

Restricted Pedestrian Flow Performance Measures during Egress from a Complex Facility

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Abstract—In this paper, we use an $M/G/C/C$ state dependent queuing model within a complex network topology to determine the different performance measures for pedestrian traffic flow. The occupants in this network topology need to go through some source corridors, from which they can choose their suitable exiting corridors. The performance measures were calculated using arrival rates that maximize the throughputs of source corridors. In order to increase the throughput of the network, the result indicates that the flow direction of pedestrian through the corridors has to be restricted and the arrival rates to the source corridor need to be controlled.

Keywords— Arrival rate, Multiple arrival sources, Probability of blocking, State dependent queuing networks, Throughput.

I. INTRODUCTION

CONGESTION may occur in transportation, telecommunication and industrial networks. An appropriate tool to model such congestion is $M/G/C/C$ state dependent queuing network. The model is used by Jain & Smith [1] to model congestion in vehicular traffic network while Yuhaski & Smith [2] to model congestion in pedestrian traffic networks. For a new or remodeled facility, in order to accommodate expected customers and optimal egress of occupants in emergency situations, pedestrian flow in the corridors of a building is one of the most commonly occurring problems in facility planning. Cruz et al. [3], [4], Mitchell and Smith [5], Smith [6], Yuhaski and Smith [2] and others had used $M/G/C/C$ state dependent queuing models to capture the congestion in pedestrian traffic flow.

Majority of these works were with arbitrary topologies and for different types of multi-storied buildings. In this paper we are concerned with a facility wherein occupants through the source corridors come from multiple sources and egress through the nearest corridor. The objective of this paper is to estimate the different performance measures namely, the blocking probability, throughput, expected number of persons in the system and expected service time during egress from the facility. These measures can be used to compute the time needed to evacuate the facility in case of an emergency and to evaluate the optimal set-up for which the maximum throughput can be obtained.

The paper is organized as follows. The next section gives an overview on pedestrian traffic flows in terms of relationship between the walking speed of a pedestrian and crowd density for different directional flows, flow relationships for different types of movements and capacity of a corridor.

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Performance measures of $M/G/C/C$ state dependent queuing models are presented in the subsequent section followed by a brief description of corridor with multiple arrival sources and topologies with multiple corridors. A complete description of the facility under study is then presented together with a methodology of computing the arrival rates and throughputs of source and exiting corridors. Next, results of the performance measures are discussed and finally, some concluding remarks are given in the last section.

II. PEDESTRIAN TRAFFIC FLOWS

Based on some empirical studies, Tregenza [7] presents a number of relationships between the walking speed of a pedestrian and the crowd density. Among these relationships that capture the linear and non-linear effect of pedestrian density, the use of linear and exponential models has been shown to be very effective [2].

Fruin [8] showed that the relationship between uni-directional, bi-directional and multi-directional flows have similar pattern as a function of pedestrian density. As such, uni-directional flow models can be used to capture the bi-directional and multi-directional flows of occupants during an evacuation. The flow relationships for stairwells are also similar to horizontal movements [8].

According to Tregenza [7], at a mean density of five pedestrians per square meter (5 ped/m²), pedestrian traffic flow comes to a halt. Corridor capacity is thus equal to the highest integer that is less than five times the area of the corridor in square meters. Thus the corridor capacity, C , is:

$$C = 5LW \quad (1)$$

where L and W are the length and width of the corridor in meters.

Linear and exponential models for uni-directional walking speed has been developed by Yuhaski and Smith [2]. The exponential model for pedestrian speed can be given as follows:

$$V_n = A \exp \left[- \left(\frac{n-1}{\beta} \right)^\gamma \right] \quad (2)$$

where

$$\gamma = \ln \left[\frac{\ln(V_a/A)}{\ln(V_b/A)} \right] / \ln \left(\frac{a-1}{b-1} \right),$$

$$\beta = \frac{a-1}{[\ln(A/V_a)]^{1/\gamma}} = \frac{b-1}{[\ln(A/V_b)]^{1/\gamma}},$$

γ, β = Shape and scale parameters for the exponential model,

V_n = Average walking speed for n occupants in a corridor,

V_a = Average walking speed when crowd density is 2 ped/m² = 0.64 m/s,

V_b = Average walking speed when crowd density is 4

$$\text{ped/m}^2 = 0.25 \text{ m/s,}$$

$A = V_l =$ Average walking speed of a lone occupant = 1.5 m/s,

$n =$ Number of occupants in a corridor,

$a = 2LW$ and $b = 4LW$.

