

Reliability-Based Ductility Seismic Spectra of Structures with Tilting

Federico Valenzuela-Beltran, Sonia E. Ruiz, Alfredo Reyes-Salazar, Juan Bojorquez

Abstract—A reliability-based methodology which uses structural demand hazard curves to consider the increment of the ductility demands of structures with tilting is proposed. The approach considers the effect of two orthogonal components of the ground motions as well as the influence of soil-structure interaction. The approach involves the calculation of ductility demand hazard curves for symmetric systems and, alternatively, for systems with different degrees of asymmetry. To get this objective, demand hazard curves corresponding to different global ductility demands of the systems are calculated. Next, Uniform Exceedance Rate Spectra (UERS) are developed for a specific mean annual rate of exceedance value. Ratios between UERS corresponding to asymmetric and to symmetric systems located in soft soil of the valley of Mexico are obtained. Results indicate that the ductility demands corresponding to tilted structures may be several times higher than those corresponding to symmetric structures, depending on several factors such as tilting angle and vibration period of structure and soil.

Keywords—Asymmetric yielding, tilted structures, seismic performance, structural reliability.

I. INTRODUCTION

STRUCTURES built in soft soil may suffer tilting due to differential settlements, leading to asymmetric yield strength which causes accumulation of plastic deformation demands in only one direction. This may significantly affect the seismic response and the seismic reliability of structures subjected to long duration intense ground motions. The asymmetric behavior may also be caused by the asymmetry of vertical loads and by the presence of adjacent buildings with different weight, height, and foundation characteristics, located in soft soils.

There are several buildings with tilting problems in the valley of Mexico City, which is due to the particular characteristics of the soft soil of the valley. The requirements for the design of tilted structures were included for the first time in Mexico City Building Code in 1987 (RCDF-1987) [1], since then, several Mexican researchers have studied this problem. In the next section, a literature review related to the seismic behavior of tilted structures is presented.

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II. PREVIOUS WORK

Ruiz et al. [2], using single degree of freedom (SDOF) systems with bilinear hysteresis behavior, found that the ductility demands of systems with asymmetric yield strength subjected to narrow band seismic motions are much higher than those corresponding to symmetric structures, and they proposed expressions to consider such increment in the ductility demands. Ruiz [3] proposed an expression to estimate the expected amplification factor of seismic design forces which takes into account the asymmetry in the structural yield strength, as well as the duration of the ground motion intensity. The author concluded that the expression was more conservative than the requirements included in RCDF-1987 and suggested to modify these. Terán-Gilmore et al. [4] studied the dynamic response of tilted SDOF systems designed in accordance with RCDF-1987 requirements. They concluded that the design of structures with asymmetric yielding should consider the hysteretic behavior and the lateral strength of the structure, as well as the frequency content and the duration of the seismic excitation. Terán-Gilmore and Arroyo-Espinoza [5] through the studying of SDOF systems with different hysteretic behavior, proposed mathematical expressions to estimate the strength amplification factor for the design of earthquake resistant structures with asymmetric yield strength.

Despite the important contributions of the studies just mentioned, all of them are limited to the analysis of SDOF systems subjected to unidirectional analyses. The influence of the two components of the seismic ground motions, the soil-structure interaction, and the implicit levels of reliability in the analysis and design of structures with asymmetric yielding has not been studied.

In this study, a methodology based on a reliability assessment to consider the increment of the ductility demands of tilted structures is proposed. It considers two horizontal components of the ground motions and soil-structure interaction.

III. RELIABILITY ASSESSMENT

In the seismic design guidelines exist several reliability based formats [6], for example: a) the semi-probabilistic [7], b) first order and second moments (FOSM) [8], c) load and resistance factors design (LRFD) format [9], d) those based on seismic hazard analysis [10], [11], and e) those based on optimization [12], [13].

In the present study the reliability is evaluated with the format based on a seismic hazard analysis, using structural

demand hazard curves, $v_D(d)$, which represent the annual rate of exceedance of certain values of the structural demand.

