

# Risk Quantification for Tunnel Excavation Process

J. Šejnoha, D. Jarušková, O. Špačková, E. Novotná

**Abstract**—Construction of tunnels is connected with high uncertainty in the field of costs, construction period, safety and impact on surroundings. Risk management became therefore a common part of tunnel projects, especially after a set of fatal collapses occurred in 1990's. Such collapses are caused usually by combination of factors that can be divided into three main groups, i.e. unfavourable geological conditions, failures in the design and planning or failures in the execution.

This paper suggests a procedure enabling quantification of the excavation risk related to extraordinary accidents using FTA and ETA tools. It will elaborate on a common process of risk analysis and enable the transfer of information and experience between particular tunnel construction projects. Further, it gives a guide for designers, management and other participants, how to deal with risk of such accidents and how to make qualified decisions based on a probabilistic approach.

**Keywords**—risk quantification, tunnel collapse, ETA, FTA, geotechnical risk

## I. INTRODUCTION

RISK analysis and management of tunnel projects are subjects of many distinguished works. Complex requirements and guidance were published, e.g., by ITA (International Tunneling Association) or ITIG (The International Tunneling Insurance Group) – see [1, 2].

Let us leave aside the risk connected with financing, with public and political interests, with operation and maintenance and many other kinds of risks threatening infrastructural projects including tunnels. This paper focuses on the construction phase of tunnel project only.

Costs of excavation itself and of construction of shotcrete lining create 40 to 75 percent of total construction costs in dependence on geotechnical conditions. Costs of other works are, on the contrary, almost not influenced by geology. Models such as DAT (Decision Aids for Tunneling) developed on Massachusetts Institute of Technology (see eg.

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[3, 4]) or simulation models based on Symphony environment (e.g. [5], [6]) are powerful tools for prediction of costs and schedule regarding the uncertainties in geotechnical predictions and assessment of unit costs and advance rates. But they do not deal in detail with risk of rare accidents that may essentially influence the success of the project.

Such a risk is often analyzed by means of different classification and rating systems, see for example [7, 9]. However, real quantification of the risk is feasible as well and it is the subject of this paper which is organized as follows: First, we use FTA (Failure Tree Analysis) for determination of intensity of occurrence of particular types of failures and for identification of most serious causes. Second, we apply ETA (Event Tree Analysis) for calculation of related risks. The suggested approach supposes sharing of information between particular projects, in order to obtain as exact inputs for analysis as possible. The Failure and Event trees were prepared to cover broad spectrum of tunnel projects, the inputs will be modified in the given range according to specific conditions.

The procedure or its parts is utilizable as the basis for decision making (e.g., about adopting costly measures in risky sections), may be used by insurers for more accurate assessment of risk specifically for a particular project etc. The paper further intends to provide a simple guidance for adopting of probabilistic approach in the construction practice, based on probabilistic data from geotechnical survey.

## II. BASIC PROBABILISTIC CONCEPT AND CATEGORIZATION OF FAILURES

For purposes of the analysis, the failures according to their nature and their consequences must be first categorized. Further types of failures were therefore defined:

- 1) Cave-in collapse
- 2) Significant exceeding of expected deformation of the tunnel tube
- 3) Exceeding of acceptable progress of subsidence trough
- 4) Disturbance of water regime in the surroundings

The basic premise of the categorization is the exclusivity of these events/failures so that the total risk might be calculated as the sum of risks arising from particular failures, e.g., large deformation of tunnel tube that immediately forerun an cave-in collapse must not be considered separately but only as part of the tunnel collapse.

For every type of hazard, the risk is calculated as product of probability of occurrence  $P[Event]$  and expected worth of loss  $D$  :

$$R = P[Event] \times D \quad (1)$$

Expression of risk used in suggested model is:

$$R = \sum_i P[Event] \times P[Consequence(i) | Event] \times D(i) \quad (2)$$

where  $D(i)$  is the expected worth of loss caused by Consequence(i). The value of probability of occurrence  $P[Event]$  might be assessed directly or might be determined on the basis of failure intensity,  $\lambda$ , indicating the number of failures on a unit length. The intensity is to be calculated using the Fault Tree diagrams described below.

