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Reduction of Rotor-Bearing-Support Finite Element Model through Substructuring

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***Abstract*—**Due to simplicity and low cost, rotordynamic system is often modeled by using lumped parameters. Recently, finite elements have been used to model rotordynamic system as it offers higher accuracy. However, it involves high degrees of freedom. In some applications such as control design, this requires higher cost. For this reason, various model reduction methods have been proposed. This work demonstrates the quality of model reduction of rotor-bearing-support system through substructuring. The quality of the model reduction is evaluated by comparing some first natural frequencies, modal damping ratio, critical speeds, and response of both the full system and the reduced system. The simulation shows that the substructuring is proven adequate to reduce finite element rotor model in the frequency range of interest as long as the number and the location of master nodes are determined appropriately. However, the reduction is less accurate in an unstable or nearly-unstable system.

***Keywords*—**Finite element model, rotordynamic system, model reduction, substructuring.

I. Introduction

ROTORDYNAMIC studies typically involve three main components: rotor, bearings, and supports. Beside these three components, many studies also have been conducted involving other components such as seals, impellers, blades, and stators. In general, the bearings can be fluid film bearings, rolling element bearings, or magnetic bearings. The stiffness and damping coefficients of the bearings usually consist of direct as well as cross-coupling coefficients. In the case of fluid film bearings, these coefficients are obtained by solving the Reynolds equation representing the fluid dynamics in the bearing, either using long bearing solution or short bearing solution. The coefficients depend on the journal eccentricity which is a function of the Sommerfeld number and ratio between length and diameter of the bearings [1]-[3].

The rotordynamic system can be modeled as a continuous or discrete system. The first mostly can be applied to simple problems because it involves partial differential equations which are difficult to solve for complex problems. The latter, on the other hand, is widely used because it is easier to solve.

The most widely used discrete modeling of rotordynamic system is the lumped parameters model and the finite element (FE) model. The lumped parameters model was widely used in the past due to its minimum computation requirement. It is even still widely used today in industry because some systems already meet acceptable accuracy using this model. The most popular numerical approach using the lumped parameters model is the Transfer Matrix Method (TMM). Along with the advance of computer hardwares, FE model recently has been widely used due to its high accuracy, particularly if the rotor has a complex geometry. Using this method, consistent mass formulation is more commonly used. Beam elements are still used today to model the shaft. In this case, either Euler-Bernoulli beam, Rayleigh beam, or Timoshenko beam is used. However, many rotor systems have geometry which is not adequate to be modeled by beam elements. Therefore, combined beam-shell, 2D axisymmetric, and cyclic elements have been used [4]. Eventually, 3D solid elements have been used to model rotors which are not adequate to be modeled with all the aforementioned elements. However, the 3D solid model can easily reach a high degree-of-freedom (DOF). Although the 3D solid elements are widely used to model the rotordynamic system, some components such as bearings and supports are often still represented by combinations of springs and dampers. In this case, only the rotor is modeled using the 3D solid elements.

Unfortunately, the use of FE model which offers high accuracy has been mainly limited to rotordynamic analysis, not in rotordynamic control due to its high cost. Hence, for the purpose of control design, the number of DOF obtained from FE model mostly has to be reduced. This is conducted through so-called model reduction (sometimes also called condensation). Many model reduction techniques have been proposed. For the purpose of control, Guyan reduction, modal analysis (MA), component mode synthesis (CMS), balanced truncation (BT), structure preserving transformations (SPT), system equivalent reduction expansion process (SEREP), and modified SEREP have been proposed [4], [5].

This work is aimed to demonstrate the quality of model reduction of rotor-bearing-support system through substructuring in ANSYS. The quality of the model reduction is evaluated by comparing some first natural frequencies, modal damping ratio, critical speeds, and response of both the full system and the reduced system.

II. Finite Element Model of Rotor-Bearing-Support System

Formulation of rotor model using beam elements is presented in many references, such as [6]-[9]. If a Timoshenko beam model is used, its stiffness matrix can be modified from

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that of Euler-Bernoulli beam by introducing shear correction factor κ. For solid circular cross section, the shear correction factor is given by:

$$\kappa = \frac{6(1+\nu)}{7+6\nu} \tag{1}$$

and for hollow circular cross section by:

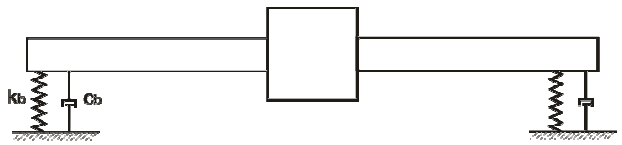
$$\kappa = \frac{6(1+\nu)(1+\Lambda^2)^2}{(7+6\nu)(1+\Lambda^2)^2 + (20+12\nu)\Lambda^2} \tag{2}$$

where Λ is ratio of the inner radius to the outer radius [10].