To obtain the structural demand hazard curves by means of the numerical integration method, an integral which includes the derivative of the seismic hazard curve multiplied by the vulnerability of the system is required, as follows [14]:

$$v_D(d) = \int_0^\infty \left| \frac{dv(S_a)}{d(S_a)} \right| P(D \geq d | S_a) d(S_a) \quad (1)$$

where

$$\frac{dv(S_a)}{d(S_a)}$$

is the derivative of the seismic hazard curve, S_a is the seismic intensity, and $P(D \geq d | S_a)$, which represents the system vulnerability, is the probability that the structural response exceeds certain specified value d , given an intensity. It must be noted that in order to calculate the demand hazard curves, it is necessary to know the seismic hazard curve associated with the site where the structure will be built and with a given fundamental vibration period (T_1). A seismic hazard curve represents the average number of occurrences of an event that exceeds per unit of time a given intensity. Fig. 1 shows the seismic hazard curves corresponding to the zone considered in this study. In Fig. 1, the abscissas are expressed in terms of the intensity, which in this case is the spectral pseudo-acceleration divided by gravity (S_a/g), and the ordinates represent the mean annual rate of exceedance of the intensity (v).

IV. METHODOLOGY

With the aim of satisfying the objectives of the study it is necessary to obtain the seismic response of the systems with and, alternatively, without tilting. Here, the global ductility of the systems (μ) is taken as the structural demand of interest, which is defined as the ratio between the maximum displacement and the yield displacement of the structure. In order to obtain μ for a given system subjected to bi-directional analyses, the ductility developed in each direction needs to be calculated, and next to be combined using (2).

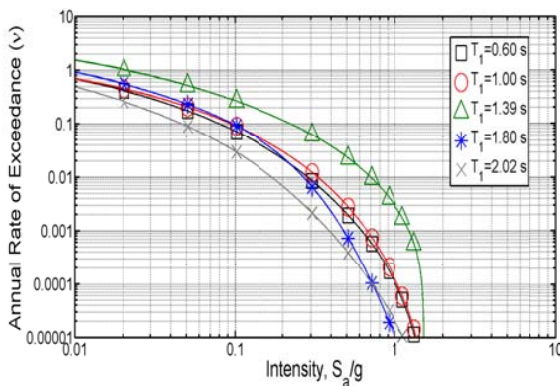


Fig. 1 Seismic hazard curves for the zone considered, corresponding to vibration periods $T_1 = 0.60$ s, 1.00 s, 1.39 s, 1.80 s, and 2.02 s

$$\mu = \sqrt{\mu_X + \mu_Y} \quad (2)$$

where μ_X and μ_Y are the ductility demands in X and Y directions, respectively.

A. Structural Systems

Tridimensional single story models (SSM) are used here to study the aspects mentioned above. Fig. 2 shows the geometrical dimensions of the models. Each system was subjected to two horizontal components of the seismic motions acting at the same time. First, they are analyzed without tilting (symmetric systems), and after, different values of asymmetry are considered. The fundamental vibration periods of the systems studied range from 0.60 s to 2.02 s.

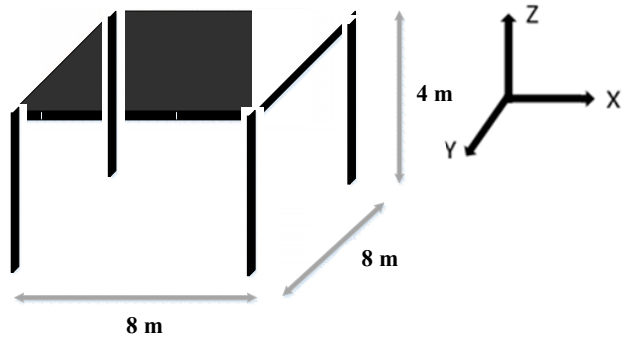
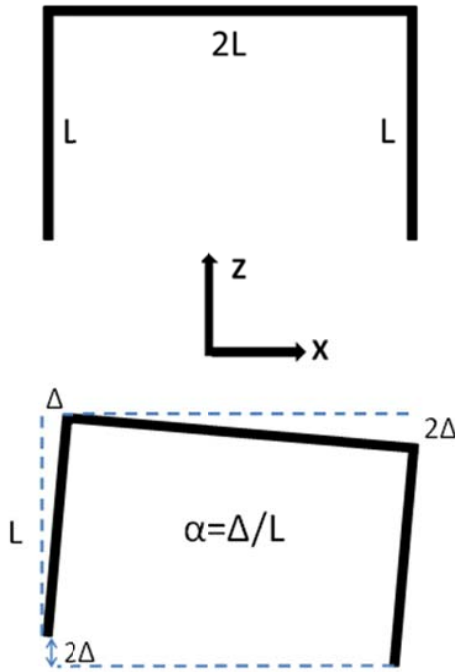


