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Remarks on Some Properties of Decision Rules

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Abstract—This paper shows that some properties of the decision rules in the literature do not hold by presenting a counterexample. We give sufficient and necessary conditions under which these properties are valid. These results will be helpful when one tries to choose the right decision rules in the research of rough set theory.

Keywords—Rough set, Decision table, Decision rule, coverage factor.

I. Introduction

In order to extract useful information hidden in voluminous data, many methods in addition to classical logic have been proposed. Rough set theory, which was proposed by Z. Pawlak in [3], plays an important role in applications of these methods. Their significance has been demonstrated by many successful applications in pattern recognition and artificial intelligence [1][2][5][9][10][11].An important application of rough set theory is to induce decision rules that indicate the decision class of an object based on its values on some condition attributes [3],[5]–[8]. In the past years, investigations for decision algorithms aroused extensive attentions of research community and some interesting results were obtained. The following proposition was given for properties of decision rules in [4][6].

Proposition 1: Let $C \rightarrow_x D$ be a decision rule, Then the following properties are valid:

$$\sum_{y\in C(x)} cer_y(C,D) = 1, \tag{1}$$

$$\sum_{y\in D(x)} cov_y(C,D) = 1, \tag{2}$$

$$\pi(D(x)) = \sum_{y\in C(x)} cer_y(C,D)\pi(C(y)) = \sum_{y\in C(x)} \sigma_y(C,D), \tag{3}$$

$$\pi(C(x)) = \sum_{y\in D(x)} cov_y(C,D)\pi(D(y)) = \sum_{y\in D(x)} \sigma_y(C,D), \tag{4}$$

$$cer_x(C,D) = \frac{cov_x(C,D)\pi(D(x))}{\pi(C(x))} = \frac{\sigma_y(C,D)}{\pi(C(x))}, \tag{5}$$

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$$cov_x(C,D) = \frac{cer_x(C,D)\pi(C(x))}{\pi(D(x))} = \frac{\sigma_y(C,D)}{\pi(D(x))}. \tag{6}$$

Remark:Property (5) and (6) come from property (1),(2),(3),(4).

We disprove Proposition 1 by a counterexample. Also, we give sufficient and necessary conditions under which Proposition 1 holds. These results will be helpful when one tries to choose the right decision rules in the research of rough set theory.

This paper is organized as follows. Section 2 recalls some basic concepts from rough set theory. Section 3 presents a counterexample. Section 4 gives sufficient and necessary conditions under which these properties in Proposition 1 are valid.

II. Preliminaries

In this section, we recall some basic concepts from rough set theory[6]. Let $S = (U, C, D)$ be a decision table, where U is a universe of discourse, C and D are disjoint sets of condition and decision attributes. Every $x \in U$ determines a sequence $c_1(x), c_2(x), ..., c_n(x), d_1(x), d_2(x), ..., d_m(x)$ where $\{c_1, c_2, ..., c_n\} = C$ and $\{d_1, d_2, ..., d_m\} = D$. The sequence will be called a decision rule induced by x and denoted by $c_1(x), c_2(x), ..., c_n(x) \rightarrow d_1(x), d_2(x), ..., d_m(x)$ or in short $C \rightarrow_x D$, $C(x)$ and $D(x)$ are referred to as the condition granule and the decision granule induced by x, respectively.

Definition 1: The number

$$supp_x(C,D) = |C(x) \cap D(x)|$$

is called the support of the rule $C \rightarrow_x D$ in S. where $|C|$ denotes the cardinality of C.

Definition 2: The number

$$cer_x(C,D) = \frac{|C(x) \cap D(x)|}{|C(x)|}$$

is called the certainty factor of the decision rule $C \rightarrow_x D$ in S, where $C(x) \neq \phi$.

Definition 3: The number

$$cov_x(C,D) = \frac{|C(x) \cap D(x)|}{|D(x)|}$$

is called the coverage factor of the decision rule $C \rightarrow_x D$ in S, where $D(x) \neq \phi$.

Definition 4: The number

$$\sigma_x(C,D) = \frac{supp_x(C,D)}{|U|}$$

is called the strength of the rule $C \rightarrow_x D$ in S.

For $x \in U$, let $\pi(C(x)) = \frac{|C(x)|}{|U|}$ and $\pi(D(x)) = \frac{|D(x)|}{|U|}$.

III. A COUNTEREXAMPLE

In this section, we exemplify that the properties (1)-(6) in Proposition 1 are invalid by a counterexample.

