International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:1, No:5, 2007

Robust Control of a High-Speed Manipulator in State Space

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***Abstract*—**A robust control approach is proposed for a high speed manipulator using a hybrid computed torque control approach in the state space. The high-speed manipulator is driven by permanent magnet dc motors to track a trajectory in the joint space in the presence of disturbances. Tracking problem is analyzed in the state space where the completed models are considered for actuators. The proposed control approach can guarantee the stability and a satisfactory tracking performance. A two-link elbow manipulator driven by electrical actuators is simulated and results are shown to satisfy conditions under technical specifications.

***Keywords*—**Computed torque, manipulator, robust control, state space.

I. INTRODUCTION

ROBOTIC manipulators are extensively used in industrial tasks such as materials transfer, welding, paint spraying. In these tasks, end-effecter is commanded to move from one point to another, or to follow a given trajectory. Various control methods were developed to control manipulators such as PID [1], feed forward [2], adaptive [3], sliding mode [4], neural networks [5], and fuzzy control [6]. Traditional PID controllers are perfectly used for low speed manipulators to perform industrial tasks. Researches are motivated by requirements such as a high degree of automation and fast speed operation from industry in the past decades. Robot performance degrades quickly as speed increases. High velocity causes dynamic problems.

Computed torque method compensates in a feedforward manner the nonlinear coupling inertial, coriolis, centripetal and gravitational forces arising due to motion of the manipulator [7]. Its operation is improved using other control methods as a hybrid approach to perform a task. Based on the Lyapunov stability a hybrid control system was proposed to control the position of a slider of the motor–toggle servo mechanism [8]. This approach combines the computed torque controller, the fuzzy neural network uncertainty observer and a compensated controller. A computed torque control approach was introduced using the sliding mode technique such that uncertainty bounds in which are estimated by an adaptive scheme [9]. Computed torque controller was used to linearize nonlinear equation of robot by canceling nonlinear terms [10]. However, it requires precise dynamical model of the manipulator.

Robust control of robotic manipulators has attracted considerable research over the past decade. The main reason for this work is that robust control can be used to handle nonlinear unknown dynamics. Several researchers addressed the problem of designing robust control for robot manipulators. For example, see [11-16] for detailed discussions of this problem.

In this paper, a robust hybrid computed torque approach is proposed in the state space. This approach is implemented to control a high speed manipulator driven by permanent magnet dc motors. An analytical consideration is then presented in the state space for tracking problem including complete models of actuators. A linear state feedback control low is applied to cancel disturbances that are not compensated by computed torque. A feedforward control path is designed for both regulating and tracking designs. After that simulation results are presented to confirm the analytical approach and the system performance is then considered to improve the system behavior.

II. LINEAR STATE FEEDBACK

A linear system in the state space is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \tag{1}$$

where $\mathbf{x}$ is a state vector, $\mathbf{A}$ is a state matrix, $\mathbf{b}$ is an input coefficient vector and u is an input. We can achieve any state in finite time using a suitable control law if the given system to be controllable. To control a linear system, we can use a linear state feedback control law of the form

$$u = -\mathbf{k}^T\mathbf{x} + u_d \tag{2}$$

where $\mathbf{k}^T$ is a coefficient vector and u_d is a desired input. Substituting (2) into (1) yields

$$\dot{\mathbf{x}} = (A - \mathbf{b}\mathbf{k}^T)\mathbf{x} + \mathbf{b}u_d \tag{3}$$

The coefficient vector $\mathbf{k}$ is determined such that the system poles are placed in desired places. The closed loop poles are

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eigenvalues of the matrix $A-\mathbf{bk}^T$. The linear state feedback provides possibility of achieving a wide range of closed loop poles. One solution to determine $\mathbf{k}$ is given by the optimum linear control law of the form

$$k = R^{-1}b^T P \tag{4}$$

where P and Q are the symmetric positive definite $n\times n$ matrixes and R>0 which satisfy the Riccaty equation as follows

$$A^T P + PA - PbR^{-1}b^T P + Q = 0 \tag{5}$$

III. STABILITY

For regulating purpose, we choose

$$u_d = 0 \tag{6}$$

Thus

$$\dot{\mathbf{x}} = (A-\mathbf{bk}^T)\mathbf{x} \tag{7}$$

Consequently, $\mathbf{x}$ asymptotically approaches zero. For tracking design, we choose