The probability of failure for a tunnel or tunnel section of a given length must then be calculated. In the case of very rare events, e.g., cave-in collapse, we might use approximation by Poisson distribution and calculate the probability of occurrence of at least one failure:

$$P[N(\text{Events}) > 1] = 1 - e^{-\lambda L} \quad (3)$$

where  $L$  is the length of examined tunnel tube and  $N(\text{Events})$  is the number of failures.

In other cases, the mean value of events on a given length will be determined:

$$N(\text{Events}) = \lambda \times L \quad (4)$$

The probabilities of particular possible consequences on the condition of the event  $P[Consequence(i) | Event]$  are to be analyzed by the help of Event Trees.

Not all types of failures need to be examined for each case. For instance, where the size and shape of subsidence trough is not the subject of observation because it cannot cause any damage, we do not even need to consider it in the analysis.

### III. FAULT TREE ANALYSIS

Within the suggested procedure, the Fault Tree Analysis (FTA) was used for determination of intensity of several types of failures as defined at the beginning of par II.

In other studies the FTA has been usually utilized for quantification of probability of particular type of collapse for the tunnel as a whole, e.g. in [9], the concept of intensity of failures  $\lambda$  is therefore innovative. This approach was chosen because it makes it possible to simply compare analyses of different tunnels, to transfer data and experiences amongst various projects.

#### A. Arrangement of the Failure Tree

The geotechnical conditions have a major role in the tunnel construction. However, they are rarely the only reason of the failure. Usually a combination of factors leads to the collapse, often a mistake of management or designers. Within the analysis we identify three main groups of causes and all of their possible combinations, as follows:

1) Unfavourable geotechnical conditions

- 2) Incorrect design and planning (including preliminary geotechnical survey and project of geotechnical monitoring)
- 3) Incorrect execution (mistake of a construction company – both of managers or workers, mistake of the surveyors, geotechnicians etc.)
- 4) Unfavourable geotechnical conditions combined with the incorrect design and planning
- 5) Unfavourable geotechnical conditions combined with the incorrect execution
- 6) Incorrect design and planning in combination with the incorrect execution
- 7) Combination of all three factors

The resulting intensity of the top event (i.e. cave-in collapse, exceeding of deformations etc.) is then determined as the union of intensities obtained on the conditions 1) – 7).

As an example let us expect unfavourable geotechnical conditions (branch 1) at 3% of the length of the tunnel and the intensity of failure (cave-in collapse), assessed as  $0.001 \text{ m}^{-1}$ , this can be interpreted as one expected cave-in collapse caused just by the unfavourable geology on the length of 1000 m, on the condition that design, planning, management and all other processes run well. The contribution of this branch to the total intensity is then  $\lambda_1 = 0.03 \times 0.001 = 3 \times 10^{-5} \text{ m}^{-1}$ .

For branch 4) the probability of concurrence of unfavourable geotechnical conditions (occurring within 3% of the length) and incorrect design and planning (with probability of 0.02) will be lower. It equals the product of both probabilities, i.e.  $0.03 \times 0.02 = 6 \times 10^{-4}$ . The intensity of failures on condition that both these factors are combined is considerably higher, we assess it as  $0.1 \text{ m}^{-1}$ . The contribution of branch 4) to the total intensity is then  $\lambda_4 = 0.0006 \times 0.1 = 6 \times 10^{-5} \text{ m}^{-1}$ , and therefore it is more significant compare to  $\lambda_1$

The branches are further structured individually for every type of tunnel failure, as shown in App.1 for the example of cave-in collapse.

#### B. Estimation of inputs

Essential condition for proper analysis is the right assessment of input probabilities/ intensities of the basic events. The primary values published in this paper were determined on the basis of expert estimates and should provide some idea of their size. The initial estimate was verified by backward analysis on the basis of given intensity of the top failure (e.g. for the case of cave-in collapse the total number of collapses occurred in the Czech Republic since 1990 divided by the total length of tunnel tubes).

