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Refitting Equations for Peak Ground Acceleration in Light of the PF-L Database

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***Abstract*—**The number of Ground Motion Prediction Equations (GMPEs) used for predicting peak ground acceleration (PGA) and the number of earthquake recordings that have been used for fitting these equations has increased in the past decades. The current PF-L database contains 3550 recordings. Since the GMPEs frequently model the peak ground acceleration the goal of the present study was to refit a selection of 44 of the existing equation models for *PGA* in light of the latest data. The algorithm Levenberg-Marquardt was used for fitting the coefficients of the equations and the results are evaluated both quantitatively by presenting the root mean squared error (RMSE) and qualitatively by drawing graphs of the five best fitted equations. The RMSE was found to be as low as 0.08 for the best equation models. The newly estimated coefficients vary from the values published in the original works.

***Keywords*—**Ground Motion Prediction Equations, Levenberg-Marquardt algorithm, refitting PF-L database.

I. Introduction

ONE of the important goals in structural engineering is to properly design a structure bearing in mind that a devastating earthquake could occur during its lifetime. The ground-motion prediction equations (GMPEs) help the engineer to estimate the possible earthquake loading by providing the correlation between seismically important variables, such as the peak ground acceleration (*PGA*) and significant seismological aspects, such as magnitude and distance from source to site. In order to address these needs, over the past decades a great number of equations for *PGA* have been developed by researchers world-wide which are systematically presented in [1], which contains 360 such equations. The purpose of all these equations have varied from general purpose to restricted, e.g. these equations have applied for different geographic areas or magnitude intervals.

The databases with earthquake recordings that have been available to researchers when designing equations for *PGA* have grown and improved in many respects in the past decades. The currently available database PF-L consists of 3550 earthquake recordings [2] and has been used in several studies in the recent years (e.g. [3]-[5]).

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The goal of the present work is therefore to explore the general applicability, physical plausibility and prediction accuracy of the adopted functional forms of existing equations in light of the latest PF-L database. At this point it should be noted that not all of the existing equations could be used for the study due to various reasons, therefore, a selection of the existing equations was made. Following this, the study uses the Levenberg-Marquardt algorithm in the Matlab program in order to fit the coefficients of the selected equations. Finally, five best equations in both quantitative and qualitative terms are selected and discussed.

II. Method

A. PF-L Database

The data used for the study is the PF-L database [2] consisting of 3550 earthquake recordings from Europe and the Americas. The PF-L database contains data about the earthquake moment magnitude M_w, the source-to-site Joyner-Boore distance R_{jb} (3550 recordings), rupture to site distance R_{rup} (3083 recordings), the style-of-faulting F (with values of F =0 for normal, F =0.5 for strike-slip and F =1 for reverse faults type of fault F, respectively), soil class (characterized by the average shear-wave velocity in the top 30 meters of soil, $V_{s,30}$ and the peak ground acceleration *PGA*. The *PGA* can be considered a dependent variable (usually expressed in g-units), defined as the geometrical average of both horizontal components. The actual data is relatively sparse at high magnitudes and short distances.