$$\mathbf{b}u_d = \dot{\mathbf{x}}_{\mathbf{d}} - (A-\mathbf{bk}^T)\mathbf{x}_{\mathbf{d}} \tag{8}$$

where $\mathbf{X}_d$ is a desired trajectory. Substituting Equation (8) into (3), yields

$$\dot{\mathbf{x}} = (A-\mathbf{bk}^{\mathbf{T}})\mathbf{x} + \dot{\mathbf{x}}_{\mathbf{d}} - (A-\mathbf{bk}^{\mathbf{T}})\mathbf{x}_{\mathbf{d}} \tag{9}$$

$$\dot{\mathbf{x}}_{\mathbf{d}} - \dot{\mathbf{x}} = (A-\mathbf{bk}^{\mathbf{T}})(\mathbf{x}_{\mathbf{d}} - \mathbf{x}) \tag{10}$$

Laplace transform of Equation (10) is calculated as

$$X_d - X = (sI - A + \mathbf{bk}^T)^{-1}(\mathbf{x}_{\mathbf{d}}(0) - \mathbf{x}(0)) \tag{11}$$

where X_d and X are Laplace transform of $\mathbf{x}_d$ and x, respectively. Thus, assuming stability, despite any differences given by $\mathbf{x}_{\mathbf{d}}(0)-\mathbf{x}(0)$, the error approaches asymptotically zero. If for an initial state $\mathbf{x}(t_0) = \mathbf{x}_{\mathbf{d}}(t_0)$, then the tracking error will be zero all the time. Here, k plays a significant role to vanish the tracking error. Taking Laplace transform of (9) and rearranging, leads to

$$X = \left(sI - A\right)^{-1}[\left(sI - A\right)X_d + bk^T\left(X_d - X\right)] \tag{12}$$

A block diagram of the tracking design is then represented in Fig. 1. The block diagram comprises a system given by $\left(sI - A\right)^{-1}$, a feed forward path $\left(sI - A\right)$, and a linear controller $\mathbf{bk}^{\mathbf{T}}$. The closed loop system will be stable if satisfying the following conditions.

1. The plant represented by A to be stable and non minimum phase since the system poles are the zeros of the feedforward path.
2. The closed loop system represented by $A-\mathbf{bk}^T$ to be stable.

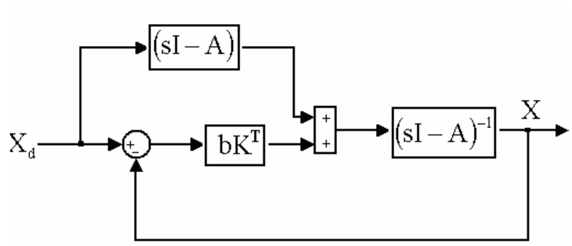

Fig.1 Tracking design

The system stability involves problems subject to disturbances. Disturbances are such inputs which are not under control. Disturbances can be assumed to be a difference between the physical system and its model. Therefore, system equation is modified as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{b}u + \mathbf{d} \tag{13}$$

Where $\mathbf{d}$ is a disturbance vector. Thus, for regulating purpose, (7) is modified as

$$\dot{\mathbf{x}} - (A-\mathbf{bk}^T)\mathbf{x} = \mathbf{d} \tag{14}$$

Assume the system is stable in the lack of disturbances. Based on the stability analysis of a LTI system stability, we can conclude that the state vector $\mathbf{x}$ is bonded if $\mathbf{d}$ is bonded. However, there will be steady state error. There is the same discussion about regulating problem subject to disturbances. Equation (10) is modified as

$$\dot{\mathbf{e}} - (A-\mathbf{bk}^T)\mathbf{e} = \mathbf{d} \tag{15}$$

where $\mathbf{e} = \mathbf{x}_{\mathbf{d}} - \mathbf{x}$ is the tracking error. It is concluded that tracking error is not zero subject to disturbances. The tracking error is reduced by selecting a suitable $\mathbf{k}$. In addition, as much as disturbances to be smaller, the tracking error will be less. A control law is proposed to cancel the tracking error as follows

$$\mathbf{b}u_d = \dot{\mathbf{x}}_d - (A-\mathbf{bk}^T)\mathbf{x}_d - \mathbf{d} \tag{16}$$

Substituting (16) into (3) yields (10) with no disturbances. This is why we apply the computed torque method.

