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Reliability Analysis of Press Unit using Vague Set

S.P. Sharma, Monica Rani

Abstract—In conventional reliability assessment, the reliability data of system components are treated as crisp values. The collected data have some uncertainties due to errors by human beings/machines or any other sources. These uncertainty factors will limit the understanding of system component failure due to the reason of incomplete data. In these situations, we need to generalize classical methods to fuzzy environment for studying and analyzing the systems of interest. Fuzzy set theory has been proposed to handle such vagueness by generalizing the notion of membership in a set. Essentially, in a Fuzzy Set (FS) each element is associated with a point-value selected from the unit interval [0, 1], which is termed as the grade of membership in the set. A Vague Set (VS), as well as an Intuitionistic Fuzzy Set (IFS), is a further generalization of an FS. Instead of using point-based membership as in FS, interval-based membership is used in VS. The interval-based membership in VS is more expressive in capturing vagueness of data. In the present paper, vague set theory coupled with conventional Lambda-Tau method is presented for reliability analysis of repairable systems. The methodology uses Petri nets (PN) to model the system instead of fault tree because it allows efficient simultaneous generation of minimal cuts and path sets. The presented method is illustrated with the press unit of the paper mill.

Keywords—Lambda - Tau methodology, Petri nets, repairable system, vague fuzzy set.

I. INTRODUCTION

For fast technology innovation, new product development is getting much complicated not only on its system functions, but also on its system components. Therefore, reliability analysis of a system is an important issue in the successful development of a system and it becomes tedious, time consuming and vague for a complex industrial system. It is well known that most databases, on which most of the reliability analyses depend, are either out of date or collected under different operating and environmental conditions. The two most important concepts for quantifying system availability are the failure and repair rates of hardware and human errors. It is common knowledge that large amount of data is required in order to estimate more accurately, the failure, error or repair rates. However, it is usually impossible to obtain such a large quantity of data in any particular plant. From this point of view, fuzzy probabilities or possibilities are better suited to model such judgement in a flexible and efficient manner.

For failure analysis of these systems, various methods available in literature are reliability block diagrams (RBD's), Monte Carlo simulation, Markov modeling, Failure mode effective analysis, root cause analysis, Fault tree analysis (FTA), and Petri nets (PN) etc [1]–[4]. Both Petri nets and fault trees are used for software reliability analysis [5], [6], analysis of coherent fault trees [7] and fault diagnosis [8].

The use of fuzzy set theory and fuzzy arithmetic to determine components or system reliability can be found in the literature. Singer [3] proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. Cheng and Mon [9] used interval of confidence for analyzing the fuzzy system reliability. Through theoretical analysis and computational results they have shown that their proposed approach is more general and straight-forward as compared to [3]. Chen [10] presented a new method for analyzing the fuzzy system reliability using fuzzy number arithmetic operations and used simplified fuzzy arithmetic operations rather than complicated interval fuzzy arithmetic operations of fuzzy numbers [9] or the complicated extended algebraic fuzzy numbers [3].

The present paper outlines a novel approach for determining the reliability of repairable technical system making use of Petri nets (PNs) and utilizes the fuzzy Lambda-Tau methodology (FLTM) in conjunction with vague set theory to obtain quantitative results. The PN methodology of reliability modeling is similar to that of fault tree modeling using graphical representation of the relation between conditions and events. The PNs have been used in reliability engineering for reliability evaluation, fault-tolerant analysis, safety- analysis and reliability-growth. The Lambda-Tau method is a technique for dealing with repairable systems in fault tree analysis [11].

Knezevic and Odoom [11] used quantified failure and repair data in the form of triangular fuzzy numbers and analyzed the behavior of a general system by using formulas of some reliability indices (measures). These indices include failure rate, repair time, mean time between failures (MTBF), expected number of failures (ENOF), availability and reliability of the system which gave more sound idea about the behavior of the system. In their approach, PN is used to model the system while fuzzy set theory is used to quantify the uncertain, vague, and imprecise data. By using this, some researchers [12]–[15] have analyzed the behavior of some complex repairable industrial system by using PN and fuzzy approach.