The suggested input values are to be specified for a particular analyzed tunnel. In the case the conditions vary significantly along its length, it is suitable to analyze different sections of the tunnel separately. Total risk is than the sum of risks connected with particular sections.

In the future, the analysis should be elaborate for every tunnel project. A database of Failure Trees (FTs) for tunnels

under different conditions will be established. The FTA might than be used for evaluation of different options, where the influence of following factors might be studied:

- Geotechnical conditions determined by chosen tunnel line
- Technology of the excavation
- Quality of the contractor
- Quality of the designer
- Quality of other involved parties

The suggested FTs are not limited to one technology of excavation in spite of the fact, that the primary values were assessed almost entirely based on tunnels made by NATM (New Austrian Tunneling Method), which has significantly prevailed in the Czech Republic since 1990. For consideration of other technologies, some adjustments of the FTs might be needed (adding or deleting of some basic events), however, the structure remains the same.

The intensities of basic events relating to design and execution, i.e. to the human factor, as well as the conditional probabilities (failure on the condition of some basic events) must be assessed based on an expert opinion. For the evaluation of geotechnical conditions different probabilistic models might be used as described in the following section.

#### C. Determination of input values for geotechnical basic events with help of probabilistic models

Where the occurrence of unfavourable geotechnical conditions is hard to assess or the accuracy of the estimate has a significant impact, it may be worth using some probabilistic models.

MODEL 1: Stochastic modeling of geotechnical parameters with homogenous probabilistic characteristics with help of Markov chains

A powerful tool is the Geologic module of DAT (Decision Aids for Tunneling) that generates probabilistic geotechnical profiles along the tunnel and enables updating of the results based on more accurate observations obtained during the excavation or additional survey. It predicts the states of defined parameters (e.g. degree of jointing, level of water inflow etc.) on the basis of Markov processes, in which the inputs are both the transition probabilities and the average lengths where particular parameters do not change. The combination of parameters then defines a ground class entering as the input into other modules of DAT. However, for purpose of our FT analysis, the state probabilities of chosen parameters are needed. The basic principle of the prediction is following:

Let us find the probability that the parameter  $U$  is in the state  $j$  at a distance  $x$  behind a starting position (e.g. tunnel face) assuming that  $U$  is in state  $i$  at the starting position. This task can be solved by means of the interval transition probability matrix

$$\mathbf{P}_U(x) = [P_{Uij}(x)], \quad (5)$$

where  $P_{Uij}(x)$  is the probability that  $U$  will be in state  $j$  after an interval  $x$  given the present state is  $i$ .

The matrix  $\mathbf{P}_U$  satisfies the forward Chapman-Kolmogorov equation

$$\frac{d\mathbf{P}_U(x)}{dx} = \mathbf{P}_U(x)\mathbf{Q}, \quad q_{ij} = -\sum_{\substack{k=1 \\ k \neq j}}^n q_{jk}, \quad (6)$$

where  $\mathbf{Q}$  is the transition intensity matrix.

The closed form solution of (6) can be written as

$$\mathbf{P}_U(x) = \exp[\mathbf{Q}x]. \quad (7)$$

If  $x$  approaches infinity,  $d\mathbf{P}_U/dx \rightarrow 0$  and (6) converts to

$$\mathbf{P}_U\mathbf{Q} = \mathbf{0}. \quad (8)$$

The vector of limiting state probabilities  $\mathbf{v} = \{v_1, v_2, \dots, v_n\}^T$  satisfies a system of equations

$$\begin{aligned} \mathbf{Q}^T \mathbf{v} &= \{0\} \\ \{1, 1, \dots, 1\} \mathbf{v} &= 1. \end{aligned} \quad (9)$$

For further details see [10].

For the purpose of the FTA, knowing state probabilities vector  $\mathbf{v}$  of a particular parameter, we will usually use the probability of the most unfavourable state (e.g., high degree of jointing, high water inflow etc.) as the input value.