Fig. 2 Dimensions of the SSM

The level of asymmetry of the systems is characterized by the parameter α , which is the displacement in the horizontal direction (Δ) divided by the height of the structural system (L), as shown in Fig. 3. The soil-structure interaction is considered assuming a "fake story" in which the translational and rotational stiffness of the columns are obtained in accordance with Appendix A of Mexico City Building Code (RCDF-2004) [15]. The models are excited by thirteen pairs of accelerograms recorded in soft soil of the valley of Mexico, with dominant periods ranging between 1.26 s and 1.74 s. The records characteristics are shown in Table I.

TABLE I
CHARACTERISTICS OF THE SEISMIC MOTIONS

Event	Date	Magnitude	Epicenter coordinates		PGA (m/s ²)	Dominant period T_s (s)	
			Lat. N	Long. W		E-W	N-S
1	95/09/14	7.3	16.31	98.88	0.311	1.74	1.58
2	97/01/11	6.9	17.90	103.00	0.164	1.70	1.54
3	97/01/11	6.9	17.91	103.04	0.199	1.32	1.28
4	89/04/25	6.9	16.60	99.40	0.554	1.25	1.19
5	95/09/14	7.3	16.31	98.88	0.373	1.29	1.38
6	89/04/25	6.9	16.60	99.40	0.397	1.29	1.38
7	89/04/25	6.9	16.60	99.40	0.239	1.35	1.41
8	95/09/14	7.3	16.31	98.88	0.287	1.57	1.40
9	95/09/14	7.3	16.31	98.88	0.278	1.34	1.42
10	97/01/11	6.9	17.90	103.00	0.130	1.35	1.35
11	97/01/11	6.9	17.91	103.04	0.260	1.36	1.48
12	95/09/14	7.3	16.31	98.88	0.287	1.45	1.28
13	97/01/11	6.9	17.91	103.04	0.175	1.26	1.27


 Fig. 3 Definition of the α parameter

B. Ground Motions

Both components of the seismic motions are scaled in order to obtain the structural demands in terms of the structural ductility as a function of the intensity S_a , which is obtained via the *quadratic mean* as shown in (3):

$$S_a = \sqrt{\frac{S_{aEW}^2 + S_{aNS}^2}{2}} \quad (3)$$

where S_{aEW} and S_{aNS} are the pseudo-acceleration spectra ordinates associated to the fundamental period of vibration of the system under consideration, for 5% of critical damping, corresponding to *E-W* and *N-S* components, respectively.

After obtaining the ductility demands as a function of the intensity, the median and the standard deviation of the demand logarithms are estimated. Next, the vulnerability curves of the systems are calculated using (4):

$$P(D \geq d | S_a) = 1 - \Phi(\ln(\frac{d}{\hat{D}}) / \sigma_{\ln D}) \quad (4)$$

where, $P(D \geq d | S_a)$, as mentioned before, is the probability that the ductility of the structure exceeds certain specified value d , for a given level of intensity S_a , and \hat{D} and $\sigma_{\ln D}$ are the median and the standard deviation of the structural demand logarithm, respectively. Once the vulnerability curves are obtained, the ductility demand hazard curves are calculated using (1).