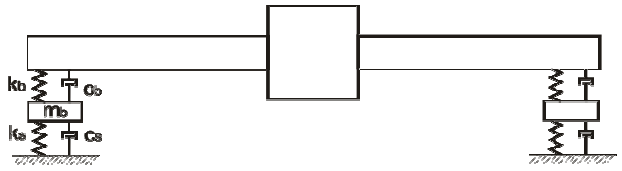
To make easier, a non-dimensional shear correction term φ which is equivalent to shear correction factor κ is usually used in the formulation.

In recent modeling of rotordynamic system, disks can be modeled either as rigid bodies or flexible bodies. In the first case, the disks cannot undergo deformation. To represent the dynamic behavior of the disks, their mass and moment inertias are used. In the latter case, the disks may undergo deformation. In this case, either a 3D solid shell or Timoshenko beam is used to model the disks.

Similarly, bearing supports may also be considered either rigid or flexible. The choice usually depends on whether the flexibility of the supports is significantly large or not. If it is not significantly large, then considering the supports flexible will only increase the complexity and cost of the modeling without giving quite different results in the analysis. In contrary, if the flexibility of the supports is significantly large, then considering the supports flexible will give an advantage. In such a case, mass of each bearings is also usually modeled as point mass located at the connectivity between the bearing and its support. Figs. 1 (a) and (b) show both rotor-bearing-support models with rigid and flexible bearing supports.



(a)



(b)

Fig. 1 Rotor-bearing-support models with (a) rigid bearing supports and (b) flexible bearing supports

The bearing, including fluid film bearing which is used in the current work, is usually modeled by eight linear dynamic coefficients: four stiffness and four damping coefficients, as shown in Fig. 2. Two spring-damper pairs are called direct coefficients, whereas the other two pairs are called cross-coupling coefficients. The latter is the main cause of instability in a rotordynamic system. Some types of fluid film bearings such as plain, fixed journal bearing (PFJB) have large cross-coupling coefficients, while some others have less cross-coupling coefficients. Tilting pad journal bearing (TPJB) is fluid film bearing type which has almost no cross-coupling coefficients, therefore it is more stable.

A bearing support can be a bearing housing while considering the foundation rigid. It also can be combination of bearing housing along with the foundation, considered as a single unit. The bearing supports considered flexible are usually modeled by direct coefficients only in two perpendicular directions, as there are no significant cross-coupling coefficients. Furthermore, in many cases, the direct coefficients are isotropic.

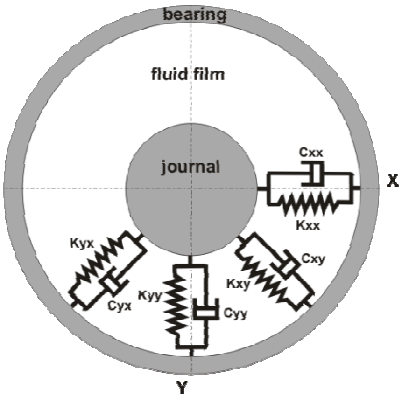


Fig. 2 Model of fluid-film bearing

III. Model Reduction through Substructuring

There are many model reduction methods having been proposed as mentioned earlier. The reduction methods are required to provide as low computational cost as possible but retain physical interpretability as well as accuracy in the frequency range of interest. In this current work, Guyan reduction and component mode synthesis (CMS) are used to reduce the size of system matrices as well as force vectors. Both of the Guyan reduction and CMS are used through ANSYS as the package provides both the techniques. The ANSYS Guyan reduction can include mass, stiffness, and damping matrices, but not gyroscopic matrix. To provide gyroscopic matrix in the reduced system, manual assembly of gyroscopic matrix is conducted based on gyroscopic matrix formulation for finite beam rotor elements. The ANSYS CMS is only used for the reduction of free-free non-rotating rotor as CMS in ANSYS cannot handle unsymmetric system (which is the case in the current rotor-bearing system due to asymmetry of bearings) as well as cannot include damping and gyroscopic matrices (which are important in rotordynamic system).

A. Guyan Reduction

Guyan reduction method reduces the system matrices of a system by dividing the DOF into master and slave DOF. The slave DOF are assumed to have low inertia relative to stiffness and are constrained to displace as dictated by elastic properties and displacements associated with other coordinates defined as the masters. The master DOF is then retained while the slave DOF is dropped.

Firstly, the matrix to be reduced is partitioned accordingly to the choice of master and slave DOF. For example, stiffness matrix [K] will be partitioned as follows:

$$[K]=\begin{bmatrix}[K_{11}] & [K_{12}]\\ [K_{21}] & [K_{22}]\end{bmatrix} \tag{3}$$

where subscript 1 refers to master DOF whereas subscript 2 refers to slave DOF.