B. Existing Equation Models for PGA

While the summary of Douglas [1] contains 360 equations not all of their functional forms could be used for refitting in connection to the PF-L database due to various reasons (e.g. in many occasions the equations for *PGA* use additional parameters, which are not contained in the *PGA* database or the documentation of some of the equation forms lacks important details). Following this initial analysis of the summary of Douglas, a list of 44 functional forms for *PGA* was made that are appropriate for refitting. These selected forms of equations for *PGA* are the following (presented in order of year published): Ambraseys [1, Section 2.12], Faccioli [1, Section 2.18], Faccioli [1, Section 2.22], Faccioli & Agalbato [1, Section 2.23], PML [1, Section 2.34], Schenk [1, Section 2.35], PML [1, Section 2.46], Sabetta & Pugliese [1, Section 2.50], Ambraseys [1, Section 2.67], Ambraseys & Bommer [1, Section 2.74], Garcìa-Fernàndez & Canas [1, Section 2.76], Ambraseys et al. [1, Section 2.86], Theodulidis & Papazachos [1, Section 2.92], Musson et al. (2 models) [1, Section 2.108], Ambraseys [1, Section 2.113], Sarma & Free [1, Section 2.118], Ambraseys et al. & Simpson [1, Section 2.119], Sarma & Srbulov [1, Section 2.146], Smit [1, Section 2.148], Ólafsson & Sigbjörnsson [1, Section 2.152], Ambraseys & Douglas [1, Section 2.157], Gülkan & Kalkan [1, Section 2.175], Tromans & Bommer [1, Section 2.181], Boomer et al. [1, Section 2.187], Halldórsson & Svelnsson (2 models) [1, Section 2.189], Skarlatoudis et al. [1, Section 2.192], Bragato [1, Section 2.195], Kalkan & Gülkan [1, Section 2.197], Özbey et al. [1, Section 2.202], Ambraseys et al. [1, Section 2.207], Bragato [1, Section 2.209], Bragato & Slejko [1, Section 2.210], Akkar & Boomer [1, Section 2.235], Danciu & Tselentis [1, Section 2.242], Cauzzi & Faccioli [1, Section 2.254], Cotton et al. [1, Section 2.256], Massa et al. [1, Section 2.260], Akyol & Karagöz [1, Section 2.266], Pétursson & Vogfjörd [1, Section 2.275] and Faccioli et al. [1, Section 2.283].

An example of these equation models is the model of Ambraseys [1, Section 2.12] as follows:

$$\log(Y) = b_1 + b_2 * M_l + b_3 * \log(R) \tag{1}$$

The original values of the coefficients b_1, b_2 and b_3, are 0.46, 0.63 and -1.10, respectively. These coefficients were fitted again by using the PF-L database along with the Levenberg-Marquardt algorithm.

C. Refitting Coefficients with Levenberg-Marquardt

The Levenberg-Marquardt algorithm in Matlab was used for refitting the existing coefficients of the functional forms of equations for *PGA*. This algorithm is very adequate as it does not assume range bounds when it is fitting coefficients. Nevertheless, the actual coefficient fitting procedure was restarted 100 times for each functional form of the selected equations and also by choosing initial values for the coefficients from different value ranges [-1, 1], [-10, 10] and [-100, 100]. From each set of trials the best fitted form of the equation for *PGA* was selected.

Once properly fitted, functional forms of the equations can predict the *PGA* and the prediction accuracy can be compared by using the value of the Root Mean Square Error (RMSE) (2). The RMSE can be calculated by comparing the measured and the predicted values for *PGA* as follows:

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(m_i - p_i)^2} \tag{2}$$

It should be noted also that all equations were refitted two times: the first time by using the R_{jb} distance and the second time by using the R_{rup} distance. Both distances are frequently used in equation models for *PGA* and the aim was to investigate the use of which distance contributes to more accurate prediction.

III. Results

A. New Coefficients

The newly fitted coefficients for the 44 chosen equation forms are presented in Table I. The coefficients of the equation functional forms are marked as a_1, $a_2 \ldots$ a_{11}, bearing in mind that the most elaborate equation forms contain as many as 11 coefficients. The coefficients are presented in the order of appearance in the respective equation form. Should we consider (1) (above) for example, a_1 stands for coefficient b_1, a_2 stands for b_2 and a_3 stands for b_3.

When comparing the use of the distances R_{jb} and R_{rup} it appeared that in all cases except for the equations García-Fernández & Canas Canas [1, Section 2.76], Musson et al. [1, Section 2.108] (first model), Smit [1, Section 2.148] and Halldőrsson & Sveinsson [1, Section 2.189] (second model) the use of the R_{jb} distance contributed to better prediction accuracy. For the equation Pétursson & Vogfjörd [1, Section 2.275] it is necessary to calculate two special coefficients that should be calculated separately after the fitting procedure by using the formulae: $g = a_3/a_1$ and $e = a_4/a_1$.