Cheah [9] provided exponential walking speed models for bi- and multi-directional corridor, which are similar in the form to the uni-directional model, except that the values for two parameters are slightly changed. For bi-directional flows, $V_a = 0.60$ and $V_b = 0.21$, and for multi-directional flows, $V_a = 0.56$ and $V_b = 0.17$ [9]. In this paper, the analysis is carried out solely for uni-directional traffic flows through corridors using the exponential pedestrian speed model.

III. PERFORMANCE MEASURES OF M/G/C/C STATE DEPENDENT QUEUING SYSTEM

We assume that pedestrians enter the corridor with Poisson rate λ and the total number of pedestrians that can enter the system is equal to the capacity of the corridor. The average time needed for a pedestrian to pass across the entire length of the corridor, is the service time of the queuing model of a corridor. The rate at which this traversal occurs, the service rate, $f(n)$ is dependent on the number of occupants (n) within the corridor. Hence, the queuing model is state dependent.

Yuhaski and Smith [2] developed the limiting probabilities for the number of pedestrians in an M/M/C/C queuing model. Cheah [9] showed that M/M/C/C and M/G/C/C state dependent queues are stochastically equivalent and developed the limiting probabilities for the number of pedestrians in an M/G/C/C state dependent queuing model as follows:

$$P_n = \frac{[\lambda E(S)]^n}{n! f(n) f(n-1) \dots f(2) f(1)} P_0, n = 1, 2, 3, \dots, C. \quad (3)$$

where $P_0^{-1} = 1 + \sum_{n=1}^C \left[\frac{\{\lambda E(S)\}^n}{n! f(n) f(n-1) \dots f(2) f(1)} \right]$.

In this model, $E(S)$ is the expected service time of a lone occupant in a corridor of length L , that is, $E(S) = \frac{L}{1.5}$.

P_n is the probability when there are n occupants in the corridor and P_0 is the probability of the corridor being empty. The service rate, $f(n)$, is the ratio of the average walking speed of n pedestrians (V_n) in the corridor to that of a lone pedestrian (V_l), that is, $f(n) = \frac{V_n}{V_l}$.

Balking occurs when a pedestrian attempts to enter a corridor, but cannot because the corridor is currently at capacity. The probability of such blocking (P_{balk}) is equal to P_n where n equals C , the capacity of the corridor. The steady state throughput through corridor is computed as

$$\theta = \lambda(1 - P_{balk}) \quad (4)$$

The expected number of pedestrians in the system (also known as work in process, WIP), is computed as

$$E(N) = \sum_{n=1}^C n P_n \quad (5)$$

and the expected service time in seconds, which is derived

from Little's formula, is given by

$$W = E(T) = \frac{E(N)}{\theta} \quad (6)$$

A. Corridor with Multiple Arrival Sources

Consider a single corridor with k arrival sources with rates $\lambda_1, \lambda_2, \dots, \lambda_k$ whose traveling distance to exit of the corridor are L_1, L_2, \dots, L_k respectively. Following the work of Yuhaski and Smith [2], this corridor is modeled as another single corridor of length L' and arrival rate λ' such that

$$\lambda' = \sum_{i=1}^k \lambda_i \text{ and } L' = \frac{\sum_{i=1}^k \lambda_i L_i}{\sum_{i=1}^k \lambda_i}.$$

That is, L' is the weighted average of the distance travelled by all the arrivals considering $\lambda_1, \lambda_2, \dots, \lambda_k$ as weights.

B. Topology with Multiple Corridors

In a topology with multiple corridors, the corridors may be in series, or in such an arrangement that one corridor is split to other corridors, or two or more corridors are merged to one corridor. Hence, a facility could be represented as a network of series, split, merge or a combination of any of these topologies. In cases when there is a split, the throughput from the splitting corridor is divided according to the probabilities of the branches. The arrival rate to a merging corridor equals the sum of the throughputs of the previous corridors.

