Reduced Order Modelling of Linear Dynamic Systems using Particle Swarm Optimized Eigen Spectrum Analysis

G. Parmar, *Life Member SSI, AMIE*, S. Mukherjee, *FIE*, and R. Prasad

Abstract—The authors present an algorithm for order reduction of linear time invariant dynamic systems using the combined advantages of the eigen spectrum analysis and the error minimization by particle swarm optimization technique. Pole centroid and system stiffness of both original and reduced order systems remain same in this method to determine the poles, whereas zeros are synthesized by minimizing the integral square error in between the transient responses of original and reduced order models using particle swarm optimization technique, pertaining to a unit step input. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. The algorithm is illustrated with the help of two numerical examples and the results are compared with the other existing techniques.

Keywords—Eigen spectrum, Integral square error, Order reduction, Particle swarm optimization, Stability.

I. INTRODUCTION

EVERY physical system can be translated into mathematical model. The mathematical procedure of system modelling often leads to comprehensive description of a process in the form of high order differential equations which are difficult to use either for analysis or controller synthesis. It is hence useful, and sometimes necessary, to find the possibility of finding some equation of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Some of the reasons for using reduced order models of high order linear systems could be:

(i) To have a better understanding of the system.
(ii) To reduce computational complexity.
(iii) To reduce hardware complexity.
(iv) To make feasible controller design.

Numerous methods are available in the literature for order-

G. Parmar is QIP Research Scholar with the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India.
(E-mail: gp555dee@iitr.ernet.in; girish_parmar2002@yahoo.com).
S. Mukherjee is Professor with the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India.
(E-mail: shmeefee@iitr.ernet.in).
R. Prasad is Associate Professor with the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, India.
(E-mail:rpdeefee@iitr.ernet.in).

reduction of linear continuous systems in time domain as well as in frequency domain [1]-[8]. Further, several methods have also been suggested by combining the features of two different methods [9]-[14]. In spite of the significant number of methods available, no approach always gives the best results for all systems. Almost all methods, however, aim at accurate reduced models for a low computational cost.

Further, numerous methods of order reduction are also available in the literature [15]-[22], which are based on the minimization of the integral square error (ISE) criterion. However, a common feature in these methods [15]-[21] is that the values of the denominator coefficients of the low order system (LOS) are chosen arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the LOS are determined by minimization of the ISE. In [22], Howitt and Luss suggested a technique, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses.

Recently, particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [23]. PSO shares many similarities with Genetic Algorithm (GA); like initialization of population of random solutions and search for the optimal by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. One of the most promising advantage of PSO over GA is its algorithmic simplicity, as it uses a few parameters and easy to implement. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

In the present work, the authors present an algorithm for order reduction in which both the pole centroid and system stiffness of the original and reduced order systems are kept exactly same to obtain the reduced order system poles, while the zeros are synthesized by minimizing the integral square error between the transient responses of original and reduced order systems using particle swarm optimization technique, pertaining to a unit step input. The algorithm is developed only for the systems with real distinct poles and is illustrated with the help of two numerical examples. The comparison between the proposed and other well known existing order

reduction techniques is also shown in the present work.

II. DESCRIPTION OF THE ALGORITHM

Let the transfer function of the high order system (HOS) of order 'n' is :

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_o + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1}}{a_o + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1} + s^n} \quad (1)$$

or,

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_o + b_1 s + b_2 s^2 + \dots + b_{n-1} s^{n-1}}{(s+\lambda_1)(s+\lambda_2) \dots (s+\lambda_n)} \quad (2)$$

where, $-\lambda_1 < -\lambda_2 < \dots < -\lambda_n$ are poles of the HOS.