The main characteristic of fuzzy sets is that: the membership function assigns to each element u in a universe of discourse (U) a membership degree ranging between zero and one and the non-membership degree equals one minus the membership degree, i.e., this membership degree combines the evidences for against u. The single number tells us nothing about the lack of knowledge. In real applications, however, the information of an object corresponding to a fuzzy concept may be incomplete, i.e., the sum of the membership degree and the non-membership degree of an element in a universe corresponding

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to a fuzzy concept may be less than one. In fuzzy set theory, there is no means to incorporate the lack of knowledge with the membership degrees. A possible solution to investigate the uncertainty in a more realistic manner is to use intuitionistic fuzzy set (IFS) theory [16] and vague set theory [17] which are the extensions of the classic fuzzy set theory. These are more generalized concepts for representing uncertainty in data. Bustince and Burillo [18] showed that vague sets are the same as IFSs. When using vague set, a decision maker concentrates on both membership and non-membership function and so it can model unknown information by using the degree of hesitation.

Chen [19] proposed fuzzy system reliability analysis based on vague set theory, where the reliability of the components of a system are represented by vague sets defined in the universe of discourse [0, 1]. Chen method has the advantages of modeling and analyzing fuzzy system reliability in a more flexible and more intelligent manner. In 1995, Chen [20] presented the measures of similarity between vague sets. Shu et al. [21] proposed an algorithm to evaluate the fault interval of the system components using vague FTA and applied this method to the failure analysis problem of Printed Circuit Board Assembly (PCBA). Chang and Cheng [22] obtained fault interval and reliability interval of the PCBA with different membership functions using FTA. Chang et al. [23] proposed an algorithm of vague FTA to calculate fault interval of system components from integrating expert's knowledge and experience in terms of providing the possibility of failure of bottom events. Zarei et al. [24] investigated the Bayesian estimation based on vague lifetime data.

In the present paper, the vagueness and incomplete information of the system is dealt by using vague set theory. The presented methodology involves qualitative modeling using PN and quantitative analysis using vague lambda-tau methodology with the basic events represented by vague numbers of triangular membership function. The presented approach is applied to the press unit of the paper mill situated in northern part of India producing approximately 200 tons of paper per day.

The paper is organized in three sections. In section II, methodology for reliability analysis is discussed in which some introduction to Petri nets theory and Lambda-tau methodology followed by the definition of vague set and its operations is given. In order to illustrate the presented approach, section III considers an example of a press unit of a paper mill and the results are compared with the lambda-tau approach. The final section IV presents the conclusions.

II. METHODOLOGY FOR RELIABILITY ANALYSIS

The proposed methodology involves qualitative modeling using PNs (for modeling the system) and quantitative analysis using vague fuzzy set (for handling the uncertainties). In brief, these are briefly described below.

A. System modeling with Petri net theory

Similar to fault tree model, PN also represents graphically the cause and effect relationship and interaction among the working units of the system to be modeled [25]. PN has a static part as well as a dynamic part. The static part is made of only three objects: places, transitions and arrows. The dynamic part is the marking of the graph and it is made of various 'tokens' which are present, or not present in the various places and evolves dynamically according to the 'firing' of the various valid transitions. In PN model, places (events) correspond to discrete states represented by circles while and the transition (gates) are represented by bars.

In the graphical model places and transitions are connected by arcs. A directed arc form the transition to a place is said to be input arc and the one from place to transition is called an output arc, with respect to the place and vice versa with respect to the transition. The number of places as well as number of transition are finite but not zero. In this study only the static part of PN is used i.e. the tokens are omitted and it is assumed that transitions are not timed, i.e. they are immediate transitions. Fig. 1 illustrate the equivalent PN models, corresponding to the logical basic AND and OR gates.

