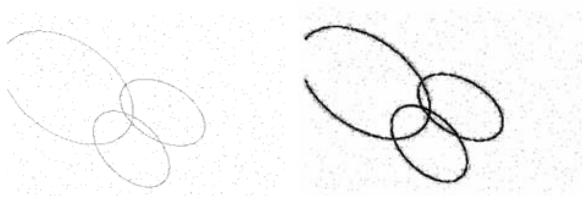
TABLE II
TIMES FITTING OF PRANSAC AND OUR ALGORITHM WITH AN ELLIPSE BY VISUAL C++ 6.0 (PENTIUM IV 2.4 GH ., 1 GB. MEMORY)

Image Resolution (pixels)	PRANSAC (Milliseconds.)	Proposed Method (Milliseconds.)
250x323	100	142
480x640	194	142
580x480	265	142

B. Ellipse fitting Results

In all experiments presented in this work, the bound of shortest distance to the ellipse perimeter in (4) was set to two pixels and the fitness threshold f_{th} to 0.5. The settings mean noises allowed in edge location are upto two pixels and the ellipses to be detected must not be occluded more than 50%. The parameters of the randomization process are =0.99, err=50%, s=6, and r=10.

The accuracy of our algorithm is tested by synthesis images. The results on the synthesis images of ellipse with isolated noise are shown in Fig. 2, 3 and Table III.



(a) (b)
Fig. 2 (a) The synthesis ellipses with isolated noise,
(b) The result superimposed on the image data.

(a) (b)
Fig. 3 (a) The synthesis ellipses with isolated noise,
(b) The result superimposed on the image data.

TABLE III
THE ACCURACY OF DETECTION ALGORITHM IS TESTED WITH SYNTHESIS ELLIPSE.

	Synthesis parameters ($h\ k\ a\ b\ \theta$)	Detected parameters ($h\ k\ a\ b\ \theta$)
1	200,10,50,30,45°	200.5, 10.6, 49.4, 29.8, 44.3°
	200,10,50,30,30°	200.6, 9.3, 49.5, 29.9, 30.5°
	100,30,80,50,35°	100.2, 29.3, 79.9,49.7, 35.4 °
2	250,10,50,30,17 °	250.3, 9.7, 50.1,29.6, 17.2°
	220,10,70,30,30 °	220.0, 10.4, 69.9, 29.8,29.9°
	100,30,80,50,35 °	100.8, 29.6, 80.0, 49.0,35.9 °

Table III shows the accuracy and time consumption to detect the ellipses. Another set of testing image is real-world images. Gaussian filter (size= 5, σ^2=1) and Canny edge detection are applied to gray scale images to obtain an edge map from these images.

An image of kitchen-wares is used to show the process of our algorithm as in Fig. 4 and 5.

(a) (b) (c)
Fig. 4 (a) Three cup image,
(b) Edge map of (a),
(c) The result superimposed on the image data.

It can be seen from the results that the method performs very well and is robust against noise and spurious edges. However, small ellipses such as the number "100" and the bicycle wheels inside the traffic signs in the last two image sets are not detected. This is because their major axes are less than 20 pixels; therefore, their samples do not pass the distance test of $2r$ in the Randomization process. From separate running a number of experiments (results not shown in this paper), the major axes of ellipses detected were ranging roughly between 20-300 pixels. This parameter determines detectable ellipses from our method. The fitness is the second important variable. It is used to classify good or bad models by the number of supporters around the ellipse perimeter.

(a) (b)

Fig. 5 (a) Edge maps of different road sign images,
(b) Results superimposed on the brightness-scaled images.

IV. Conclusion

In this paper, a novel method to detect multiple ellipses was presented. It is an iterative technique that finds and removes the best ellipse until no reasonable ellipse is found. At each run, the best ellipse is extracted, its fitness calculated and compared to a fitness threshold. RANSAC algorithm is applied as a sampling process together with the Direct Least Square fitting of ellipses (DLS) as the fitting algorithm. The method performs very well and is robust against noise and spurious edges. There are also some adjustable parameters affecting the size and completeness of ellipses detectable by our method. The selected area parameter (r) of two selected points for an ellipse fitting is an important parameter. Each selected area is likely to contain pixels from the same underlying curve. Thus, the result for the ellipse fitting is more accurate than PRANSAC which has no selected area. Although, the framework of the proposed method is close to PRANSAC, our algorithm is more efficient.

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