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IV. Hybrid Computed Torque

Dynamic equation of a permanent magnet dc geared motor is derived as

$$J_m L\dddot{\theta}+(J_m R+LB_m)\ddot{\theta}+(RB_m+K_m K_b)\dot{\theta}=rK_m v-r^2(R\tau_l+L\frac{d\tau_l}{dt}) \tag{17}$$

where $\dot{\theta}$ is the load angular velocity, v is the motor voltage, τ_l is the torque load, L is the armature inductance, R is the armature resistance, K_b is the back emf constant, K_m is the torque constant and r is the gear ratio, J_m is the moment of inertia, B_m is the damping coefficient and r is the gear ratio. The load torque τ_l is applied on motor shaft by the manipulator. The load torque vector is calculated by dynamic robot equation as

$$M(\mathbf{q})\ddot{\mathbf{q}}+C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+G(\mathbf{q})=\boldsymbol{\tau}_\mathbf{l} \tag{18}$$

where $\boldsymbol{\tau}_\mathbf{l}$ is the $n\times 1$ joint torque vector, $\mathbf{q}$ is the joint position vector, $M(\mathbf{q})$ is the $n\times\mathrm{n}$ inertia matrix, $C(\mathbf{q},\dot{\mathbf{q}})$ is the $n\times 1$ Coriolis/centripetal matrix, $G(\mathbf{q})$ is the $n\times 1$ gravity vector. Equation (1) is formed as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & \dfrac{-\left(RB_m+K_mK_b\right)}{J_mL} & \dfrac{-\left(J_mR+LB_m\right)}{J_mL}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\frac{rK_m}{J_mL}v-\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\frac{r^2\left(R\tau_l+L\dot{\tau}_l\right)}{J_mL} \tag{19}$$

According to (13), we have

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & \dfrac{-\left(RB_m+K_mK_b\right)}{J_mL} & \dfrac{-\left(J_mR+LB_m\right)}{J_mL}\end{bmatrix},$$

$$\mathbf{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}\theta\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix},\mathbf{b}=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},u=\frac{rK_m}{J_mL}v \tag{20}$$

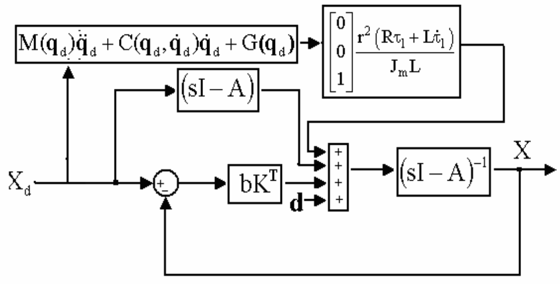

Fig. 2 Hybrid computed torque control of manipulator

$$\mathbf{d}=-\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\frac{r^2\left(R\tau_l+L\dot{\tau}_l\right)}{J_mL} \tag{21}$$

Vector $\boldsymbol{\tau}_\mathbf{l}$ comprises of motor load torques. Now, we present an algorithm to form the control law shown in Fig. 2.

1. Computed torque is calculated at desired trajectory by (18) in off-line case as

$$M(\mathbf{q}_d)\ddot{\mathbf{q}}_d+C(\mathbf{q}_d,\dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d+G(\mathbf{q}_d)=\boldsymbol{\tau}_{lc} \tag{22}$$

   where $\mathbf{q}_\mathrm{d}$ is the desired trajectory and $\boldsymbol{\tau}_\mathbf{lc}$ is the computed torque vector. The computed torque of each motor τ_{lc} is given by vector $\boldsymbol{\tau}_\mathbf{lc}$.
2. Equation (21) is then calculated for each motor. We named it $\mathbf{d}_\mathbf{c}$.
3. The control law given by Equation (16) is applied to cancel the tracking error.