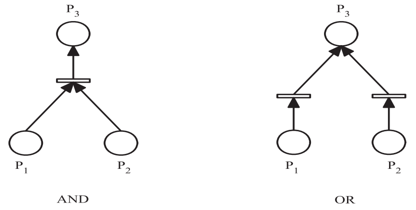


Fig. 1. Petri Net Model of Logical AND and OR Operations

B. Minimal cut set and path sets using PN model

A cut set is a set of components whose failure will result in a system failure and a minimal cut set is one in which all the component must fail in order that the system fails. Similarly, a path set is a set of components whose functioning ensures that the system functions and a minimal path set is one in which all the components within the set must function for the system to function. Minimal cut and path sets can be derived from a PN model more efficiently than from an equivalent fault tree model. It was demonstrated through a matrix method [25] that the determination of minimal path sets could be achieved in PN model without transforming the PN to its dual and thus ensured that the minimal cut and path sets can be determined at the same time.

C. Lambda-Tau methodology

Lambda-Tau methodology is a traditional method in which fault tree is used to model the system. The constant failure rate model is adopted in this method and the basic expressions used to evaluate failure rate and repair time associated with logical AND and OR-gates of fault tree model are given in Table I and reliability parameters for the system are given in Table II.

TABLE I
BASIC EXPRESSIONS OF LAMBDA TAU METHODOLOGY

Gate	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expression	$\prod_{j=1}^{n} \lambda_j \left[\sum_{i=1}^{n} \prod_{\substack{j=1 \\ i \neq j}}^{n} \tau_j \right]$	$\dfrac{\prod_{i=1}^{n} \tau_i}{\sum_{j=1}^{n} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n} \tau_i \right]}$	$\sum_{i=1}^{n} \lambda_i$	$\dfrac{\sum_{i=1}^{n} \lambda_i \tau_i}{\sum_{i=1}^{n} \lambda_i}$

TABLE II
SOME RELIABILITY PARAMETERS

Parameters	Expressions
Failure rate	$\text{MTTF}_s = \frac{1}{\lambda_s}$
Repair time	$\text{MTTR}_s = \frac{1}{\mu_s} = \tau_s$
MTBF	$\text{MTBF}_s = \text{MTTF}_s + \text{MTTR}_s$
ENOF	$\text{W}_s(0,t) = \frac{\lambda_s \mu_s t}{\lambda_s + \mu_s} + \frac{\lambda_s^2}{(\lambda_s + \mu_s)^2}[1 - e^{-(\lambda_s + \mu_s)t}]$
Reliability	$\text{R}_s = e^{-\lambda_s t}$
Availability	$\text{A}_s = \frac{\mu_s}{\lambda_s + \mu_s} + \frac{\lambda_s}{\lambda_s + \mu_s} e^{-(\lambda_s + \mu_s)t}$

D. Vague set theory

In 1965, Zadeh proposed fuzzy sets to describe situations in which the data are imprecise or vague. Fuzzy set theory has been shown to be a useful tool to handle such situations by attributing a degree to which a certain object belongs to a set. In real life, a person may assume that an object belongs to a set to a certain degree, but it is possible that he is not so sure about it. In other words, there may be a hesitation or uncertainty about the membership degree of an object. Gau and Buehrer [17] presented the concepts of vague sets. A vague set $\widetilde{A}$ in U is characterized by a truth membership function t_A and a false membership function f_A such that $t_A : U \to [0,1]$; $f_A : U \to [0,1]$ where $t_A(u_i)$ is a lower bound on the grade of membership of u_i derived from the evidence for u_i, $f_A(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i and $t_A(u_i) + f_A(u_i) \leq 1$. The grade of membership of u_i in the vague set $\widetilde{A}$ is bounded to a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of [0, 1]. The vague value $[t_A(u_i), 1 - f_A(u_i)]$ indicates that the exact grade of membership $\mu_A(u_i)$ of u_i may be unknown, but is bounded by $t_A(u_i) \leq \mu_A(u_i) \leq 1 - f_A(u_i)$.