IV. DESCRIPTION OF DTSP NETWORK

In this paper, the Dewan Tuanku Syed Putra (DTSP) hall room of Universiti Sains Malaysia is considered. Fig. 1 presents the simplified graphical representation of the internal set up of the hall room. The numbers represent the corridors, the alphabets S, T, U, V, W, X, Y and Z represent the different seating arrangements and A', B', C' and D' are the exits to other corridors. There are eight rows of chairs in each of arrangements S and T and ten rows of chairs in each of arrangements U and V. Each of the rows represents a source to corridors 10 and 11.

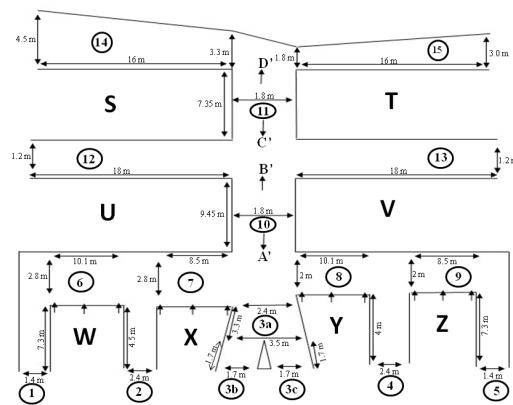


Fig. 1 Simplified Representation of Corridors of DTSP Hall

The chairs in *W*, *X*, *Y* and *Z* are arranged in such a way that each of the arrangement provide three sources to corridors 6, 7, 8 and 9 respectively.

In total there are 17 corridors inside the hall room. Corridors 6, 7, 8, 9, 10 and 11 are denoted as source corridors, since from sitting down position the occupants first need to come to these corridors. After entering into these corridors the occupants choose their nearest corridor to exit. The other corridors, except for 3a which we denote as intermediate corridor, are denoted as exiting corridors, since occupants can exit from the hall room through these corridors.

Table I presents the dimension, number of sources and average travelling distance of the source corridors. Using λ' and L' , the different performance measures of these corridors can be calculated. The different dimensions of the exiting corridors are presented in Table II.

TABLE I
DIMENSIONS, NUMBER OF SOURCES AND AVERAGE TRAVELLING DISTANCE OF SOURCE CORRIDORS

Corridor	Length (in meter)	Width (in meter)	No. of Sources	Average Travelling Distance (L')
6	10.1	2.8	3	2.156
7	8.5	2.8	3	1.78
8	10.1	2.0	3	2.156
9	8.5	2.0	3	1.78
10	9.45	1.8	20	2.7
11	7.35	1.8	16	2.275

The occupants sitting surrounding corridors 10 and 11 first come to these corridors and then from corridors 10 and 11 they choose the other corridors for a way out. Here we can assume that occupants will choose their nearest corridor. Each of source corridors 10 and 11 has two exits into other corridors, which are denoted as *A'* and *B'* for corridor 10, and *C* and *D* for corridor 11. For arrangements *U* and *V*, we assume that occupants from the 1st five rows will choose exit *A'*, as it is the nearest exit. Similarly, the occupants from the next five rows will choose exit *B'*. For arrangements *S* and *T*, occupants from the first four rows will choose exit *C'*, and those from the next four rows will choose exit *D'*.

TABLE II
DIMENSIONS OF THE EXITING CORRIDORS

Corridor	Length (in meter)	Width at Entrance (in meter)	Width at Exit (in meter)
1	7.3	1.4	1.4
2	4.5	2.4	2.4
3a	3.3	2.4	3.5
3b	1.7	1.7	1.7
3c	1.7	1.7	1.7
4	4.0	2.4	2.4
5	7.3	1.4	1.4
12	18	1.2	1.2
13	18	1.2	1.2
14	16	3.3	4.5
15	16	1.8	3

To calculate the throughput of corridors 10 and 11, we modeled corridor 10 as a single corridor with 20 arrival sources and corridor 11 with 16 arrival sources. Here we assume the same arrival rate from each arrival source.