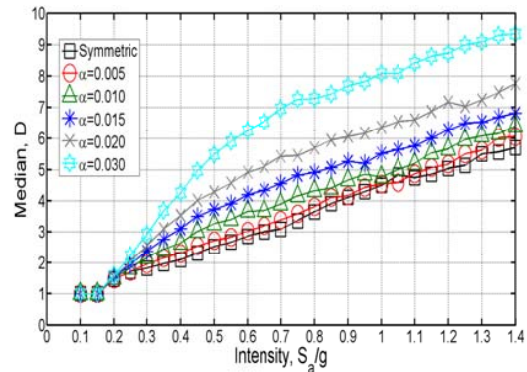
Demand hazard curves are obtained for several fundamental vibration periods of the systems and different values of α . With the demand hazard curves, it is possible to obtain Uniform Exceedance Rate Spectra (UERS), which can be developed for several α and ductility values, associated with a mean annual rate of exceedance.

V. RELIABILITY ASSESSMENT OF THE SEISMIC PERFORMANCE OF TILTED STRUCTURES

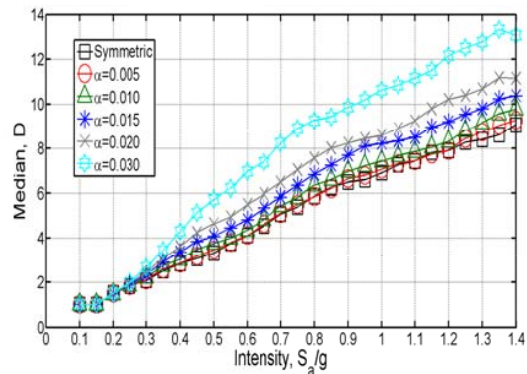
A. Median and Standard Deviation of the Structural Demands

In this section each of the steps of the proposed methodology are described more clearly. The procedure is illustrated by means of the analysis of two structural systems having a seismic coefficient $C_s=0.20$, and vibration periods of 1.39 s and 2.02 s, subjected to the thirteen pairs of seismic motions shown in Table I.

The first step of the proposed methodology is to obtain the median values of the ductility demands, as well as the standard deviations of the logarithm of the ductility demands as functions of S_a/g , as shown in Figs. 4 and 5, respectively, for several values of α . It is noted that the results shown in Figs. 4 and 5 are only for two systems ($T_1=1.39$ s and $T_1=2.02$ s, respectively); however, the study was performed for several structural systems with different T_1 values.



(a)



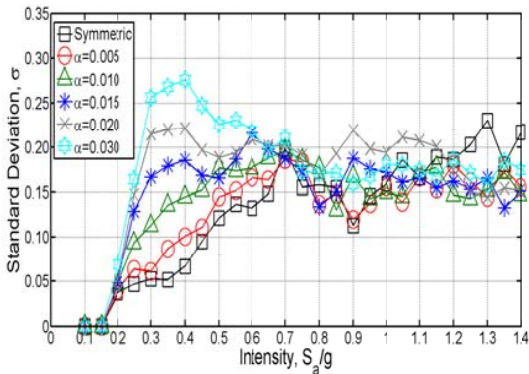
(b)

Fig. 4 Median values of the ductility demands for systems with vibration periods of (a) 1.39 s and (b) 2.02 s

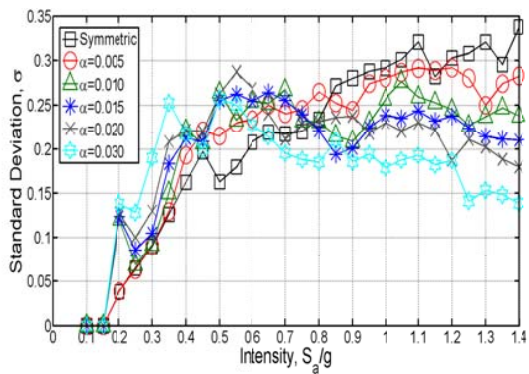
Fig. 4 shows that the median of the ductility demand increases significantly as the values of α grow. The increment in the ductility demand is more important for the system with $T_1=1.39$ s than for those with $T_1=2.02$ s. For example, for an intensity $S_a/g=0.70$ and $\alpha=0.030$, the system with $T_1=1.39$ s has an increment of the mean ductility demand of 125% with

respect to the symmetric system, while the system with $T_1=2.02$ s has only an increment of 62%. It indicates that the ratio between the fundamental period of vibration of the system and the dominant period of the excitation has a significant influence on the structural demand of tilted structures.