Example 1: In Table 1, 7 facts concerning 7 cases of driving a car in various driving conditions are presented. In the table columns labeled weather, road and time, are called condition attributes, which represent driving conditions. The table columns labeled accident are called decision attributes.

Table 1: An example of information system

$Fact$	$Weather$	$Road$	$Time$	$Accident$
No_1	$misty$	icy	day	yes
No_2	$foggy$	icy	$night$	yes
No_3	$misty$	$not\ icy$	$night$	yes
No_4	$sunny$	icy	day	no
No_5	$foggy$	icy	$night$	no
No_6	$misty$	$not\ icy$	$night$	no
No_7	$sunny$	icy	day	no

The certainty and coverage factors for the decision algorithm are shown in Table 2.

Table 2: Certainty and coverage factors

$Fact\ No.$		$Certainty$	$Coverage$	$Strength$
No_1	x_1	1.0000	0.3333	0.1429
No_2	x_2	0.5000	0.3333	0.1429
No_3	x_3	0.5000	0.3333	0.1429
No_4	x_4	1.0000	0.5000	0.2857
No_5	x_5	0.5000	0.2500	0.1429
No_6	x_6	0.5000	0.2500	0.1429
No_7	x_7	1.0000	0.5000	0.2857

Now we check the properties (1)-(4) in Proposition 1.

For No_4, we obtain following results.

$$\sum_{y\in C(x_4)} cer_y(C,D) = 2 \neq 1.$$

$$\sum_{y\in D(x_4)} cov_y(C,D) = 1.5 \neq 1.$$

These show that the property (1) and (2) in Proposition 1 are not true.

For No_1, we get following results.

$$\pi(D(x_1)) = 0.4286$$

$$\sum_{y\in C(x_1)} cer_y(C,D)\pi(C(y)) = 0.1429.$$

It is clear that

$$\pi(D(x_1)) \neq \sum_{y\in C(x_1)} cer_y(C,D)\pi(C(y)),$$

thus the property (3) in Proposition 1 is not true.

For No_4, we have the results as below.

$$\pi(C(x_4)) = 0.2857$$

$$\sum_{y\in D(x_4)} cov_y(C,D)\pi(D(y)) = 0.8571.$$

It is clear that

$$\pi(C(x_4)) \neq \sum_{y\in D(x_4)} cov_y(C,D)\pi(D(y)),$$

thus the property (4) in Proposition 1 is not true.

But for No_4,

$$\pi(D(x_4)) = 0.5714$$

$$\sum_{y\in C(x_4)} cer_y(C,D)\pi(C(y)) = 0.5714.$$

These show that the property (3) in Proposition 1 is true.

IV. SUFFICIENT AND NECESSARY CONDITIONS OF THE PROPERTIES

In this section we give the sufficient and necessary conditions such that the properties (1)-(6) in Proposition 1 are true.

Lemma 1: Let $x, y \in U$, we have

(1).$C(x) = C(y)$ if and only if $y \in C(x)$.

(2).$D(x) = D(y)$ if and only if $y \in D(x)$.

Firstly, we study the property (1) and the property (2) in Proposition 1.

Theorem 1: Let $x \in U$ and $C \to_x D$ be a decision rule, then the following are equivalent.

(1) $\sum\limits_{y\in C(x)} cer_y(C,D) = 1$.

(2)$C(y)\bigcap D(y) = \{y\}$ for each $y \in C(x)$.

Proof: (1) $\Rightarrow$ (2) :If $\sum\limits_{y\in C(x)} cer_y(C,D) = 1$,

i.e. $\sum\limits_{y\in C(x)} \frac{|C(y)\cap D(y)|}{|C(y)|} = 1$.

By Lemma 1(1), we have $C(y) = C(x)$ for each $y \in C(x)$. so

$$\sum_{y\in C(x)} |C(y)\cap D(y)| = |C(x)|,$$

For each $y \in C(x)$, we have $y \in C(y)\cap D(y)$, hence $|C(y)\cap D(y)| \geq 1$.

In fact, if $|C(y')\cap D(y')| > 1$ for some $y' \in C(x)$, then

$$\sum_{y\in C(x)} |C(y)\cap D(y)| > |C(x)|,$$

This is a contradiction.

Thus we obtain that $|C(y)\cap D(y)| = 1$ for each $y \in C(x)$, i.e. $C(y)\cap D(y) = \{y\}$ for each $y \in C(x)$.