The corresponding DOF vector is (x_1, x_2) where x_1 contains master DOF whereas x_2 contains slave DOF. The reduced stiffness matrix [K_{red}] is given by:

$$[K_{red}]=[K_{11}]-[K_{12}][K_{22}]^{-1}[K_{12}]^T \tag{4}$$

Being derived from kinetic energy expression, the reduced mass matrix [M_{red}] is given by:

$$[M_{red}]=[M_{11}]-[M_{12}][K_{22}]^{-1}[K_{12}]^T-\left([M_{12}][K_{22}]^{-1}[K_{12}]^T\right)^T +[K_{12}][K_{22}]^{-1}[M_{22}][K_{22}]^{-1}[K_{12}]^T \tag{5}$$

Similarly, being derived from the part of kinetic energy due to gyroscopic effect, the reduced gyroscopic matrix [G_{red}] is given by:

$$[G_{red}]=[G_{11}]-[G_{12}][G_{22}]^{-1}[G_{12}]^T-\left([G_{12}][G_{22}]^{-1}[G_{12}]^T\right)^T +[G_{12}][G_{22}]^{-1}[G_{22}][G_{22}]^{-1}[G_{12}]^T \tag{6}$$

Also in similar way, being derived from dissipation energy expression, the reduced damping matrix [C_{red}] is given by:

$$[C_{red}]=[C_{11}]-[C_{12}][C_{22}]^{-1}[C_{12}]^T-\left([C_{12}][C_{22}]^{-1}[C_{12}]^T\right)^T +[C_{12}][C_{22}]^{-1}[C_{22}][C_{22}]^{-1}[C_{12}]^T \tag{7}$$

Finally, the reduced force vector due to unbalance [F_{red}] is given by:

$$\{F_{red}\}=\{F_1\}-[K_{12}][K_{22}]^{-1}\{F_2\} \tag{8}$$

where {F1} is force vector which corresponds to DOF vector {x1} whereas {F2} is force vector which corresponds to DOF vector {x2}.

B. Component Mode Synthesis (CMS)

In the CMS, the structure is divided into some substructures. Some DOF located at the intersection between the substructures are called boundary DOF (also called interface DOF or constraint DOF), whereas some other DOF are called internal DOF. The simplest scheme is to assign all of the boundary DOF as master DOF while assigning all the internal DOF as slave DOF. Another scheme can be to assign some internal DOF as additional master DOF.

The CMS is basically Guyan reduction which includes some of modal data in the reduced matrix to increase the accuracy. Beside the master DOF, some modal DOF are added to the reduced matrix. Exact results are obtained if all the modes are retained, but this will lead to similar size of the reduced matrix to that of the full system, and therefore it is no more a reduction. Fortunately, retaining only some of the modes is sufficient to obtain quite accurate results. Certainly, the more modes are retained, the more accurate the results but the larger size of the reduced matrix.

It is to notice that a full matrix has (m + n) DOF where m is the number of master DOF whereas n is the number of slave DOF. After the reduction is conducted, the reduced matrix has (m + k) DOF where k is the number of modal data included in the reduction. Therefore, if no modal data is included (k = 0) then the method reduces to Guyan reduction method.

Similar to Guyan reduction procedure, firstly the full matrix is partitioned accordingly to the choice of master and slave DOF, as shown in (3). The corresponding DOF vector is defined by (x_1, x_2) where:

$$x_2 = x_2' + x_2'' \tag{9}$$

$$x_2' = -[K_{22}]^{-1}[K_{21}]x_1 \tag{10}$$

x_2'' is obtained by solving the following eigen problem:

$$\left([K_{22}]-\lambda[M_{22}]\right)x_2''=0 \tag{11}$$

Then modal transformation is conducted as follows:

$$x_2'' = \phi\eta_2 \tag{12}$$

The generalized coordinates, therefore, can be expressed as:

$$\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=\begin{Bmatrix}x_1\\ x_2'+x_2''\end{Bmatrix}=\begin{Bmatrix}x_1\\ -[K_{22}]^{-1}[K_{21}]x_1+\phi\eta_2\end{Bmatrix} =\begin{bmatrix}I & 0\\ -[K_{22}]^{-1}[K_{21}] & \phi\end{bmatrix}\begin{Bmatrix}x_1\\ \eta_2\end{Bmatrix} \tag{13}$$

or can be written as:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = [\psi] \begin{Bmatrix} x_1 \\ \eta_2 \end{Bmatrix} \tag{14}$$

where is called transformation matrix and given by:

$$[\psi] = \begin{bmatrix} I & 0 \\ -[K_{22}]^{-1}[K_{21}] & \phi \end{bmatrix} \tag{15}$$

by using the transformation matrix Ψ, the reduced matrices are defined by:

$$[M_{red}] = [\psi]^T [M][\psi] \tag{16}$$

$$[K_{red}] = [\psi]^T [K][\psi] \tag{17}$$

$$[G_{red}] = [\psi]^T [G][\psi] \tag{18}$$

$$[C_{red}] = [\psi]^T [C][\psi] \tag{19}$$

$$\{F_{red}\} = [\psi]^T \{F\} \tag{20}$$

C. Substructuring in ANSYS

Guyan reduction and CMS can be conducted using substructuring in ANSYS. Substructuring is a procedure that condenses a group of finite elements into one element represented as a matrix. The single-matrix element is called a superelement. Substructuring reduces computer time and allows solution of large problems with limited computer resources. The substructure analysis uses the technique of matrix reduction to reduce the system matrices to a smaller set of DOF. In the substructuring, the superelements are created to result in reduced number of DOF.

General substructuring in ANSYS uses Guyan reduction procedure to calculate the reduced matrices. The key assumption in this procedure is that for the lower frequencies, inertia forces on the slave degrees of freedom (those degrees of freedom being reduced out) are negligible compared to elastic forces transmitted by the master degrees of freedom (MDOF). Therefore, the total mass of the structure is apportioned among only the MDOF. The net result is that the reduced stiffness matrix is exact, whereas the reduced mass and damping matrices are approximate. Guyan reduction in ANSYS is improved so that it can include damping matrix.

A special substructuring using CMS is also available in ANSYS. However, CMS in ANSYS now does not yet include damping matrix although in theory the CMS can include the damping matrix. Furthermore, CMS in ANSYS only work with symmetric matrix. In general, CMS gives better reduction for higher frequencies because it includes the modal data in the reduction process. However, the reduced matrices can be larger as some additional DOF coming from the modal data are added in the matrices.

In superelements, some master nodes have to be selected. Selecting the master nodes is an important step in a reduced analysis. The accuracy of the reduced mass matrix (and hence the accuracy of the solution) depends on the number and location of masters. The master nodes are selected based on the following guidelines:

1) The total number of MDOF should be at least twice the number of modes of interest.
2) Selected MDOF should be in directions in which the structure or component is expected to vibrate.
3) Zero lateral displacement are assigned as master nodes.
4) Nodes at which forces are applied are assigned as master nodes.
5) Nodes at which inertia is relatively large and stiffness is relatively low are assigned as master nodes.

Substructuring involves three distinct steps called passes: (1) generation pass, (2) use pass, and (3) expansion pass. The generation pass is condensing a group of "regular" finite elements into a single superelement. The condensation is done by identifying a set of MDOF. The procedure to generate a superelement consists of two main steps: (1) building the model and (2) applying loads and creating the superelement matrices.

The use pass is using the superelement in an analysis by making it part of the model. The entire model may be a superelement, or, the superelement may be connected to other nonsuperelements. The solution from the use pass consists only of the reduced solution for the superelement and complete solution for nonsuperelements.

The expansion pass is starting with the reduced solution and calculating the results at all DOF in the superelement. If multiple superelements are used in the use pass, a separate expansion pass will be required for each superelement. The backsubstitution method uses the reduced solution from the use pass and substitutes it back into the available factorized matrix file to calculate the complete solution.

Based on how the substructuring is conducted, there are two kinds of substructuring which can be conducted in ANSYS. The first is called bottom-up substructuring, meaning that each superelement is separately generated in an individual generation pass, and all superelements are assembled together in the use pass. This method is suitable for very large models which are divided into smaller superelements so that they can "fit" on the computer.

The second is called top-down substructuring. This is suitable for substructuring of smaller models. An advantage of this method is that the results for multiple superelements can be assembled in postprocessing. The procedure for top-down substructuring is in general similar to that of bottom-up substructuring. What makes the top-down substructuring different is that the whole model is built first.

IV. Numerical Example

In the following example, a rotor-bearing-support system of a steam turbine, which has been analyzed in a previous work [11], is used. Some data and results of the previous work are

used as reference for validation purpose.

A. Component Specifications

The geometry of the rotor is shown in Fig. 3. The shaft and the disks are a single piece machined, forged steel. As the disks are not shrunk into the shaft, the system does not undergo stress stiffening. The overall length of the rotor is 524.56cm (206.52in). The properties of the rotor steel are: density of 8450 kg/m^3 (0.30531 lb/in^3), Young modulus of 190 GPa (isotropic), and Poisson ratio of 0.3.

There are two bearings in the system: one at the high pressure end (HPE) and another one at the low pressure end (LPE). The distance between the bearings is 410.36cm (161.56 in). The ratio of length over diameter of the bearings is assumed to be L/D = 1 as required by standard for steam turbine rotor-bearing system. The oil used as bearing fluid film has a constant viscosity = 2.3206 x 10^{-6} reyns. Two types of bearings are evaluated in this work: TPJB and PFJB. The two different bearings are used in order to evaluate the quality of the model reduction with different stability level. It is important to notice that the bearing dynamic coefficients vary with the rotor speed. Furthermore, the behavior of the bearings makes the overall system unsymmetric.