TABLE III
ARRIVAL RATES AND THROUGHPUTS OF CORRIDORS

Corridor	Arrival Rate from each Source	Total Arrival Rate	Throughput
Source Corridors	6	$\lambda'_{6i} = \sum_{i=1}^3 \lambda_{6i}$	θ_6
	7	$\lambda'_{7i} = \sum_{i=1}^3 \lambda_{7i}$	θ_7
	8	$\lambda'_{8i} = \sum_{i=1}^3 \lambda_{8i}$	θ_8
	9	$\lambda'_{9i} = \sum_{i=1}^3 \lambda_{9i}$	θ_9
	10	$\lambda'_{10i} = \sum_{i=1}^{20} \lambda_{10i}$	θ_{10}
11	$\lambda'_{11i} = \sum_{i=1}^{16} \lambda_{11i}$	θ_{11}	
Intermediate Corridor	3a	$\lambda_{3a} = \frac{\theta_7}{2} + \frac{\theta_{10}}{2} + \frac{\theta_8}{2}$	θ_{3a}
Exiting Corridors	1	$\lambda_1 = \frac{\theta_6}{2}$	θ_1
	2	$\lambda_2 = \frac{\theta_6}{2} + \frac{\theta_7}{2}$	θ_2
	3b	$\lambda_{3b} = \frac{\theta_{3a}}{2}$	θ_{3b}
	3c	$\lambda_{3c} = \frac{\theta_{3a}}{2}$	θ_{3c}
	4	$\lambda_4 = \frac{\theta_8}{2} + \frac{\theta_9}{2}$	θ_4
	5	$\lambda_5 = \frac{\theta_9}{2}$	θ_5
	12	$\lambda_{12} = \frac{\theta_{10}}{4} + \frac{\theta_{11}}{4}$	θ_{12}
	13	$\lambda_{13} = \frac{\theta_{10}}{4} + \frac{\theta_{11}}{4}$	θ_{13}
	14	$\lambda_{14} = \frac{\theta_{11}}{4}$	θ_{14}
	15	$\lambda_{15} = \frac{\theta_{11}}{4}$	θ_{15}

Occupants from seating arrangements *W*, *X*, *Y* and *Z* come to corridor 6, 7, 8 and 9 respectively, through three sources. Each of corridors 6, 7, 8 and 9 is modeled as a corridor with three arrival sources. The different arrival rates and throughputs of the corridors when the facility is considered as a network are presented in Table III.

V. RESULTS AND DISCUSSION

The capacity of the corridors varies because of their varying sizes. The maximum throughput of these corridors can be achieved from different arrival rates. Considering the whole facility as a network of corridors and using Table III, the performance measures of the source corridors for the arrival rates that maximize the throughput of source corridors are presented in Table IV.

Using the throughput of the source corridors obtained in Table IV as the arrival rates of intermediate corridor and

exiting corridors, their performance measures are presented in Table V. From Table IV, we observe that for corridor 6, the maximum value of throughput is 14.043559 ped/s. Half of this throughput (7.021779 ped/s) is the arrival rate for corridor 1. The probability of blocking is 0.848372 and the throughput is 1.064696 ped/s. The expected number of occupants in corridor 1 is 51.820205, which is approximately equal to its capacity. The expected service time is approximately 49 seconds, which is about ten times of the expected service time of a lone occupant. For corridor 2, the arrival rate is 14.166975 ped/s, the blocking probability is 0.868129 and throughput is 1.868206 ped/s. Thus, only 13.2 percent of the arrivals can pass through this corridor at each time epoch.

Also the expected number of occupants in the system is approximately equal to the capacity of the corridor. Expected service time is approximately 29seconds, which is about ten times of that for a lone occupant.

The arrival rate for corridor 3a is high since corridors 7, 8 and 10 are merging to corridor 3a, and as a result there is a high probability of blocking (0.852509) and low throughput (2.279254 ped/s) in this corridor. This low throughput is further divided into corridors 3b and 3c. Hence, the blocking probabilities in these two corridors are very small with an almost equal arrival rate and throughput. In addition, these small blocking probabilities are due to the short lengths of these corridors.