(2) $\Rightarrow$ (1) : If $C(y)\bigcap D(y) = \{y\}$ for each $y \in C(x)$, then

$$\begin{aligned}\sum_{y\in C(x)} cer_y(C,D) &= \sum_{y\in C(x)} \frac{|C(y)\cap D(y)|}{|C(y)|} \\ &= \sum_{y\in C(x)} \frac{|\{y\}|}{|C(x)|} \\ &= \frac{\sum_{y\in C(x)} 1}{|C(x)|} = 1.\end{aligned}$$

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Similarly, we have

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Theorem 2: Let $x \in U$ and $C \rightarrow_x D$ be a decision rule, then the following are equivalent.

(1) $\sum_{y \in D(x)} cov_y(C, D) = 1$.

(2)$C(y) \bigcap D(y) = \{y\}$ for each $y \in D(x)$.

Secondly, we look at the property (3) and the property (4) in Proposition 1.

Lemma 2: Let $x \in U$ and $C \rightarrow_x D$ be a decision rule, if

$$\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$$

and

$$\pi(C(x)) = \sum_{y \in D(x)} cov_y(C, D)\pi(D(y)),$$

then the following hold.

(1). $|C(x)| = |D(x)|$.

(2). $\sum_{y \in C(x)} cer_y(C, D) = 1$.

(3). $\sum_{y \in D(x)} cov_y(C, D) = 1$.

Proof: If $\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$, then $|D(x)| = \sum_{y \in C(x)} |C(y) \bigcap D(y)|$.

In fact $\sum_{y \in C(x)} |C(y) \bigcap D(y)| \geq |C(x)|$, so we have

$$|D(x)| \geq |C(x)|.$$

If $\pi(C(x)) = \sum_{y \in D(x)} cov_y(C, D)\pi(D(y))$ then

$$|C(x)| = \sum_{y \in D(x)} |C(y) \bigcap D(y)|.$$

In fact $\sum_{y \in D(x)} |C(y) \bigcap D(y)| \geq |D(x)|$, so we have

$$|C(x)| \geq |D(x)|.$$

Thus, we get

$$|C(x)| = |D(x)|.$$

If $\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$, then by Lemma 1,

$$\pi(D(x)) = \pi(C(x)) \sum_{y \in C(x)} cer_y(C, D)$$

i.e.

$$|D(x))| = |C(x)| \sum_{y \in C(x)} cer_y(C, D).$$

Thus,we get

$$\sum_{y \in C(x)} cer_y(C, D) = 1.$$

Similarly,we have

$$\sum_{y \in D(x)} cov_y(C, D) = 1.$$

■

Similarly, we have

Lemma 3: Let $x \in U$ and $C \rightarrow_x D$ be a decision rule, if two of the following conditions hold, then other holds.

(1).$\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$.

(2). $\sum_{y \in C(x)} cer_y(C, D) = 1$.

(3).$|C(x)| = |D(x)|$.

Theorem 3: Let $x \in U$ and $C \rightarrow_x D$ be a decision rule, then

$$\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$$

and

$$\pi(C(x)) = \sum_{y \in D(x)} cov_y(C, D)\pi(D(y)),$$

hold if and only if the following expressions hold.

(1). $|C(x)| = |D(x)|$.

(2). $\sum_{y \in C(x)} cer_y(C, D) = 1$.

(3). $\sum_{y \in D(x)} cov_y(C, D) = 1$.

Proof: The sufficiency can be obtained by Lemma 2 and Lemma 3.

We prove the necessity as follows.

If $\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$, then

$$|D(x)| = \sum_{y \in C(x)} |C(y) \cap D(x)|.$$

Since

$$\sum_{y \in C(x)} |C(y) \cap D(x)| \geq |C(x)|,$$

we have

$$|D(x)| \geq |C(x)|.$$

If $\pi(C(x)) = \sum_{y \in D(x)} cov_y(C, D)\pi(D(y))$, then

$$|C(x)| = \sum_{y \in D(x)} |C(x) \cap D(y)|.$$

Since

$$\sum_{y \in D(x)} |C(x) \cap D(y)| \geq |D(x)|,$$

we have

$$|C(x)| \geq |D(x)|.$$

Thus, we have

$$|C(x)| = |D(x)|.$$

If $\pi(D(x)) = \sum_{y \in C(x)} cer_y(C, D)\pi(C(y))$, then by Lemma 1(1),

$$\pi(D(x)) = \pi(C(x)) \sum_{y \in C(x)} cer_y(C, D),$$

by definition, we have

$$|D(x)| = |C(x)| \sum_{y \in C(x)} cer_y(C, D).$$

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Hence

$$\sum_{y \in C(x)} cer_y(C, D) = 1.$$

Similarly, we have

$$\sum_{y \in D(x)} cov_y(C, D) = 1.$$

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