Bearing supports are taken into account because they are quite flexible (their stiffness is worthy compared to the bearing stiffness). The stiffness of the bearing supports is 1.4 GN/m (8.0 x 10^6 lb/in) at both LPE and HPE. The stiffness in horizontal and vertical direction are the same. The damping of the bearing supports is negligible as its value is very small compared to the stiffness of the supports as well as the damping of the bearings.

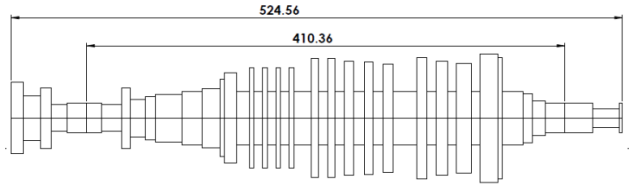


Fig. 3 Geometry of the rotor (length in cm)

B. Finite Element Modeling

The finite element modeling of the system is conducted by using non-commercial ANSYS package. The following elements are used for modeling:

1) The rotor is modeled by using Timoshenko beam elements. Both the shaft and the disks are flexible.
2) The bearings as well as their supports are modeled by ANSYS element called COMBI214 which supports both direct and cross-coupling coefficients.
3) Mass of the bearings is modeled by ANSYS element called MASS21 which is a point mass.

The Timoshenko rotor elements have 6 DOF per node: three of them are translational and three others are rotational. The springs and dampers in COMBI214 have only 1 translational DOF per node per axis, or 3 translational DOF per node in 3 axis. Due to different number and/or type of DOF per node between the beam rotor elements and the bearing elements, some special elements called multi-point constraints (MPC) are added at connection points between the rotor and the bearings. By adding the MPC elements, the energy in all DOF of the rotor can be transferred well to the bearings.

The boundary conditions consist of some constraints and loads. The constraints are as follows:

1) The nodes where the bearings are attached on the axis are prevented from longitudinal translation.
2) The bearings are attached to certain positions of the axis.
3) The mass of the bearings can move in radial direction.
4) The bearing supports are attached to ground.

The loads are comprised of rotational velocity of the rotor and unbalance forces. The latter is used only in the harmonic unbalance force analysis.

Fig. 4 shows the mesh of the rotor model. It has been evaluated that the mesh gives identical response to more refined mesh. Hence, the mesh is adequate. The dimensionality of the model is shown in Table I. Finally, the system model has been validated by comparing its critical speeds to those of the reference work. A quite good agreement is achieved.

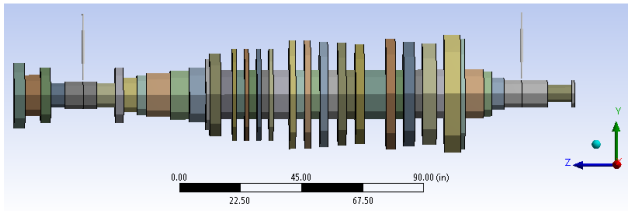


Fig. 4 Mesh of the system using Timoshenko beam rotor model

TABLE I
DIMENSIONALITY OF THE FINITE ELEMENT ROTOR MODEL

Quantity	Value
Number of nodes	99
Number of elements	49
Size of M, C, K matrix	1230 x 1230
Size of F vector	1230 x 1

C. Model Reduction Results

1. Model Reduction of Free-Free Rotor

By ignoring rotating (hysteretic) damping, both Guyan reduction and CMS can be used to create reduced mass and stiffness matrix. The reduction results show that CMS gives better accuracy. Table II shows some first natural frequencies of the rotor model after reduction into 30 DOF using Guyan reduction as well as CMS with 12 additional modal DOF, compared to those of the full system. As the mode increases, the accuracy of the Guyan reduction decreases whereas that of the CMS remains good. Furthermore, reducing the rotor model into less number of DOF using Guyan reduction is evidenced to reduce the model accuracy, particularly at higher modes.

2. Model Reduction of Rotor-Bearing-Support System

Since damping in the system cannot be ignored and the assembled global matrices are unsymmetric, ANSYS CMS

cannot be used for reduction. Therefore, the reduction was only conducted by using ANSYS Guyan reduction feature. As the rotor is rotating, gyroscopic effect is included. The selection of master nodes is shown in Fig. 5. Master nodes are indicated by circles. The total reduced DOF is 30, as calculated in Table III.