MODEL 2: Stochastic modeling of continuously changing geotechnical parameters with non-homogenous probabilistic characteristics

For the assessment of parameters that do not have homogenous characteristics along the tunnel axis (e.g. significantly weakened rock overburden  $V(x)$  where its thickness tends to decrease) another models must be utilized. As an example let us consider a randomly varying depth of the rock overburden which tends to decrease (see Fig 1). To guarantee reliable excavation of the tunnel, the depth of the rock layer should not be smaller than a given limit  $h-a$ .

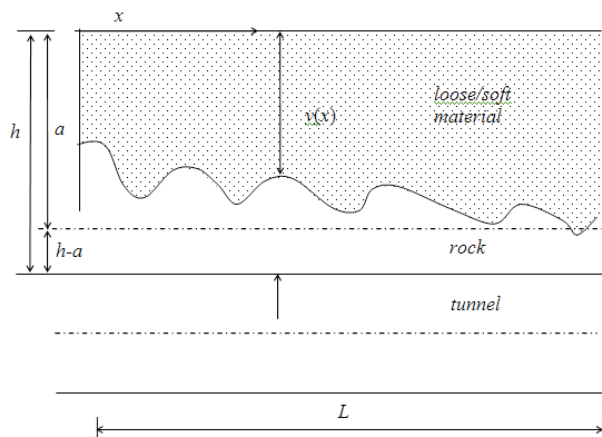


Fig. 1 Decreasing level of tunnel overburden.

There is a variety of theoretical models suitable for the prediction of the first barrier upcrossing probability, see e.g., [10]. Because of their mathematically tractable properties, the most common continuous process is the differentiable normal (Gaussian) process  $V = V(x)$  with the derivative  $\dot{V}(x)$ . For a constant level  $a$ , the barrier upcrossing rate,  $v_a^+$ , is given by Rice's formula

$$v_a^+ = \int_0^\infty \dot{v} f_{V, \dot{V}}(a, \dot{v}) d\dot{v}, \tag{10}$$

where  $f_{V, \dot{V}}(v, \dot{v})$  is the joint probability density function of  $V, \dot{V}$ .

In the case of non-stationary process (i.e. thickness of the bearable rock boundary along the tunnel axis) with a mean  $\mu_V(x)$  and a constant standard deviation  $\sigma_V$ , the intensity of upcrossing the level  $a$  can be calculated as

$$v_a^+(x) = \frac{1}{2\pi} \frac{\sigma_{\dot{V}}}{\sigma_V} \exp\left[-\frac{(a - \mu_V(x))^2}{2\sigma_V^2}\right]. \tag{11}$$

Further, introduce the spectral density function  $S_V(\omega)$  and corresponding relations

$$\sigma_{\dot{V}}^2 = \int_{-\infty}^\infty S_V(\omega) d\omega, \quad \sigma_V^2 = \int_{-\infty}^\infty \omega^2 S_V(\omega) d\omega. \tag{12}$$

After some manipulation and considering a narrow band process characterized by frequency  $\omega_o$  we arrive at

$$\frac{\sigma_{\dot{V}}}{\sigma_V} \approx \omega_o.$$

It should be evident that  $v_a^+$  expresses the intensity of Poisson process. It is then possible to use (11) to obtain the first-passage probability of the failure probability

$$P[N_f > 1] = 1 - \exp(-\tilde{v}_a^+ L), \tag{13}$$

where  $\tilde{v}_a^+ = \frac{1}{L} \int_0^L v_a^+(x) dx$ .

The number of failures on given length, which is needed for determination of the intensity for FTA, is then

$$N_f(v_a^+(x), L) = \int_0^L v_a^+(x) dx. \tag{14}$$

MODEL 3: Stochastic modeling of discrete geotechnical parameters with non-homogenous probabilistic characteristics

Another but similar problem is outlined in Fig. 2 Jointed rock interferes in the compact rock mass of the depth  $h - a$ . The basic question is what is the probability that a joint will disrupt compactness of the rock layer forming the

intermediate overburden of the tunnel. We can observe the location of the lower end of particular joints  $V$ . Let  $N_f$  be the number of joints breaking the barrier to compact layer with probability

$$p_f = P_r[V > a]. \tag{15}$$

If the joints occur with a constant intensity  $\lambda$ , the probability that the number of barrier upcrossings is  $j$  on the length of  $L$  reads

$$P[N_f = j] = \frac{(\lambda L p_f)^j \exp(-\lambda L p_f)}{j!} \tag{16}$$

for  $p_f$  being constant.