The computed torque will not be the same as load torque if exits any tracking error. We can modify computed torque control by measuring $\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}$ and calculating computed torque from (18). In practice, we can measure $\mathbf{q}$, and $\dot{\mathbf{q}}$ precisely and then calculate $\ddot{\mathbf{q}}$ from them. This control law reduces tracking error as much as possible.

V. Simulation

Specifications of the two-link manipulator driven by permanent magnet dc motors are given in Table I.

TABLE I

Manipulator	Motors
$l_1=1\mathrm{m}$	$R=0.1\Omega$, $L=0.001\mathrm{H}$
$l_2=1\mathrm{m}$	$J_m=0.0001\mathrm{kg.m}^2$
$m_1=1\mathrm{kg}$	$B_m=0.001\mathrm{Nm.s/rad}$
$m_2=1\mathrm{kg}$	$K_m=K_b=0.02$ Nm/A
	$r=0.02$, 24V

The desired trajectory shown in Fig. 3 is designed to be smooth. It starts at zero with a velocity of zero and will finish at time 1sec with a zero velocity. The manipulator moves 1rad for operating time of 1sec that is a short time if we compare it with operating range of industrial robots. The trajectory is required to be smooth since the first and the second derivative of the trajectory are used in the control system. This point should be considered for planning the trajectory. In the case of existing jumps, derivatives of variables may cause the infinity problem in simulation. In addition, it causes responses that maybe out of the specifications and limits of the system. We have applied a model of dc motor which comprises inductance as well. The system order has been increased one order due to including inductance. It is also noted that the trajectory is started in where the joint angle is positioned. This yields a zero initial tracking error which is a significant factor to reduce the tracking error. Therefore, the trajectory is planned to satisfy the above conditions. An example of such trajectory is proposed as follows

$$\theta = -a\cos(\frac{\pi}{T}t) + a, \quad \mathrm{t} \geq 0$$

The trajectory begins at zero and ends at T where the position angle is 2a. Here we set $a = 0.5rad$, and $\mathrm{T} = 1\sec$ shown in Fig. 3. We consider the motor specifications to operate well under the technical limits such as voltage limit, current limit and torque limit. A gear ratio of 0.02 is used in this simulation to provide the operation requirements.

The first simulation is performed using the hybrid computed torque control of manipulator in state space shown in Fig. 2. Substituting parameters into Equation (20) yields

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5000 & -110 \end{bmatrix} \Rightarrow \begin{matrix} \lambda_1 = 0 \\ \lambda_2 = -55 + i\,44.441 \\ \lambda_3 = -55 - i\,44.441 \end{matrix}$$

Where λ_1, λ_2 and λ_3 are eigenvalues of matrix A that are system poles. System involves stability problem due to existing a pole at origin which should be moved to the left hand side of S plan. It is required to consider controllability by forming

$$\phi = \begin{bmatrix} b & Ab & A^2b \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -110 \\ 1 & -110 & 7100 \end{bmatrix}$$

The determinant of ϕ is

$$\det(\phi) = -1$$

where ϕ is the controllability matrix. The system is controllable since $\det(\phi) \neq 0$. A linear feedback control law **k** of the form $\mathbf{k} = \begin{bmatrix} 100 & 16 & 0 \end{bmatrix}$ can replace the closed loop poles at

$$A - bk^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -100 & -5016 & -110 \end{bmatrix} \Rightarrow \begin{matrix} \lambda_1 = -0.0199 \\ \lambda_2 = -54.99 + i44.6083 \\ \lambda_3 = -54.99 - i44.6083 \end{matrix}$$

Such that all poles are placed in the left hand side of S plan. The acceleration coefficient in linear state feedback **k** is zero. Therefore, the motor acceleration is not required to be feedback which is a practical facility. Fig. 4 shows tracking error in all over the operating range. The tracking error is limited under about 0.0008rad that is acceptable due to mechanical resolution.