E. Arithmetic operations between two triangular vague set

A simple triangle vague set is represented as $\widetilde{A} =< [(a,b,c); \mu, \nu] >$, as shown in Fig. 2.

The four basic arithmetic operation i.e., addition, subtraction, multiplication and division, on two triangular vague sets $\widetilde{A} =< (a_1, b_1, c_1); \mu_1, \nu_1 >$ and $\widetilde{B} =< (a_2, b_2, c_2); \mu_2, \nu_2 >$ are shown in Table III with $\mu = \min(\mu_1, \mu_2)$ and $\nu = \min(\nu_1, \nu_2)$

TABLE III
BASIC OPERATIONS ON INTUITIONISTIC FUZZY NUMBERS

Operation	Crisp	Intuitionistic fuzzy
Addition	$A + B$	$\widetilde{A} + \widetilde{B} = [a_1 + a_2, b_1 + b_2, c_1 + c_2]$
Subtraction	$A - B$	$\widetilde{A} + \widetilde{B} = [a_1 - c_2, b_1 - b_2, c_1 - a_2]$
Multiplication	$A \cdot B$	$\widetilde{A} + \widetilde{B} = [a_1 \cdot a_2, b_1 \cdot b_2, c_1 \cdot c_2]$
Division	$A \div B$	$\widetilde{A} + \widetilde{B} = [\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}]$, if $0 \notin [a_2, b_2, c_2]$

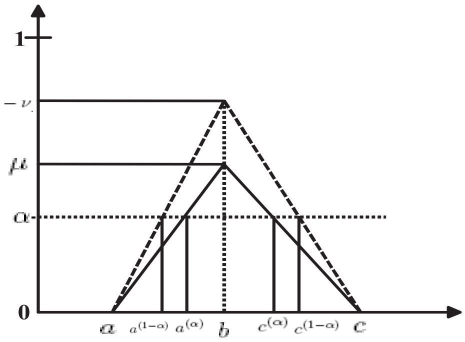


Fig. 2. $\alpha-$ cut of IFS $\widetilde{A}$

In fuzzy Lambda-Tau methodology, the highest level of confidence of domain experts is taken as 1. In real life problems, the highest level of confidence of domain experts lies between [0, 1] according to the expert's knowledge. To sort out this difficulty, vague lambda-tau methodology for reliability analysis of a system is presented. The major advantage of using vague sets over fuzzy sets is that the vague sets separate the positive and negative evidences for membership of an element in the set and also, in vague sets, the level of confidence of domain experts lies between [0, 1] and can describe the uncertainty of confidence level. The presented method can model and analyse the vague reliability in a more flexible and intelligent manner. The basic parameters for reliability analysis as given in Table II are modified using arithmetic operation given in Table III . For the reliability analysis of the repairable system, the minimal cut sets of the system are obtained by using Petri net model. Based on these cut sets, various reliability parameters of interest such as failure rate, repair time, reliability, availability, MTBF, ENOF are evaluated at top event of the system at various α-cuts.

III. ILLUSTRATIVE EXAMPLE

The presented approach for analyzing the system failure behavior is illustrated by taking, a case from a paper mill, which produces approximately 200 tons of paper per day. The paper mills are large capital-oriented engineering systems [2], comprise of subsystems namely chipping, feeding, pulping, washing, screening, bleaching, production of paper consisting of press unit and collection, arranged in predetermined configuration. The present paper considers the most important functionary unit namely press unit as a subject of discussion. It consists of felt (synthetic belt) and top and bottom rolls as its main components. The unit receives wet paper sheet from the forming unit on to the felt, which is then carried through press rolls and thereby reducing the moisture content to the extent. The system consists of

- **Synthetic Belt (A):** One unit only. Its failure causes the complete failure of the system.
- **Upper Roller (B) and Lower Roller (C):** These consist of bearing, bending and rubber wear.