B. Prediction Accuracy

The prediction accuracy was analyzed by comparing the RMSE errors for all selected equation functional forms regardless of the distance used R_{jb} or R_{rup}. The results are presented in Fig. 1. As it can be seen from the figure most of the refitted equations have approximately the same RMSE regardless of the year when the equations were published and regardless of the data which was available at that time. It can also be noted that most of the equations have RMSE in the range between 0.08 and 0.1. This suggests that there exists a lower bound on the prediction error for this method (of fitting equation forms).

C. Graphs of Five Most Accurate Equations

Five equations for which the RMSE was the lowest are the following: Pétersson & Vogfjörd [1, Section 2.275] – 1 with RMSE = 0.082, Bragato & Slejko [1, Section 2.210] – 2 with RMSE = 0.084, Akkar & Boomer [1, Section 2.235] – 3 with RMSE = 0.085, Faccioli [1, Section 2.18] – 4 with RMSE = 0.085 and Bragato [1, Section 2.195]– 5 with RMSE = 0.085.

Graphs of these equation forms are drawn in Figs. 2-4. The graphs are drawn for three different magnitudes (M_w=5, 6 and 7), for F=strike-slip fault type and shear wave velocity $V_{s,30}$=520 m/s.

TABLE I
REFITTED COEFFICIENTS

Equation	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11
[1, Section 2.12]	1,52	0,2	-0,68								
[1, Section 2.18]	26226	0,12	1,84								
[1, Section 2.22]	3,18	0,27	-1,72								
[1, Section 2.23]	1,43	0,21	-0,68								
[1, Section 2.34]	-3,17	0,88	-1,33	0,1	0,74						
[1, Section 2.35]	0,21	0,68	1,43								
[1, Section 2.46]	-1,64	0,62	-1,31	0,5	0,48	0,05					
[1, Section 2.50]	-1,34	0,26	-5,72	0,05							
[1, Section 2.67]	-1,31	0,27	6,43	-0,001							
[1, Section 2.74]	-0,89	0,2	6,3	-0,0005							
[1, Section 2.76]	2,75	0,57	0,01								
[1, Section 2.86]	-1,18	0,27	-0,00002	7,59	-1,1						
[1, Section 2.92]	6,14	0,47	-1,28	9,81	-0,14						
[1, Section 2.108]	4,29	0,55	-0,001								
[1, Section 2.108]	-0,55	0,54	0,002								
[1, Section 2.113]	-0,76	0,2	0,00004	7,5	-1,1						
[1, Section 2.118]	-3,06	0,88	-0,05	-1,15	-7,83	0,0003	0,07				
[1, Section 2.119]	-0,82	0,2	-1,1	-7,33	0,02	0,12					
[1, Section 2.146]	-1,57	0,19	-0,004	-0,44							
[1, Section 2.148]	7,93	0,14	0,23								
[1, Section 2.152]	2,56	0,14	0,68								
[1, Section 2.157]	-1,14	0,2	-0,01	-0,02	0,07						
[1, Section 2.175]	2,08	0,68	-0,11	-1,1	-7,13	-0,19	1,81				
[1, Section 2.181]	2,17	0,2	-1,1	-7,33	0,02	0,12					
[1, Section 2.187]	-0,84	0,2	-1,09	-7,06	0,01	0,12	0,03	0,05			
[1, Section 2.189]	0,21	0,68	-1,56								
[1, Section 2.189]	0,24	0,00028	-1,13								
[1, Section 2.192]	1,73	0,27	-1,12	-7,67	-0,01	0,08					
[1, Section 2.195]	-0,43	0,28	-0,02	-1,76	0,1	-7,72					
[1, Section 2.197]	0,91	0,89	-0,03	-0,13	-1,12	7,39	0,19	0,27			
[1, Section 2.202]	3,4	0,3	-0,05	-1,1	-7,11	0,11	0,18				
[1, Section 2.207]	0,88	0,08	-1,79	0,11	7,57	0,12	0,01	0,03	0,06		
[1, Section 2.209]	-1,11	0,2	-0,01								
[1, Section 2.210]	-3,12	1,2	-0,1	-1,44	0,0013	-7,76					
[1, Section 2.235]	0,73	0,84	-0,07	-2,06	0,15	-7,45	0,13	0,03	0,04	0,05	
[1, Section 2.242]	1,74	0,27	1,12	-7,65	0,08	0					
[1, Section 2.254]	-0,58	0,21	-0,69	-0,03	0,07	0,13					
[1, Section 2.256]	-2,44	0,84	-0,04	0	0,01						
[1, Section 2.260]	0,31	0,26	-1,1	-7,64	-1,43	-1,36					
[1, Section 2.266]	-0,28	0,21	-0,68	0,11							
[1, Section 2.275]	-1,32	0,02	0,83	-0,04	-2,71						
[1, Section 2.283]	-3,05	0,37	-1,31	0,09	0,33	0,02	0,12	0,19	4,61	4,64	3,59