TABLE IV
PERFORMANCE MEASURES OF THE SOURCE CORRIDORS USING THE ARRIVAL RATES THAT MAXIMIZE THROUGHPUT OF SOURCE CORRIDORS

Corridor	Arrival Rate from each Source (λ_i)	Total Arrival Rate ($\lambda' = \sum \lambda_i$)	Blocking Probability (P_n)	Expected No. of Occupants ($E(N)$)	Expected Service Time ($E(T)$)	Throughput (θ)
6	4.726667	14.18	0.009622	38.230217	2.722260	14.043559
7	4.82	14.46	0.011730	33.349923	2.333731	14.290391
8	3.37	10.11	0.013408	29.104225	2.917879	9.974444
9	3.43	10.29	0.016394	25.625759	2.531863	10.121304
10	0.3375	6.75	0.015961	25.170343	3.789424	6.642261
11	0.388125	6.21	0.020836	21.000184	3.453632	6.080608

TABLE V
PERFORMANCE MEASURES OF THE INTERMEDIATE AND EXITING CORRIDORS FOR MAXIMUM THROUGHPUT OF SOURCE CORRIDORS

Corridor	Arrival Rate from each Source (λ_i)	Blocking Probability (P_n)	Expected No. of Occupants ($E(N)$)	Expected Service Time ($E(T)$)	Throughput (θ)	
Intermediate Corridor	3a	15.453548	0.852509	48.825958	21.421903	2.279254
Exiting Corridors	1	7.021779	0.848372	51.820205	48.671382	1.064696
	2	14.166975	0.868129	53.847397	28.823050	1.868206
	3b	1.139627	0.000506	1.968010	1.727765	1.139050
	3c	1.139627	0.000506	1.968010	1.727765	1.139050
	4	10.047874	0.813384	47.768631	25.475300	1.875096
	5	5.060652	0.789085	51.730110	48.465116	1.067368
	12	3.180717	0.706918	107.582121	115.405483	0.932210
	13	3.180717	0.706918	107.582121	115.405483	0.932210
	14	1.520152	0.0	18.104994	11.909990	1.520152
	15	1.520152	0.0	19.972029	13.138179	1.520152
	Total Throughput of the Facility					13.058189

For corridors 4, 5, 12 and 13, the blocking probabilities are high, expected number of occupants in the system are approximately equal to their respective capacities, the expected service times are much higher than that for a lone occupant and throughputs are very small. For corridors 14 and 15, the blocking probabilities are zero because of less arrival rate and high capacity of each of these corridors. The expected service times are slightly greater than that for a lone occupant.

Thus if the maximum throughput is achieved from the source corridors, then there is a high probability of blocking in exiting corridors except for corridors 14 and 15. The expected numbers of occupants reach the capacity of the corridors and the expected service times become longer. That is in such situation there is a high congestion in exiting corridors except for corridors 14 and 15 and occupants need a much longer time to exit from the facility.

By summing the throughput of all the exiting corridors the total throughput of the facility is computed as 13.058189 ped/s. To increase the throughput of the entire system a restriction can be put in place. That is, we restrict occupants from corridor 11 to exit only through corridors 14 and 15, since there is no chance of blocking in these two corridors and each of these corridors has large capacity. Occupants from all other source corridors are assumed to choose their nearest corridor to exit. Under this restriction, the average travelling distance for the occupants in corridor 11 is changed to 4.095 meter. Also the arrival rates for corridors 12, 13, 14 and 15 are changed. The new arrival rate for each of corridors 12 and 13 is now one fourth of the throughput of corridor 10 ($\frac{\theta_{10}}{4}$) and that for each of corridors 14 and 15 is half of the throughput of

corridor 11 ($\frac{\theta_{11}}{2}$). Table VI presents the performance measures of the source corridors under this restriction and the arrival rates to the source corridors are adjusted such that the throughputs of the exiting corridors are the highest. From Table VI, we observe that there is a small probability of blocking on corridor 11, and for all other source corridors the

blocking probability is zero. The low probability of blocking in corridor 1 is due to the fact that the occupants need to travel an increased distance under the new restriction. Performance measures of the intermediate corridor and exiting corridors under the restriction are presented in Table VII.