The interactions among the various components of the unit is described by its Petri nets model shown in Fig. 3 where

PSF represents the top failure events of the press unit. Also, the input data i.e failure rate (λ's) and repair time (τ's) of the basic events of the press unit is given in Table IV. The minimal cut sets are singletons {A}, $\{B_1\}$, $\{B_2\}$, $\{B_3\}$, $\{C_1\}$, $\{C_2\}$ and $\{C_3\}$, obtained using matrix method.

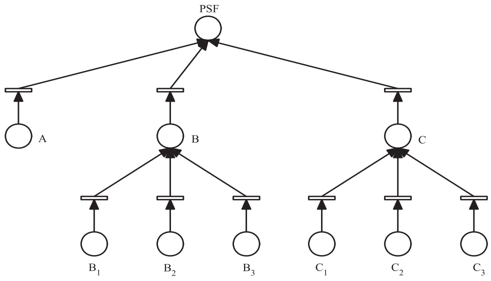


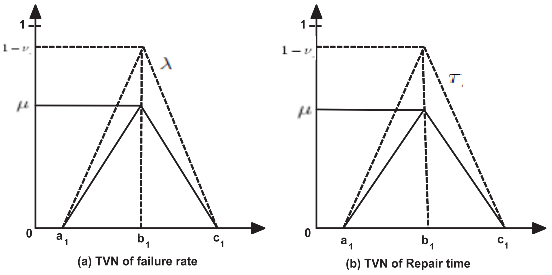
Fig. 3. Petri net model of press unit

TABLE IV
FAILURE RATE(/HRS) AND REPAIR TIME(HRS) OF THE UNIT [2].

Component	λ_i	τ_i
Felt	0.0001	5.0
Bearing (i=2,5)	0.0010	2.0
Bending (i=3,6)	0.0015	3.0
Rubber wear (i=4,7)	0.0020	4.0

The following steps are followed to calculate different parameters of the reliability analysis:

Step 1: To handle the vagueness and uncertainty in data, for all components, crisp input data (λ) and (τ) are converted into known triangular vague numbers(TVNs) using error factor of probability [26] and are represented as shown in Fig. 4 where lower triangle represents the truth membership and upper triangle represents false membership values of the vague set.



(a) TVN of failure rate
(b) TVN of Repair time

Fig. 4. Triangular vague number of the input data

Step 2: As soon as the input triangular vague number for failure rate and repair times for each of the components are known, the corresponding fuzzy value for the top event of the Petri net model of the system can be obtained using the extension principle coupled with alpha(α)-cut and interval arithmetic operations of triangular vague set.

Step 3: After calculating (λ_s) and (τ_s) values of the top event of the system, the various other reliability parameters such as ENOF, MTBF, availability and reliability are calculated at different α-cut with their left and right spread. The calculated values are tabulated in Tables V, VI and VII. And their graphical representations are given in Fig. 5. The results of the vague triangular set are more flexible than Lambda-Tau method, because the left/right end points become interval values when using vague set.

IV. CONCLUSION

Since crisp approach tends to have difficulty in conveying the imprecision or vagueness nature in system modelling in representing the failure rate of system component. Nevertheless, membership values are not always certain in real situations. There may be some degree of hesitation between membership and non-membership. To overcome this difficulty, the fuzzy set theory is applied to evaluate system reliability under fuzzy environment. Vague fuzzy sets are appropriate to deal with the many real life problems in which the available information is insufficient. It can solve the problem that experts cant express the fuzziness under confirmable confidence level when a new type of product is developed. VSs are considered to be highly useful for addressing uncertainty and vagueness and have become a popular topic of investigation in the field of fuzzy set research. In contrast with conventional fuzzy sets, VSs offer additional information about the degree of hesitation. So in this direction, in the present study, an attempt has been made by the authors to analyze the system reliability of a press unit in a paper mill by using vague set theory. To strengthen the analysis, various reliability parameters have been computed in the form of triangular membership function. The technique uses tools of PN and vague fuzzy set theory in which PN is used to model the system while fuzzy takes care of impreciseness in data of each reliability index used for behavior analysis of the system. In the nut shell, we can say that, by using the presented approach, the system reliability can be analysed in a more flexible and intelligent manner.