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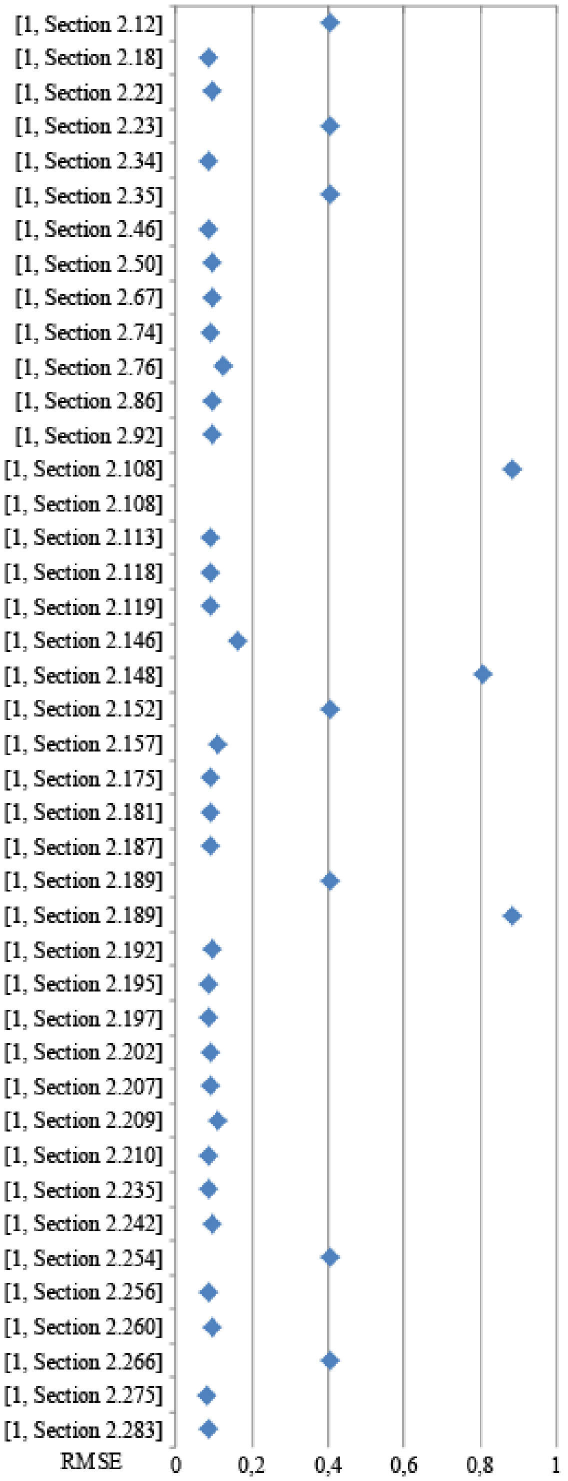