Tables IV and V show some first damped natural frequencies of the reduced system with TPJB and PFJB, respectively, using ANSYS Guyan reduction compared to those of the full system. It can be seen that the system with PFJB is unstable/nearly-unstable as some values in the real part of the complex frequency are positive or very near to zero. The first damped natural frequencies along the speeds of interest represented in Campbell diagrams of both the full and reduced system with TPJB and PFJB are shown in Figs. 6 and 7 respectively. The 1x synchronous critical speeds are determined by putting a straight line with gradient of 1 in the Campbell diagram so that it intersects with the natural frequency curves. Both the tables of natural frequencies and the Campbell diagrams show that the model reduction is quite accurate for both the systems, but less accurate for the system with PFJB. Furthermore, it is shown that the third natural frequency in the reduced system with PFJB is an additional natural frequency which is not captured in the full system.

Finally, due to a certain amount of rotating unbalance, the unbalance responses are also compared between the full and the reduced system. For both of the systems with TPJB and PFJB, the frequency response to the rotating unbalance at midspan and HPE bearing is shown in Figs. 8 to 11, comparing that of the reduced system to that of the full system.

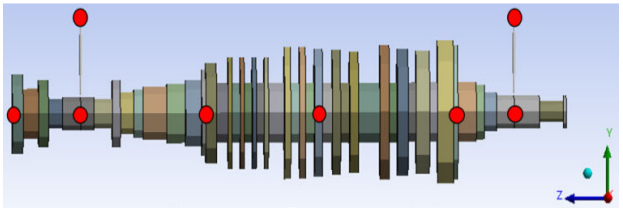


Fig. 5 Selected master nodes

TABLE II
NATURAL FREQUENCIES OF THE FULL AND REDUCED FREE-FREE ROTOR

Mode	Full system, Hz	Reduced system using Guyan reduction, Hz	Reduced system using CMS, Hz
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	66.0361	66.3230	66.0528
8	66.0361	66.3230	66.0528
9	149.4047	156.2477	149.5194
10	149.4047	156.2477	149.5194
11	187.6451	189.6003	187.9358
12	256.6236	278.6275	257.1861

TABLE III
CALCULATION OF THE TOTAL REDUCED DOF

Master nodes	Number of DOF
6 master nodes taking place at the rotor	6 x 6 = 36
2 master nodes at the ground (all DOF are fixed)	0
Elimination of DOF due to zero axial displacement at all master nodes taking place at the rotor	- 6
Total number of DOF	30

TABLE IV
COMPLEX FREQUENCY AND MODAL DAMPING RATIO OF SYSTEM WITH TPJB

Mode	Complex frequency, Hz				Modal damping ratio	
	Real part		Complex part		Full	Reduced
	Full	Reduced	Full	Reduced		
1	- 0.479	- 0.449	- 31.18	- 31.39	1.5 E-2	1.4 E-2
	- 0.479	- 0.449	+31.18	+31.39	1.5 E-2	1.4 E-2
2	- 0.007	- 0.001	- 35.40	- 35.40	2.0 E-4	3.7 E-5
	- 0.007	- 0.001	+35.40	+35.40	2.0 E-4	3.7 E-5
3	- 0.613	- 0.629	- 41.47	- 41.71	1.5 E-2	1.5 E-2
	- 0.613	- 0.629	+41.47	+41.71	1.5 E-2	1.5 E-2
4	- 0.141	- 0.154	- 87.26	- 87.70	1.6 E-3	1.8 E-3
	- 0.141	- 0.154	+87.26	+87.70	1.6 E-3	1.8 E-3
5	- 0.079	- 0.055	- 91.29	- 91.25	8.7 E-4	6.0 E-4
	- 0.079	- 0.055	+91.29	+91.25	8.7 E-4	6.0 E-4
6	- 0.644	- 0.675	- 103.9	- 104.7	6.2 E-3	6.4 E-3
	- 0.644	- 0.675	+103.9	+104.7	6.2 E-3	6.4 E-3
7	- 0.053	- 0.030	- 111.4	- 111.9	4.8 E-4	2.7 E-4
	- 0.053	- 0.030	+111.4	+111.9	4.8 E-4	2.7 E-4

TABLE V
COMPLEX FREQUENCY AND MODAL DAMPING RATIO OF SYSTEM WITH PFJB

Mode	Complex frequency, Hz				Modal damping ratio	
	Real part		Complex part		Full	Reduced
	Full	Reduced	Full	Reduced		
1	- 0.097	- 0.179	- 35.21	- 35.39	2.7 E-3	5.1 E-3
	- 0.097	- 0.179	+35.21	+35.39	2.7 E-3	5.1 E-3
2	- 0.031	- 0.032	- 36.14	- 35.50	8.6 E-4	8.9 E-4
	- 0.031	- 0.032	+36.14	+35.50	8.6 E-4	8.9 E-4
2b		- 0.000		- 68.84		1.2 E-5
		- 0.000		+68.84		1.2 E-5
3	- 0.360	- 0.815	- 88.15	- 89.57	4.1 E-3	9.1 E-3
	- 0.360	- 0.815	+88.15	+89.57	4.1 E-3	9.1 E-3
4	- 0.637	- 0.860	- 91.68	- 91.93	7.0 E-3	9.3 E-3
	- 0.637	- 0.860	+91.68	+91.93	7.0 E-3	9.3 E-3
5	- 3.940	- 7.757	- 110.9	- 112.5	3.6 E-2	6.9 E-2
	- 3.940	- 7.757	+110.9	+112.5	3.6 E-2	6.9 E-2
6	- 0.265	+ 0.491	- 114.1	- 112.9	2.3 E-3	- 4 E-3
	- 0.265	+ 0.491	+114.1	+112.9	2.3 E-3	- 4 E-3