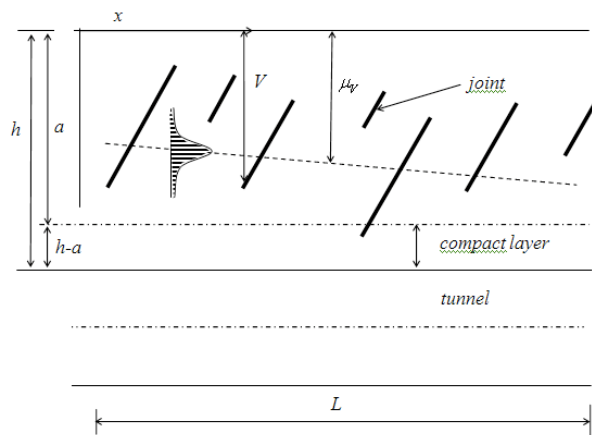


Fig. 2 Upcrossing of a barrier by joints.

If the mean of location of the bottom ends of joints  $V$  is not constant, we continue as follows.

Introduce a new random function

$$\tilde{V} = V - \mu_V(x) \tag{17}$$

such that  $\mu_{\tilde{V}} = 0, \sigma_{\tilde{V}} = \sigma_V = \text{const.}$

Then

$$p_f(x) = P_r[V(x) > a] = P_r[\tilde{V}(x) > a - \mu_V(x)] = 1 - F_{\tilde{V}}[a - \mu_V(x)]. \tag{18}$$

In case of variable  $p_f = p_f(x)$ , (16) must be replaced by relation

$$P[N_f = j] = \frac{(\Lambda_f)^j \exp(-\Lambda_f)}{j!}, \tag{19}$$

where

$$\Lambda_f = \int_0^L \lambda(x) p_f(x) dx \tag{20}$$

is the cumulative intensity and  $\lambda = \lambda(x)$  is the intensity of a non-homogenous Poisson process.

Finally, the probability that within the tunnel length  $L$  at least one joint breaks the compactness of the rock layer is

$$P[N_f > 1] = \sum_{j=1}^{\infty} \frac{(\Lambda_f)^j \exp(-\Lambda_f)}{j!} = 1 - \exp(-\Lambda_f) \quad (21)$$

This formula complies well with (13).

#### D. Software for calculation of Fault Tress

For compilation and calculation of FTs, an open source software OpenFTA was used [12]. It enables calculation of the probability of top event with the help of minimal cut sets analysis. Further, the contribution of particular basic events to the top event is analyzed which enables control of correctness of input values.

A number of commercial software products was tested, often providing also other modules such as ETA, FMEA (Failure Mode and Effect Analysis), Markov chains etc. However, their indisputable advantages (e.g., better graphical output, interconnection of modules, broader offer of functions etc.) does not offset their high prices which might discourage usage of the FTA in the construction practice.

#### IV. EVENT TREE ANALYSIS

Because the surroundings of a tunnel vary along its length, the consequences of a failure are not certain. The relating damage depends on many factors and it must be assessed on the probabilistic basis. Therefore a generic ETs (Event Trees) serving for the calculation of risk on the basis of a given probability of failure (obtained directly by expert estimates, from FTA as described above or in another manner) has been developed. For every type of primer event, an individual ET is to be used (see ETA for cave-in collapse in App 2)

The ET enables to consider all possible combinations of consequences and to evaluate damages separately for every combination. Consequences are structured as follows:

- 1) Human injury or death
- 2) Environmental damages
- 3) Damage to property and infrastructure (incl. intangible losses such as cultural heritage, suspension of businesses etc.)