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TABLE V
TRUTH INTERVAL OF CONFIDENCE OF THE RELIABILITY PARAMETER OF THE TOP EVENT

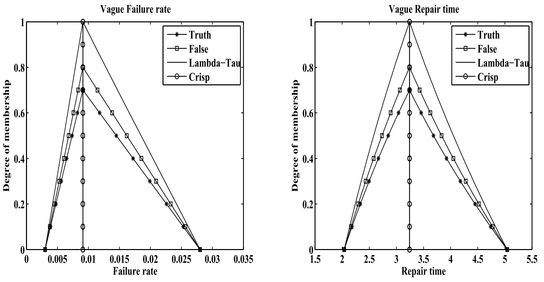
α	Left Spread values						Right Spread values					
	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF
0	0.0030099	2.03667	0.755784	0.8776349	0.0115318	37.75096	0.0280000	5.04379	0.970349	0.9943125	0.699815	337.2763
0.1	0.0038799	2.18130	0.776468	0.8945015	0.0169918	41.70699	0.0253000	4.73813	0.961944	0.9921451	0.551877	262.4734
0.2	0.0047499	2.33422	0.797718	0.9102311	0.0237743	46.58200	0.0226000	4.45047	0.953611	0.9897201	0.430700	214.9786
0.3	0.0056199	2.49591	0.819550	0.9248377	0.0321487	52.74717	0.019900	4.17955	0.945350	0.9870096	0.331623	182.1164
0.4	0.0064899	2.66691	0.841979	0.9383422	0.0424400	60.80645	0.0172000	3.92422	0.937161	0.9839804	0.250818	158.0081
0.5	0.0073599	2.84780	0.865022	0.9507721	0.0550408	71.81332	0.0145000	3.68344	0.929043	0.9805922	0.185135	139.5534
0.6	0.0082299	3.03919	0.888696	0.9621598	0.0704273	87.78495	0.0118000	3.45624	0.920996	0.9767978	0.131973	124.9631
0.7	0.0091000	3.24176	0.913018	0.9725420	0.0891792	113.1319	0.0091000	3.24176	0.913018	0.9725420	0.0891792	113.1319
0.8	–	–	–	–	–	–	–	–	–	–	–	–
0.9	–	–	–	–	–	–	–	–	–	–	–	–
1	–	–	–	–	–	–	–	–	–	–	–	–

TABLE VI
FALSE INTERVAL OF CONFIDENCE OF THE RELIABILITY PARAMETER OF THE TOP EVENT

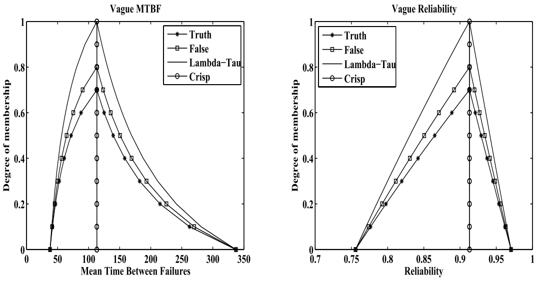
α	Left Spread values						Right Spread values					
	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF
0	0.0030099	2.0366	0.755784	0.877634	0.011531	37.7509	0.0280000	5.0437	0.970349	0.9943125	0.69981	337.276
0.1	0.0037712	2.1627	0.773852	0.892455	0.016242	41.1681	0.0256375	4.7753	0.962990	0.9924296	0.56879	269.942
0.2	0.0045324	2.2951	0.792352	0.906404	0.021940	45.2597	0.0232750	4.5207	0.955687	0.9903519	0.45875	225.152
0.3	0.0052937	2.4342	0.811294	0.919491	0.028802	50.2525	0.0209125	4.2792	0.948440	0.9880615	0.36644	193.182
0.4	0.0060549	2.5802	0.830689	0.9317262	0.037032	56.4885	0.0185500	4.0500	0.941247	0.9855372	0.28914	169.203
0.5	0.0068162	2.7335	0.850548	0.9431278	0.046871	64.5096	0.0161875	3.8322	0.934109	0.9827542	0.22456	150.540
0.6	0.0075774	2.8946	0.870881	0.9537154	0.058605	75.2273	0.0138250	3.6254	0.927025	0.9796837	0.17076	135.595
0.7	0.0083387	3.0638	0.891700	0.9635118	0.072574	90.3048	0.0114625	3.4287	0.919995	0.9762923	0.12609	123.350
0.8	0.0091000	3.2417	0.913018	0.972542	0.089179	113.131	0.0091000	3.2417	0.913018	0.9725420	0.89179	113.131
0.9	–	–	–	–	–	–	–	–	–	–	–	–
1	–	–	–	–	–	–	–	–	–	–	–	–