In Fig. 5 it can be seen that, in general, σ values are relatively small ($\sigma < 0.35$) and do not show a specific trend. Standard deviation of the structural demand logarithm increases rapidly for S_a/g values from 0 to 0.5, and for higher intensities ($S_a/g > 0.5$), it does not vary much, and in some cases (e.g. Fig. 5 (a)) it remains almost constant.



(a)



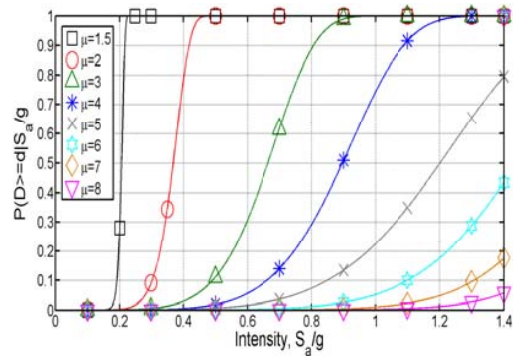
(b)

Fig. 5 Standard deviations of the logarithms of the ductility demands for systems with vibration period of (a) 1.39 s and (b) 2.02 s

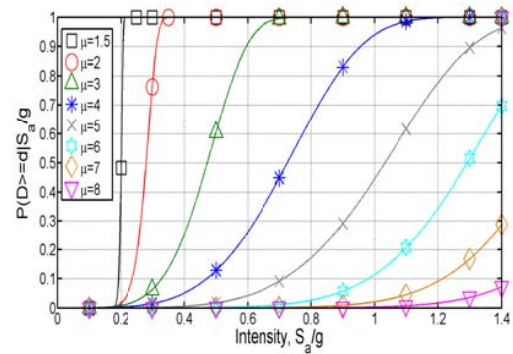
B. Vulnerability Curves

The second step is to obtain vulnerability curves for several values of structural ductility demands, using (4). The curves are shown in Fig. 6 for ductility's values ranging from 1.5 to 8. Results are shown for a few values of α . It is observed that the exceedance probability of a given ductility demand becomes higher as the level of asymmetry of the system increases. The results indicate that the increment in the ductility demand of the asymmetric systems with respect to the symmetric systems may be very important in some cases, particularly for systems with vibration periods close to the dominant period of the soil, especially in the zone of periods

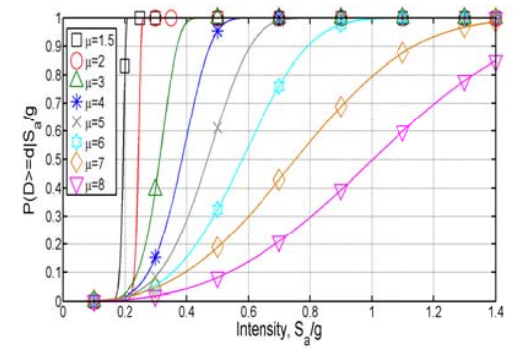
where the system may suffer structural “softening” behavior.