With the help of the ET the probability of particular combination is first calculated. Second, the probable damage for every combination is assessed (as a sum of damages to property, environment, loss caused by delay etc.). Human injury or death might be expressed in monetary terms, eventually it might be considered as a separate criterion without quantification. Finally, the total risk is calculated as a sum of risks resulting from particular combinations.

For analysis of risk to property and infrastructure, more detailed analysis is usually needed compare to other types of consequences, especially when an urban tunnel is tackled. Suitable categories of surface development must be defined. The probability to hit a particular category by the tunnel failure is than calculated as the length of the section where the

tunnel tube crosses that category to the total length of the tunnel tube/ analyzed section (see Fig. 3).

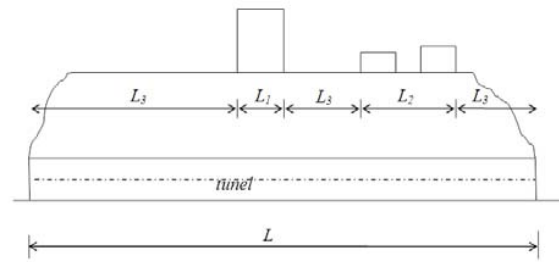


Fig. 3 Categories of development above a tunnel tube

In justified cases, when in spite of proper structuralization of the problem (consequences) it is not possible to accurately assess the damages, it is possible to enter them using a probability distribution function. The calculation of the event tree is thus made using Monte Carlo simulation.

The ET analysis is not limited to utilization within risk analysis of the tunnel as a whole. It might be used as the basis for operative decisions, e.g., to evaluate the efficiency of application of costly technology or measure on a risky section of a tunnel. The technology (measure) is efficient if the reduction of risk is higher than its costs.