Let, the transfer function of low order system (LOS) of order 'r' to be synthesized is:

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s + \dots + \alpha_{r-1} s^{r-1}}{d_o + d_1 s + d_2 s^2 + \dots + d_{r-1} s^{r-1} + s^r} \quad (3)$$

or,

$$G_r(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)} = \frac{\alpha_0 + \alpha_1 s + \dots + \alpha_{r-1} s^{r-1}}{(s + \lambda'_1)(s + \lambda'_2) \dots (s + \lambda'_r)} \quad (4)$$

where, $-\lambda'_1 < -\lambda'_2 < \dots < -\lambda'_r$ are poles of the LOS then steps are as under :

Step-1: Fixing of the eigen spectrum zone (ESZ) of the HOS as shown in Fig. 1:

If poles $-\lambda_i$ $(i = 1,\dots,n)$ are located at $-(\operatorname{Re}\lambda_i \pm \operatorname{Im}\lambda_i)$ $(i = 1,\dots,p)$ within the ESZ, then the two lines passing through the nearest $(\operatorname{Re}\lambda_1)$ and farthest $(\operatorname{Re}\lambda_p)$ real poles when cut by two lines passing through the farthest imaginary pole pairs $(\pm\operatorname{Im}(\max))$ form the ESZ.

Step-2: Quantification of pole centroid and stiffness of HOS:

Pole centroid is defined as the mean of real parts of the poles and is expressed as:

$$\lambda_m \overset{\Delta}{=} \frac{\sum_{i=1}^{p} \operatorname{Re}\lambda_i}{p} \quad (5)$$

System stiffness is defined as the ratio of the nearest to the farthest pole of a system in terms of real parts only and is put as:

$$\lambda_s \overset{\Delta}{=} \frac{\operatorname{Re}\lambda_1}{\operatorname{Re}\lambda_p} \quad (6)$$

Step-3: Determination of eigen spectral points of LOS:

If λ'_m and λ'_s are pole centroid and system stiffness of LOS such that $\lambda'_m = \lambda_m$ and $\lambda'_s = \lambda_s$ then following situation arise:

$$\lambda'_s = \frac{\operatorname{Re}\lambda'_1}{\operatorname{Re}\lambda'_{p'}} = \lambda_s$$

$$\lambda'_m = \frac{\operatorname{Re}\lambda'_1 + \operatorname{Re}\lambda'_2 + \dots + \operatorname{Re}\lambda'_{p'}}{p'} = \lambda_m \quad (7)$$

where, λ'_i $(i = 1,\dots,r)$ are the poles of LOS located at $-(\operatorname{Re}\lambda'_i \pm \operatorname{Im}\lambda'_i)_{i=1,\dots,p'}$. Now if,

$$\frac{\operatorname{Re}\lambda'_{p'} - \operatorname{Re}\lambda'_1}{p'-1} = M \quad (8)$$

i.e., $\operatorname{Re}\lambda'_1 + M = \operatorname{Re}\lambda'_2$, $\operatorname{Re}\lambda'_2 + M = \operatorname{Re}\lambda'_3$ and so on till $\operatorname{Re}\lambda'_{p'-1} + M = \operatorname{Re}\lambda'_{p'}$ then (7) can be put as :

$$\lambda_m = \frac{\operatorname{Re}\lambda'_1 + \operatorname{Re}\lambda'_{p'} + (\operatorname{Re}\lambda'_1 + M) + (\operatorname{Re}\lambda'_2 + M) + \dots + (\operatorname{Re}\lambda'_{p'-2} + M)}{p'}$$

or, $\lambda_m p' = \operatorname{Re}\lambda'_1 + \operatorname{Re}\lambda'_{p'} + (\operatorname{Re}\lambda'_1 + M) + (\operatorname{Re}\lambda'_1 + 2M) + \dots + (\operatorname{Re}\lambda'_1 + (p'-2)M)$

$= \operatorname{Re}\lambda'_1 + \operatorname{Re}\lambda'_{p'} + \operatorname{Re}\lambda'_1 (p'-2) + (M + 2M + \dots + (p'-2)M)$

or, $\quad N = \operatorname{Re}\lambda'_1 (p'-1) + \operatorname{Re}\lambda'_{p'} + QM \quad (9)$

where, $N = \lambda_m p'$ and $QM = M + 2M + \dots + (p'-2) M$.