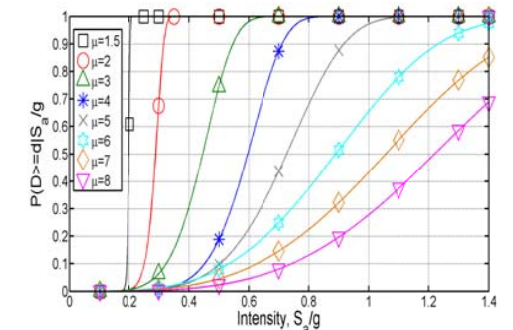
(a) $\alpha=0.0$, $T_1=1.39$ s



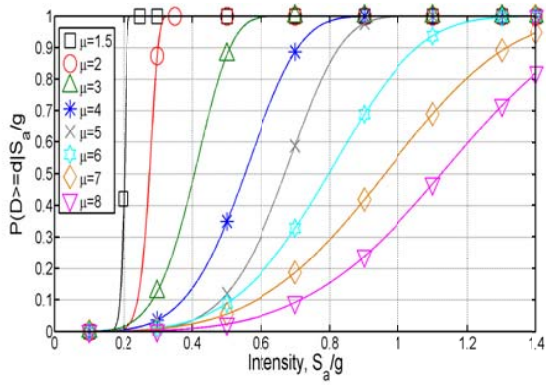
(b) $\alpha=0.010$, $T_1=1.39$ s



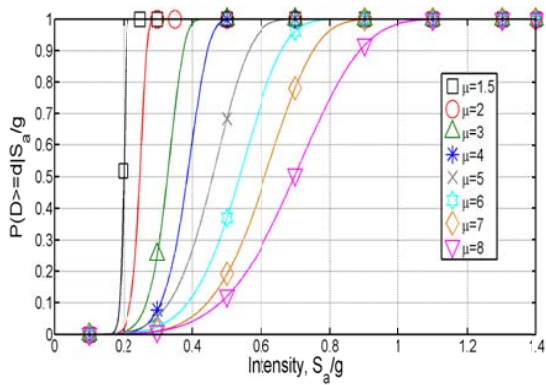
(c) $\alpha=0.030$, $T_1=1.39$ s



(d) $\alpha=0.0$, $T_1=2.02$ s



(e) $\alpha=0.010, T_1=2.02$ s



(f) $\alpha=0.030, T_1=2.02$ s

Fig. 6 Vulnerability curves for several values of ductility demands of the systems

C. Ductility Demand Hazard Curves

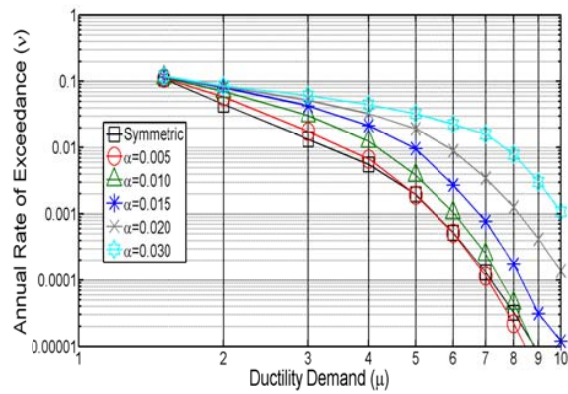
The next step is to develop ductility demand hazard curves for symmetric as well as for asymmetric systems using (1). The corresponding curves are shown in Fig. 7. In this, it is observed that the ductility demand corresponding to a given return period (T_R) is significantly higher for asymmetric than for symmetric systems, as expected. The increment in the ductility demand is more significant for systems whose vibration period is closer to that corresponding to the dominant period of the spectrum. As can be seen in Fig. 7 (a), for a value of $\nu=0.008$, it corresponds a ductility of 3.3 for the symmetric system, while, for $\alpha=0.030$ the expected ductility is 8, which represents an increment of almost 150% in this value. On the other hand, for the system with $T_1=2.02$ s (Fig. 7 (b)) and for the same value of the mean annual rate of exceedance, such increment is only of 15%.

From here it can be concluded that, the increment in the expected ductility demand of tilted structures depends on the level of asymmetry of the system as well as of the ratio between the vibration period of the system and that of the excitation.

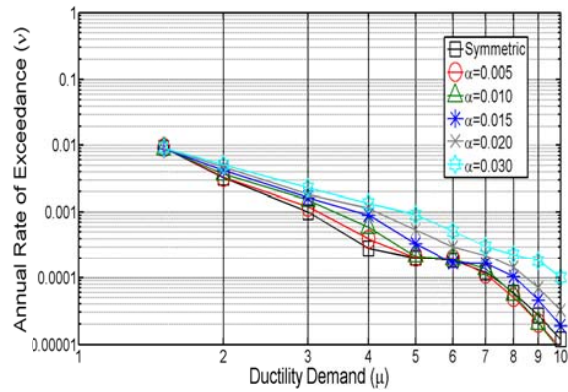
D. Uniform Exceedance Rate Spectra

Next, based on the ductility demand hazard curves, UERS are calculated for asymmetric and for symmetric systems.