#### V. RISK ANALYSIS OF TUNNEL PROJECTS IN THE CZECH REPUBLIC AND CONCLUSIONS

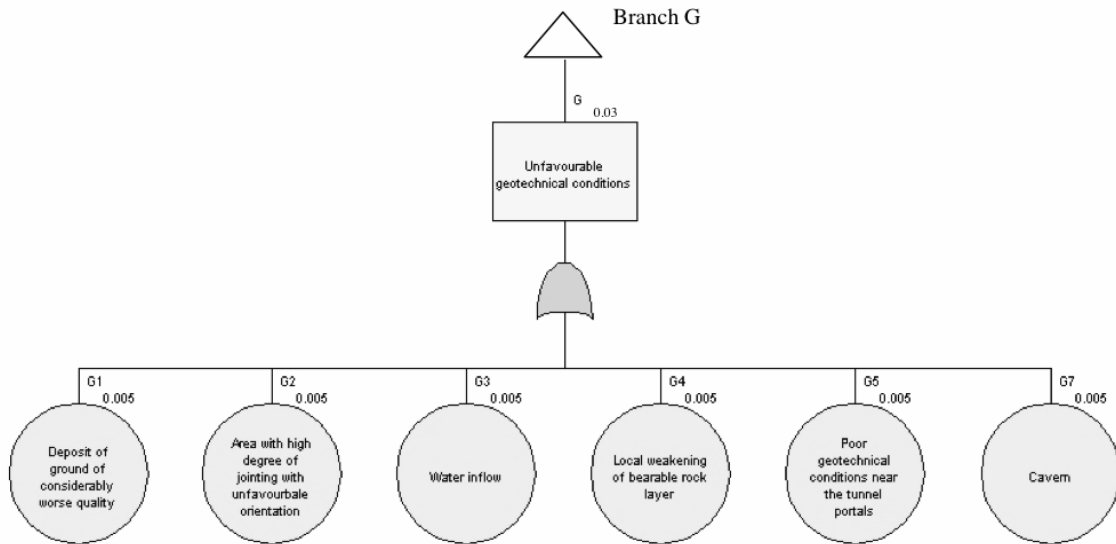
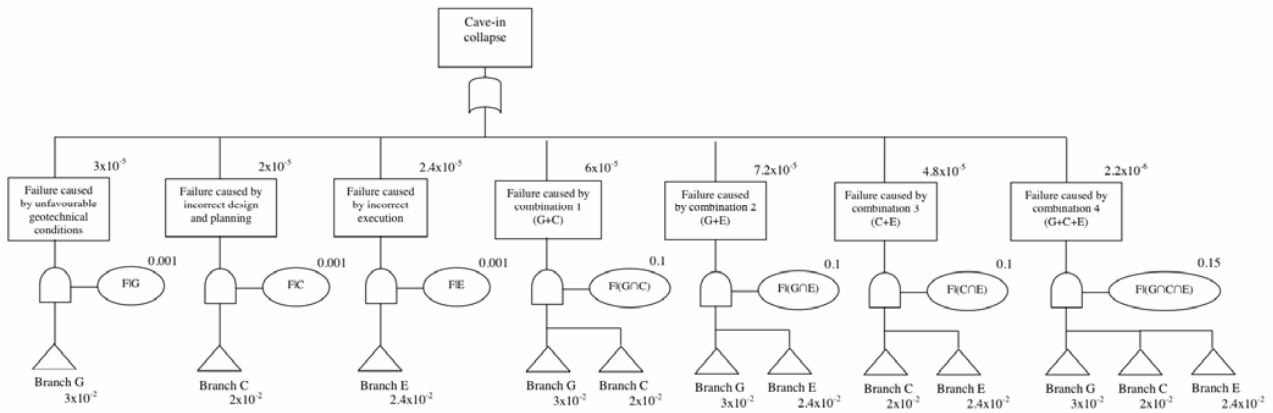
In the Czech Republic, the risk analysis has been carried out for majority of tunnel projects, mostly in the preparative phase. It served to complex identification of the most important hazards (incl. financing, operation and many other aspects) as the basis for proper risk management. Let us mention for example the risk analysis of Dobrovsky tunnel in Brno ordered by the contract authority. Another example is analysis of construction risk of submerged floating tunnels for Prague underground (IV.C1 section Holesovice railway station – Troja) elaborated for the general contractor. In the case of the largest planned tunnel project, the railway tunnel with length over 20 km between Prague and Beroun, the risk analysis was used as the base for the evaluation of options (choice of the tunnel trace and technology). In most cases, expert rating methods such as FMEA (Failure Mode and Effect Analysis) or UMRA (Universal Matrix of Risk Analysis) were used (see []).