TABLE VI
PERFORMANCE MEASURES OF THE SOURCE CORRIDORS UNDER RESTRICTION THAT OCCUPANTS FROM CORRIDOR 11 EXIT ONLY THROUGH CORRIDORS 14 AND 15

Corridor	Arrival Rate from each Source (λ_i)	Total Arrival Rate ($\lambda' = \sum \lambda_i$)	Blocking Probability (P_n)	Expected No. of Occupants $E(N)$	Expected Service Time $E(T)$	Throughput (θ)
6	1.533333	4.6	0.0	7.312731	1.589724	4.6
7	0.2	0.6	0.0	0.719834	1.199723	0.6
8	0.2	0.6	0.0	0.876448	1.460746	0.6
9	1.533333	4.6	0.0	6.371404	1.385088	4.6
10	0.26	5.2	0.0	12.987500	2.497596	5.199999
11	0.215625	3.45	0.020836	21.000184	6.216538	3.378115

TABLE VII
PERFORMANCE MEASURES OF THE INTERMEDIATE AND EXITING CORRIDORS UNDER RESTRICTION

Corridor	Arrival Rate from each Source (λ_i)	Blocking Probability (P_n)	Expected No. of Occupants $E(N)$	Expected Service Time $E(T)$	Throughput (θ)	
Intermediate Corridor	3a	3.199999	0.045718	19.797476	6.483108	3.053702
	1	2.3	0.527086	51.050416	46.934137	1.087703
	2	2.6	0.036519	20.460352	8.167638	2.505051
	3b	1.526851	0.018215	3.960972	2.642339	1.499040
	3c	1.526851	0.018215	3.960972	2.642339	1.499040
Exiting Corridors	4	2.6	0.040336	18.879285	7.566463	2.495127
	5	2.3	0.527086	51.050416	46.934137	1.087703
	12	1.3	0.015998	31.918991	24.952264	1.279202
	13	1.3	0.015998	31.918991	24.952264	1.279202
	14	1.689058	0.0	20.422796	12.091236	1.689058
	15	1.689058	0.0	22.937858	13.580269	1.689058
Total Throughput of the Facility						16.110184

From Table VII, we observe that the highest probability of blocking appear in corridors 1 and 5, which is 0.527086. Other exiting corridors have a small probability of blocking. The expected numbers of occupants for these two corridors are near to their capacity. Comparing Table V and Table VII, we can say that under the restriction the arrival rates to corridors 3b, 3c, 14 and 15 are increased a little bit, which increase their throughputs and expected service times. For all other corridors, the expected service times are less compared to that in Table V. The total throughput from the facility increased to 16.110184 ped/s. It can be observed that to increase throughput, we need to put some restrictions on the arrival rates of the source corridors. If from all the seating arrangements occupants rush to exit, then there will be a high probability of blocking and the overall throughput from the facility will decrease.

Since majority of the occupants sit surrounding corridors 10 and 11 and they need to travel a long way for the exit, the arrival rate to these corridors need to be controlled in such a way that the highest throughputs from the exiting corridors can be maintained. This may cause a high probability of blocking at corridors 1 and 5, but remarkably increases the overall throughput. Occupants from seating arrangements S and T may be suggested to go through corridors 14 and 15 in case of an emergency. While it may seem that it will take a long walk to go through these corridors, the probabilities of blocking are zero because of the larger capacities.

VI. CONCLUSION

The different performance measures of all the corridors of this hall room during egress have been computed. These measures can be used to compute the time needed to evacuate

the hall room in case of an emergency and to evaluate the optimal internal set up for which the maximum throughput can be obtained. In this paper, state dependent $M/G/C/C$ queuing model has been used to capture the bottleneck effects of pedestrian flow within circulation system of a complex pedestrian network topology involving combinations of merges and splits. We have calculated the different performance measures for this network. The higher arrival rate to the source corridor is found as a cause of higher blocking probability in the exiting corridors. It may be concluded that some restrictions need to be put on arrival to the source corridors and on travelling direction from source corridor to exiting corridors to control the blocking. Further extensions of this work will be to investigate the optimal internal set up and average evacuation time for such facilities.

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