By putting $\operatorname{Re}\lambda'_1 = \lambda_s \operatorname{Re}\lambda'_{p'}$, (8) and (9) will be as under:

$$\operatorname{Re}\lambda'_{p'} - \lambda_s \operatorname{Re}\lambda'_{p'} = M (p'-1) \quad (10)$$

$$\lambda_s \operatorname{Re}\lambda'_{p'} (p'-1) + \operatorname{Re}\lambda'_{p'} + QM = N \quad (11)$$

Equations (10) and (11) can be put as :

$$\operatorname{Re}\lambda'_{p'} (1 - \lambda_s) + M (1 - p') = 0$$

$$\operatorname{Re}\lambda'_{p'} [\lambda_s (p'-1) + 1] + MQ = N$$

or,

$$\begin{bmatrix} \lambda_s(p'-1)+1 & Q \\ (1-\lambda_s) & (1-p') \end{bmatrix} \begin{bmatrix} \operatorname{Re}\lambda'_{p'} \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \quad (12)$$

Equation (12) can be solved for $\operatorname{Re}\lambda'_{p'}$ and M enabling thereby to locate the eigen spectral points (ESP) as shown in Fig. 1.

Therefore, the denominator polynomial in (3) is now known, which is given by :

$$\tilde{D}(s) = d_o + d_1 s + d_2 s^2 + \dots + d_{r-1} s^{r-1} + s^r \quad (13)$$

(a) eigen spectrum zone (ESZ) of HOS

(b) eigen spectrum zone (ESZ) of LOS

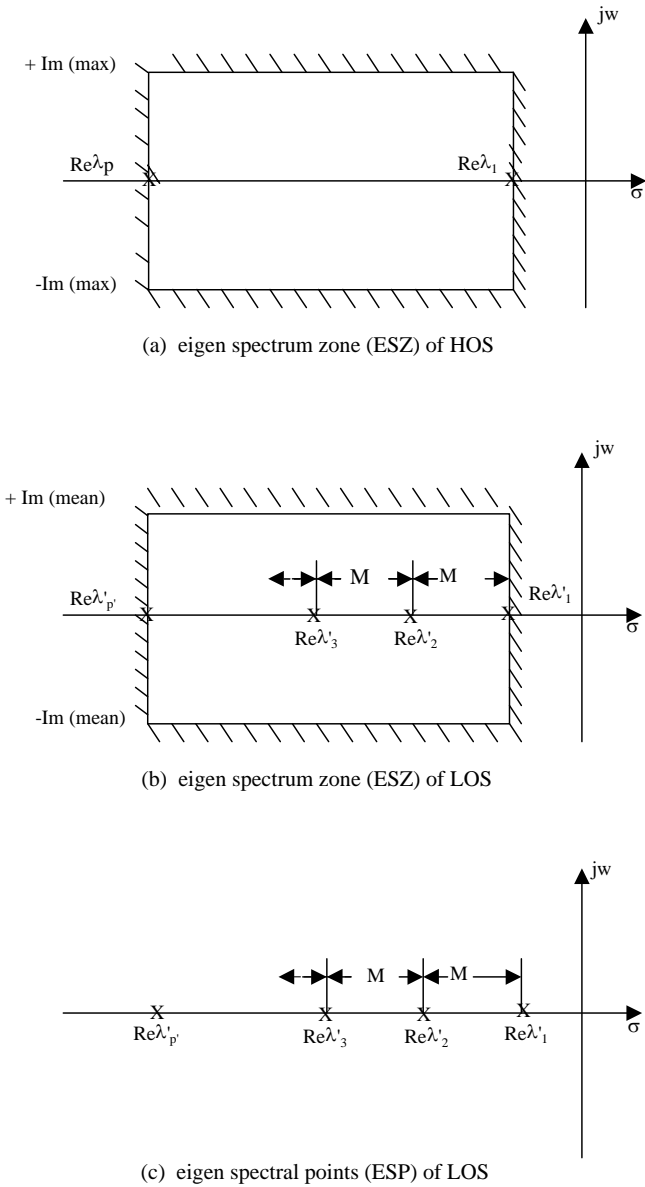(c) eigen spectral points (ESP) of LOS

Fig. 1 Eigen spectrum zones and points of system.