TABLE VII
VALUES FOR DIFFERENT RELIABILITY PARAMETERS OF THE TOP EVENT USING LAMBDA-TAU METHOD

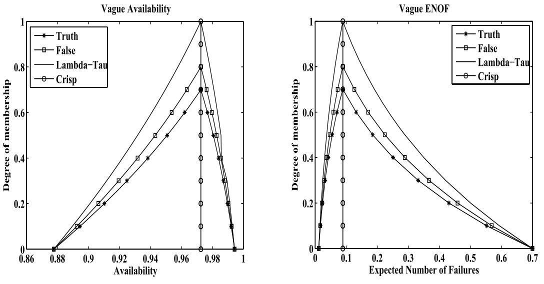
α	Left Spread values						Right Spread values					
	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF	Failure rate	Repair time	Reliability	Availability	ENOF	MTBF
0	0.00300	2.036669	0.7557837	0.8776349	0.01153	37.7509	0.0280	5.04379	0.970349	0.9943125	0.69981	337.276
0.1	0.00361	2.137061	0.7702038	0.8895614	0.01522	40.4365	0.0261	4.82787	0.964457	0.9928213	0.59321	281.151
0.2	0.00422	2.241444	0.7848991	0.9009292	0.01953	43.5296	0.0242	4.62097	0.958601	0.9912074	0.50047	241.142
0.3	0.00483	2.349981	0.7998748	0.9117420	0.02453	47.1327	0.0223	4.42264	0.952781	0.9894623	0.41986	211.164
0.4	0.00544	2.462844	0.8151362	0.9220053	0.03033	51.3865	0.0204	4.23245	0.946996	0.9857609	0.34986	187.854
0.5	0.00605	2.580213	0.8306888	0.9317261	0.03703	56.4885	0.0185	4.05000	0.941246	0.9855372	0.28914	169.203
0.6	0.00666	2.702280	0.8465381	0.9409132	0.04476	62.7262	0.0166	3.87493	0.935532	0.9833329	0.23656	153.935
0.7	0.00727	2.829248	0.8626898	0.9495776	0.05366	70.5340	0.0147	3.70689	0.929852	0.9809485	0.19110	141.202
0.8	0.00788	2.961330	0.8791497	0.9577275	0.06390	80.6010	0.0128	3.54554	0.924206	0.9783677	0.15188	130.417
0.9	0.00849	3.098753	0.89537	0.9653783	0.07567	94.0905	0.0109	3.39059	0.918595	0.9755722	0.11813	121.162
1	0.00910	3.241758	0.913018	0.9725420	0.08917	113.131	0.0091	3.24175	0.913018	0.9725420	0.08917	113.131



(a) Failure rate (b) Repair time



(c) MTBF (d) Reliability



(e) Availability (f) ENOF

Fig. 5. Vague reliability parameters plots for press system

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