These are developed for a value of $\nu=0.008$, corresponding to a return period of 125 years. Fig. 8 shows UERS for systems with T_1 varying from 0.60 s to 2.02 s, and for $T_R=125$ years.



(a) $T_1=1.39$ s



(b) $T_1=2.02$ s

Fig. 7 Ductility demand hazard curves for several values of α corresponding to systems with a) $T_1=1.39$ s and b) $T_1=2.02$ s

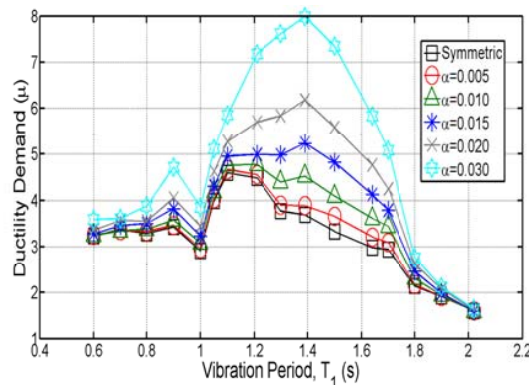


Fig. 8 Uniform Exceedance Rate Spectra for $T_R=125$ years

In Fig. 8 is observed that the expected ductility demands are significantly higher for asymmetric than for symmetric systems. This becomes more notorious for systems with vibration periods close to the dominant periods of the spectra (which in this example are ranging from 1.2 s to 1.7 s). Ratios

between UERS corresponding to asymmetric and to symmetric systems are obtained, as shown in (5):

$$R(T_1, \alpha, \nu) = \frac{\mu[UERS(T_1, \nu, \alpha)]}{\mu[UERS(T_1, \nu, \alpha=0.0)]} \quad (5)$$

where T_1 , α and ν are the parameters for the system for which the R values are calculated. Equation (5) represents the ratio between the UERS of a system with certain characteristics having a degree of asymmetry (α), with respect to a system with the same characteristics but with symmetric strength ($\alpha=0$).

The calculated values of the parameter R for a return period 125 years are shown in Fig. 9. It can be observed that the increment in the ductility demand is important for systems with vibration periods ranging from 1.2 s to 1.7 s, particularly for systems with $T_1=1.5$ s. The increment becomes more important as α (i.e. tilting of the system) increases, as expected. On the other hand, for systems with vibration periods away from the spectral dominant period of the excitation, the increment of the ductility demand becomes negligible for any value of α .

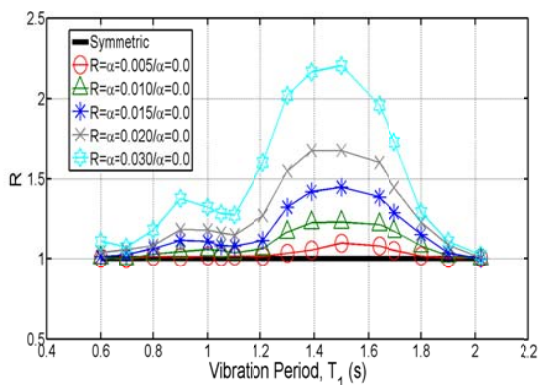


Fig. 9 R values corresponding to $T_R=125$ years

VI. CONCLUSIONS

A reliability-based methodology which uses ductility demand hazard curves and uniform exceedance rate spectra to consider the increment in the expected ductility demand of tilted structures is proposed. In the formulation, the influence of the two orthogonal components of the seismic ground motions as well as the soil-structure interaction is considered. It is concluded that the ductility demand corresponding to tilted structures with respect to symmetric structures increases as their level of asymmetry grows. This increment may be close to 200% in some cases, particularly for systems with vibration periods close to the dominant period of the spectrum. In summary, the increment in the ductility demand of tilted structures depends on several factors such as the tilting angle and the ratio between the vibration period of the system and of the soil.

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