It is ramped up to 13V when starting, then it is ramped down at once and after about 3sec it decreases continually to be negative for the final section of trajectory to reduce motor velocity. Considering the technical limits shows that motor work well. Fig. 6 shows the load torque applied on the motor shaft. The load torque is goes up to 90Nm when starting to move and it is then decreasing to be a negative value of about 20Nm at the end. It is concluded that in the last section

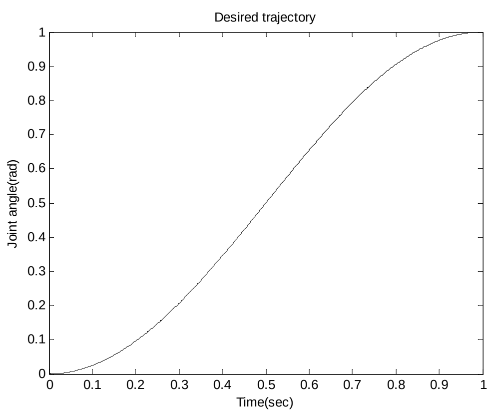

Fig. 3 Desired trajectory

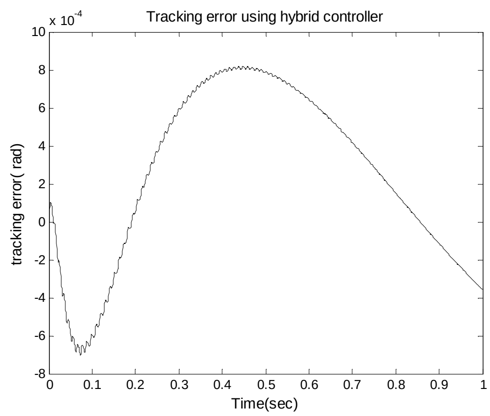

Fig. 4 Tracking error using hybrid controller

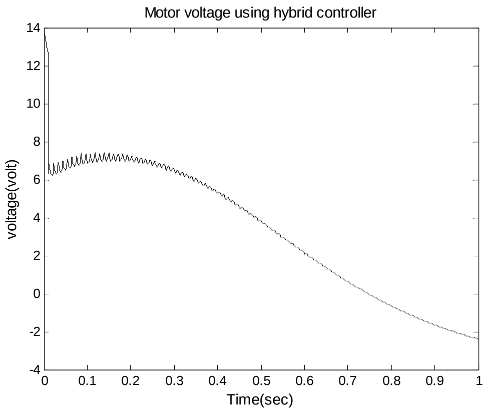

Fig. 5 Motor voltage using hybrid controller

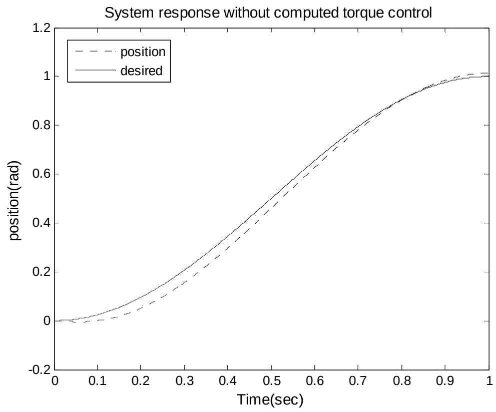

Fig. 7 System response without computed torque

of the trajectory, the acceleration is negative which cause reducing velocity. Fig. 6 represents a vibration on the motor torque due to simulation problem to compute torque load. The first and second derivatives of the load angle are required for calculating the motor torque.

In this simulation, the computed control law is omitted to find out its effect on system responses. Fig. 7 shows the system response. Tracking error is increased to a maximum of 0.05rad in Fig. 8. Comparing Fig. 4 and Fig. 8 shows that tracking error is about 62 times of one that was already. It means that computed torque control has found a significant role to reduce tracking error.

In the next simulation, the feedforward control path is removed from the control system to find out its effect on system responses. Fig. 9 represents system response. Tracking error is increased to a maximum of 0.020rad in Fig. 10. Comparing Fig. 4 and Fig. 10 shows that tracking error is about 25 times of one that was already. Therefore, the feedforward control can operate well to reduce the tracking error. Finally, we simulate the control system for tracking purpose just using the linear state feedback control. System is stable and there will be a tracking error with a maximum value of about 0.062rad shown in Fig. 11. The control system is stable. However, the tracking error is larger than other cases.