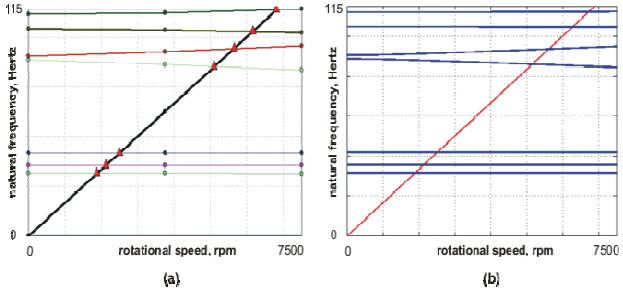


Fig. 6 Campbell diagram of (a) the full and (b) the reduced system with TPJB

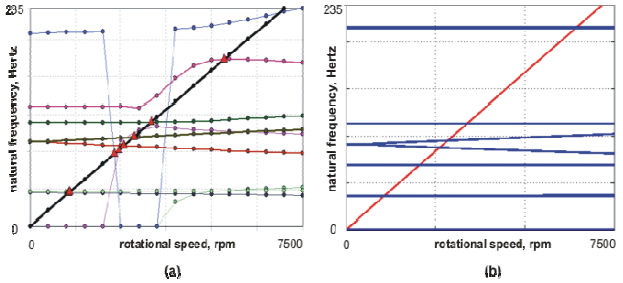


Fig. 7 Campbell diagram of (a) the full and (b) the reduced system with PFJB

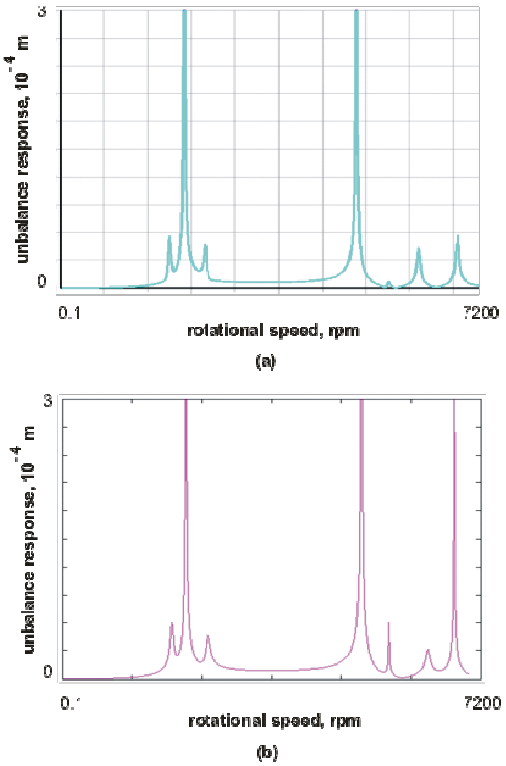


Fig. 8 Unbalance response of (a) full and (b) reduced system with TPJB at the midspan

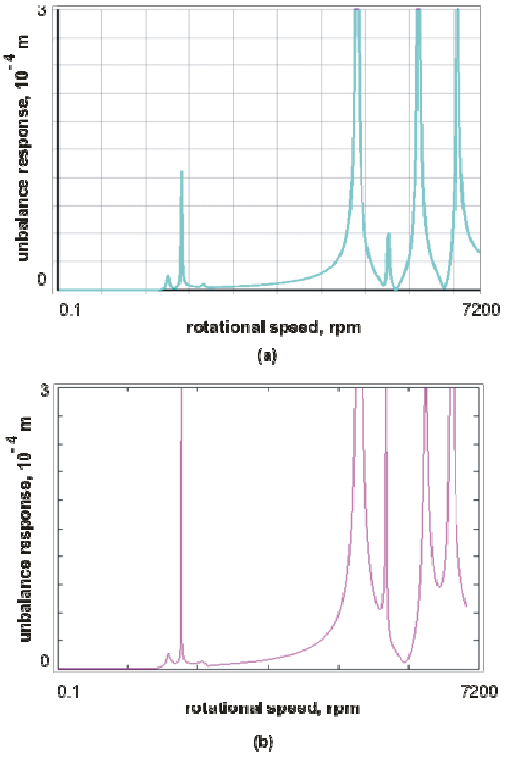


Fig. 9 Unbalance response of (a) full and (b) reduced system with TPJB at the HPE bearing