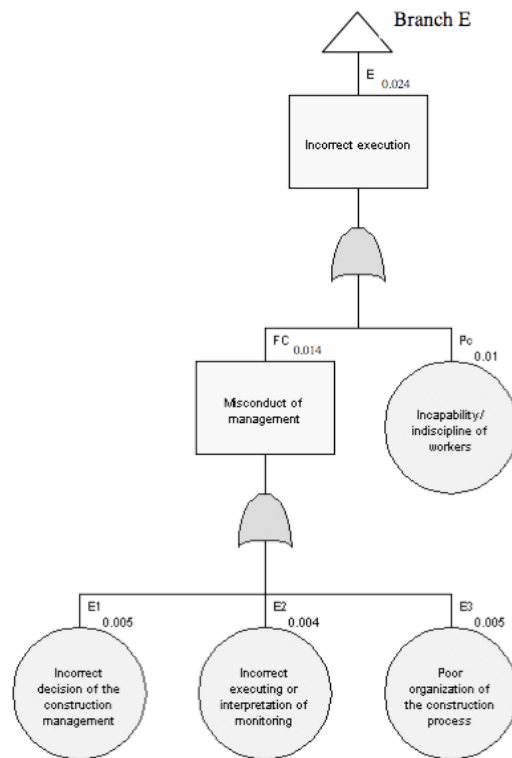
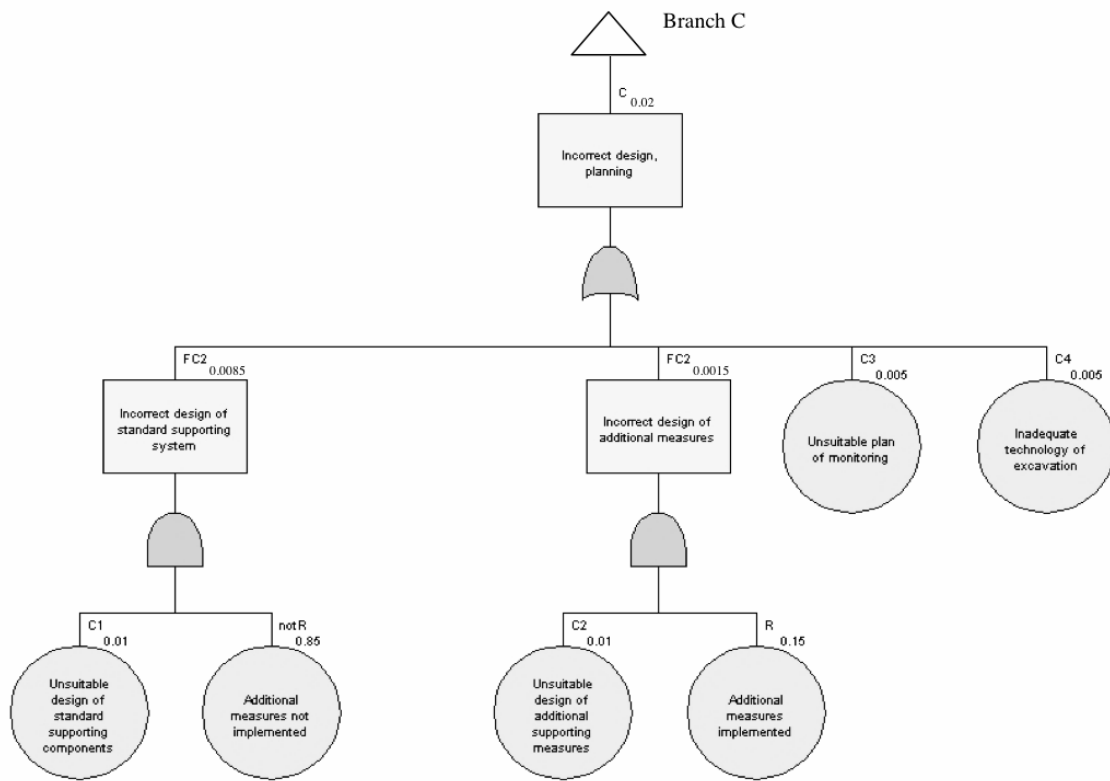
However, the results of risk analyses are not sufficiently reflected during later phases of the project, risks are not considered during the choice of general contractor. Operative decisions are not carried out based on risk analysis. In addition, the insurers of large tunnel projects do not have appropriate data for setting the insurance premium. This paper therefore intends to provide a tool for overcoming these shortcomings by introducing FTA and ETA methods enabling comparison and knowledge transfer between particular tunnel projects.

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APPENDIX 1: FTA – CAVE-IN COLLAPSE







APPENDIX 2: ETA – CAVE-IN COLLAPSE

| Will show itself on the surface? | Will cause human injury or death? | Will cause environmental losses? | Will damage buildings or infrastructure? | $P_i$     | Expected delay | Damage estimation                            |  |                                |                                       |  | Risk<br>$R_i = P_i \times D_i$ |                               |
|----------------------------------|-----------------------------------|----------------------------------|--|-----------|----------------|--|--|--------------------------------|---------------------------------------|--|--------------------------------|-------------------------------|
|                                  |                                   |                                  |  |           |                | Sanitation of the tunnel tube and overburden | Costs of the change of technology or project | Evaluation of the delay losses | Damage to property and infrastructure | Intangible damages (cultural heritage, break of business, ...) |                                | Number of people injured/dead |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  | YES<br>p                          | NO<br>1-p                        | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | YES<br>p                         | YES<br>p                                 | YES<br>p  |                |  |  |                                |                                       |  |                                |                               |
|                                  |                                   | NO<br>1-p                        | NO<br>1-p                                | NO<br>1-p |                |  |  |                                |                                       |  |                                |                               |