Fig. 1 RMSE of each refitted GMPE

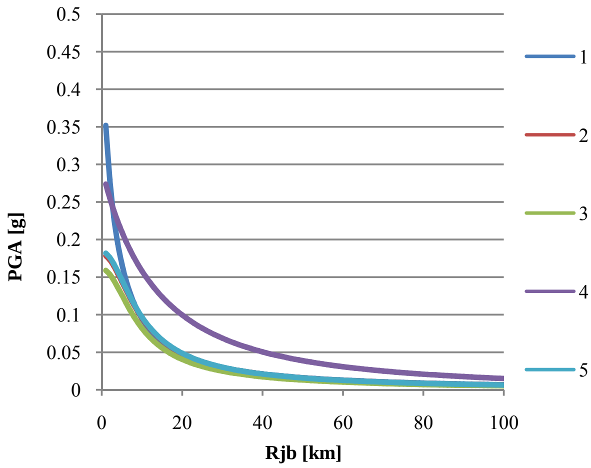

Fig. 2 Graph of *PGA* in respect of R_{jb} at M_w=5

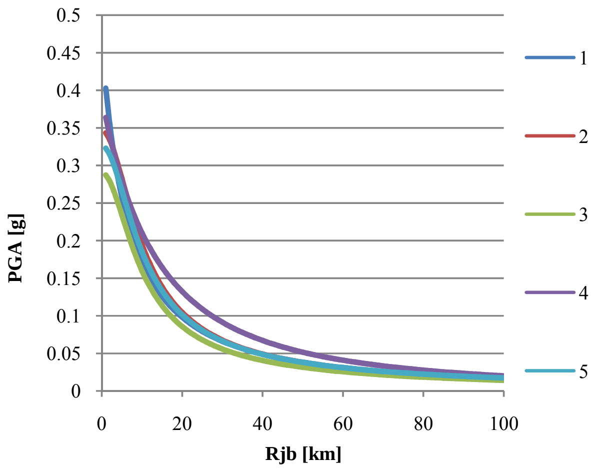

Fig. 3 Graph of *PGA* in respect of R_{jb} at M_w=6

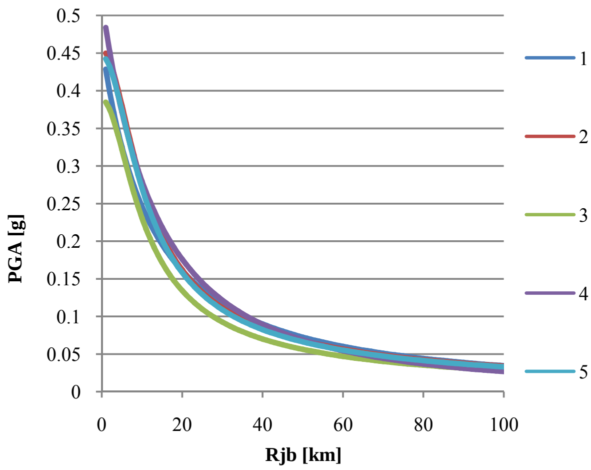

Fig. 4 Graph of *PGA* in respect of R_{jb} at M_w=7

IV. Conclusion

In this study we refitted the coefficients of 44 selected GMPEs by using the latest PF-L database. All equations coefficients were fitted two times by using the R_{jb} and R_{rup} distances. In most cases better fits were obtained when using the R_{jb} distance. The selected equation functional forms were compared quantitatively by using the RMSE error and five best fitted equations were presented. A lower bound for the RMSE was found at the value 0.08. Many of the equations functional forms had good fits with RMSE errors between the values of 0.08 and 0.1, regardless of the number of coefficients and complexity of the formulae and regardless of the year when the equation was first published. The presented graphs for the five best equations functional forms suggest that they are also adequate for engineering purposes.

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