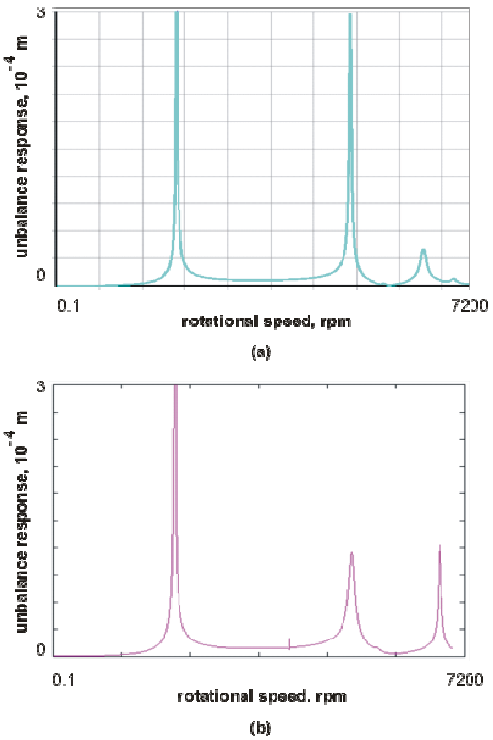


Fig. 10 Unbalance response of (a) full and (b) reduced system with PFJB at the midspan

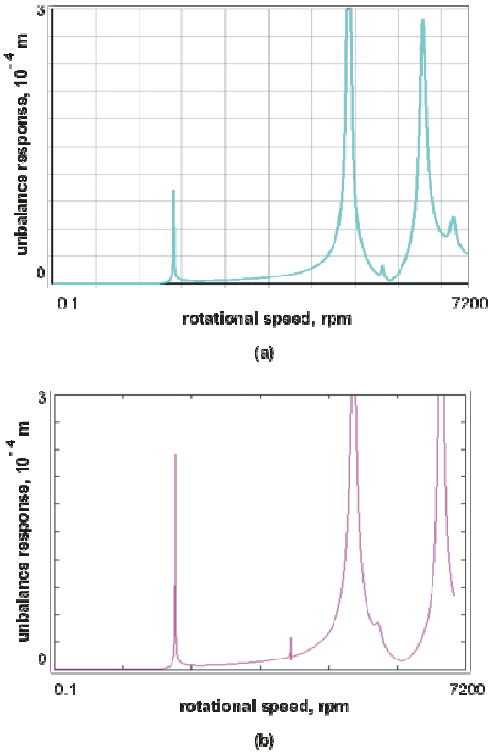


Fig. 11 Unbalance response of (a) full and (b) reduced system with PFJB at the HPE bearing

REFERENCES

[1] Lund, J.W., Review of the Concept of Dynamic Coefficients for Fluid Film Journal Bearings. Journal of Tribology, 1987. 109(1): p. 37-41.
[2] Khonsari, M.M. and E.R. Booser, Applied Tribology: Bearing Design and Lubrication, 2001: John Wiley & Sons, Inc.
[3] Childs, D., Turbomachinery Rotordynamics: Phenomena, Modeling, Analysis, 1993: John Wiley & Sons, Inc.
[4] Wagner, M.B., et al., Model Reduction Methods for Rotor Dynamic Analysis: A Survey and Review. International Journal of Rotating Machinery, 2010. 2010: p. 1-17.
[5] Das, A.S. and J.K. Dutt, A reduced rotor model using modified SEREP approach for vibration control of rotors. Mechanical Systems and Signal Processing, 2012. 26: p. 167-180.
[6] Abduljabbar, Z.S., M.M. El-Madany, and A.A. Al-Abdulwahab, On the Dynamic Analysis of Rotor-Bearing Systems Using Finite Elements, 2005.
[7] Cook, R.D., Finite Element Modeling for Stress Analysis, 1995, USA: John Wiley & Sons.
[8] Jones, S., Finite Elements for the Analysis of Rotor-Dynamic Systems that Includes Gyroscopic Effects, in Mechanical Engineering Department, 2005, Brunel University.
[9] Logan, D.L., A First Course in the Finite Element Method. 4 ed, 2007, India: Thomson.
[10] Pilkey, W.D., Geometric Properties of Planes, in Formulas for Stress, Strain, and Structural Matrices, 2008, John Wiley & Sons: NJ, USA.
[11] Bethel, L.L., et al. Critical Speeds and the Importance of Stiffness - A Case Study in the Design and Testing of a Large Mechanical Drive Steam Turbine to API Standard 612 in Proceedings of the Twenty-Second Turbomachinery Simposium.