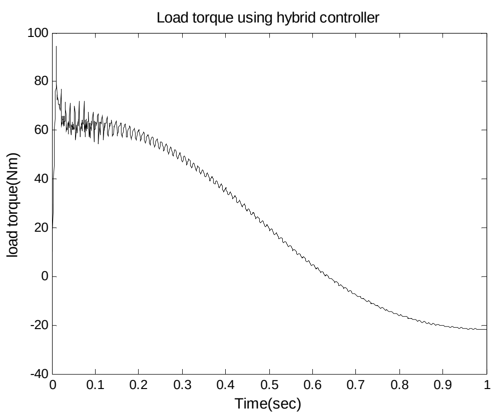

Fig. 6 Motor torque using hybrid controller

VI. CONCLUSION

A hybrid control approach was proposed for a high speed manipulator comprises computed torque control, feedforward control and linear state feedback in state space. The manipulator is driven by permanent magnet dc motors to track a trajectory in joint space. The linear state feedback control law provides a wide range of closed loop poles to stabilize the system. The computed torque control has found a significant effect on the control system to reduce the tracking error. The control system cannot work perfect without feedforward control. A smooth trajectory was proposed to satisfy the technical specification and limits. Analytical consideration in state space for tracking problem confirms that the initial error is required to be zero. The proposed control approach can guarantee the stability and satisfactory tracking performance.

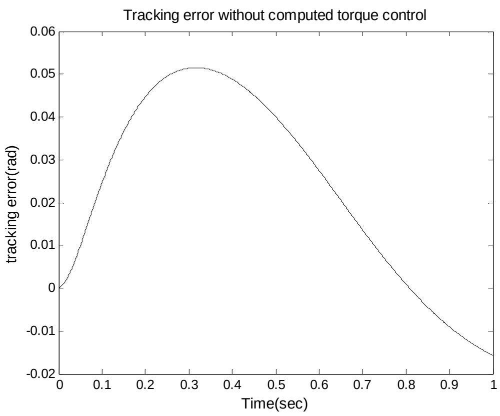

Fig. 8 Tracking error without computed torque control

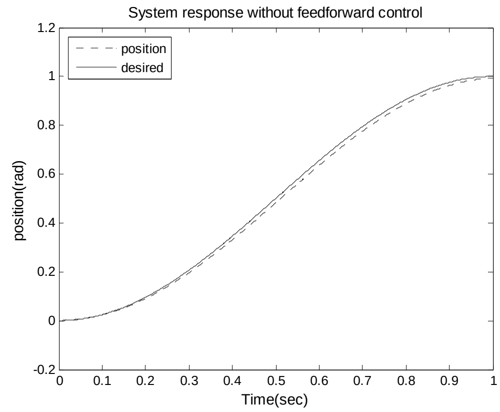

Fig. 9 System response without feedforward control

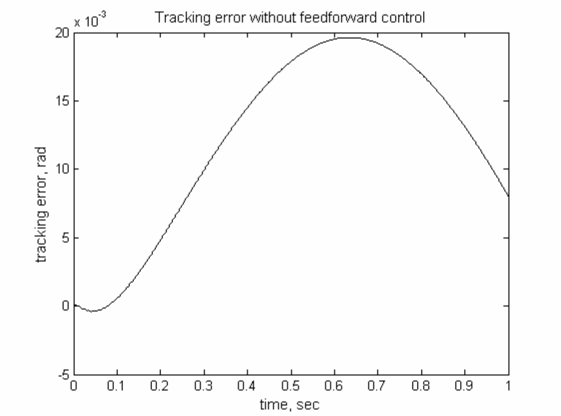

Fig. 10 Tracking error without feedforward control

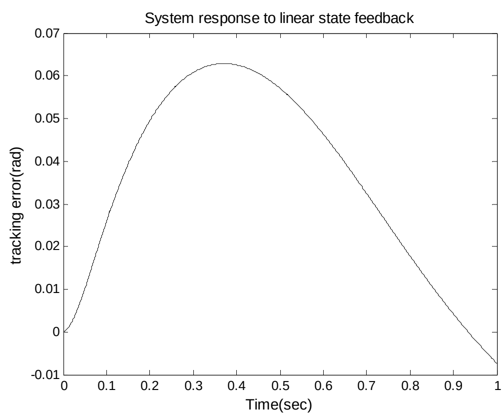

Fig. 11 Tracking by linear state feedback control

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