Step-4: Determination of the numerator coefficients of the LOS by particle swarm optimization technique:

The PSO method is a population based search algorithm where each individual is referred to as particle and represents a candidate solution. Each particle flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also the flying experience of the other particles. In PSO, each particle strives to improve itself by imitating traits from their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as pbest and the overall best out of all the particles in the population is called gbest [24].

In a d-dimensional search space, the best particle updates its velocity and positions with following equations:

$$v_{id}^{n+1} = w v_{id}^{n} + c_1 r_1^{n} (p_{id}^{n} - x_{id}^{n}) + c_2 r_2^{n} (p_{gd}^{n} - x_{id}^{n}) \qquad (14)$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1} \qquad (15)$$

where,

w = inertia weight.

c_1, c_2 = cognitive and social acceleration, respectively.

r_1, r_2 = random numbers uniformly distributed in the range (0, 1).

The i-th particle in the swarm is represented by a d-dimensional vector $X_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ and its velocity is denoted by another d-dimensional vector $V_i = (v_{i1}, v_{i2}, \ldots, v_{id})$. The best previously visited position of the i-th particle is represented by $P_i = (p_{i1}, p_{i2}, \ldots, p_{id})$.

In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group's previous best solution. The velocity update in particle swarm consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm [25]. The parameters c_1 & c_2 determine the relative pull of pbest and gbest and the parameters r_1 & r_2 help in stochastically varying these pulls. In the above equations (14) and (15), superscripts denote the iteration number. Fig. 2 shows the position updates of a particle for a two-dimensional parameter space.
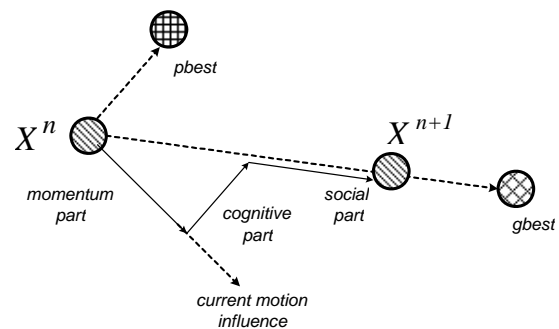

Fig. 2 Position updates in PSO for a two dimensional parameter space.

In the present study, PSO is employed to minimize the objective function 'E', which is the integral square error in between the transient responses of HOS and LOS and is given by:

$$E = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \qquad (16)$$

where, $y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems, and the parameters to be determined are the numerator coefficients of the LOS $\alpha_i \ (i = 0, 1, \ldots, (r-1))$.

In Table I, the specified parameters for the PSO algorithm

used in the present study are given. The computational flow chart of the proposed algorithm is shown in Fig. 3

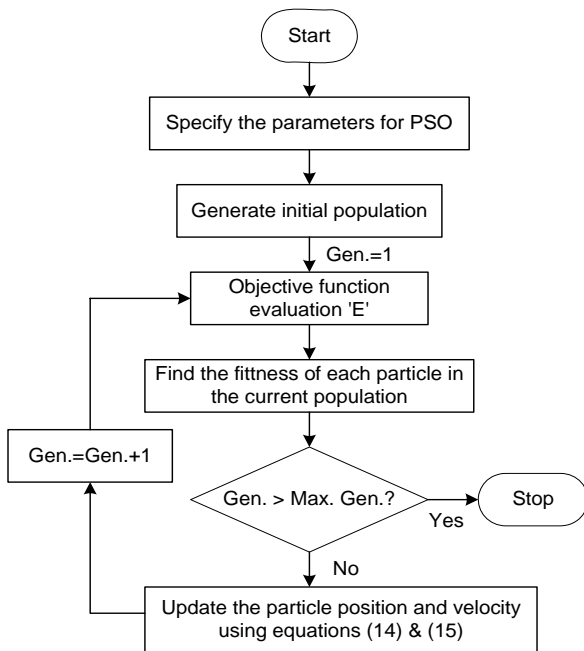| Parameters	Value
Swarm Size	20
Max. Generation	100
c_1, c_2	2.0, 2.0
w_{start}, w_{end}	0.9, 0.4

Fig. 3 Flowchart of PSO Algorithm.

III. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature for the comparison of the low order system (LOS) with the original high order system (HOS). The proposed algorithm is described in detail for one example while only the result of the other example is given.

Two error indices [26], known as relative integral square errors in between the transient parts of original and reduced order systems are also calculated to measure the goodness of the LOS, which are given by :

$$I = \int_0^\infty [\ g(t) \ - \ \tilde{g}(t)]^2 \ dt \Big/ \int_0^\infty g^2(t) \ dt \qquad (17)$$

$$J = \int_0^\infty [\ r(t) \ - \ \tilde{r}(t)]^2 \ dt \Big/ \int_0^\infty [r(t) - r(\infty)\ ]^2 \ dt \quad (18)$$

where, $g(t)$ and $r(t)$ are the impulse and step responses of original system, respectively, and $\tilde{g}(t)$ and $\tilde{r}(t)$ are that of their approximents.

Example-1. Consider a fourth-order system taken from Mukherjee and Mishra [16] and Mittal et al. [21]:

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \qquad (19)$$

The poles of the above system are all real and given by:

$$\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = -3, \lambda_4 = -4.$$

If a second-order order model $G_2(s)$ is to be synthesized using this algorithm, steps to be followed are as under:

Step-1: Fixing of ESZ of HOS:
Since all poles are real, it will be a line joining the nearest and farthest poles.

Step-2: Quantification of pole centroid and stiffness of HOS:

$$\lambda_m = \frac{\sum_{i=1}^{4} \lambda_i}{4} = 2.5$$

$$\lambda_s = \frac{\lambda_1}{\lambda_4} = 0.25$$

Step-3: Determination of eigen spectral points of LOS:
Equation (12) can be formed as under:

$$\begin{bmatrix} 1.25 & 0 \\ 0.75 & -1 \end{bmatrix} \begin{bmatrix} \text{Re}\,\lambda'_{p'} \\ M \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \qquad (20)$$

where, the values of λ_s, Q, p' and N are to be put as 0.25, 0, 2, 5 respectively.

Solution of (20) gives the location of the farthest pole $\text{Re}\,\lambda'_{p'}$ and M.

where, M = (Farthest pole-Nearest pole)/(p'-1),
and since p' = 2; Re$\lambda'_{p'}$ = 4; M = 3, ESPs of LOS are its two poles as $\lambda'_1 = 1$ and $\lambda'_2 = 4$.

Therefore, $\tilde{D}(s) = s^2 + 5s + 4$.

**Step-4:** By using PSO to minimize the objective function 'E', as described earlier, we have $\tilde{N}(s) = 0.6349s + 4$.

Therefore, finally $G_2(s)$ is given as:

$$G_2(s) = \frac{0.6349s + 4}{s^2 + 5s + 4} \qquad (21)$$

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model is given in Table II. Fig. 4(a)–(c) presents
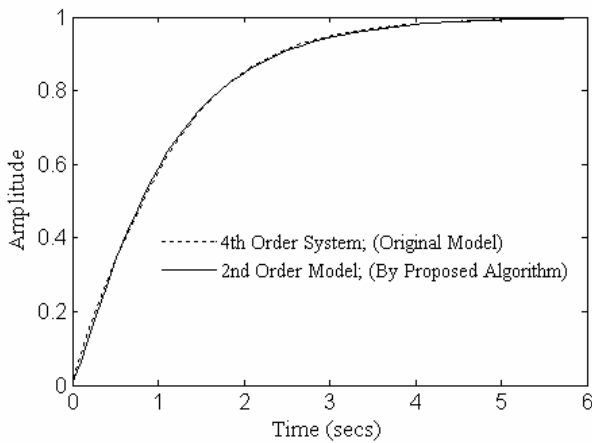
diagrams of convergence of the objective function 'E' for gbest, movement of the particles in the PSO algorithm, step responses of $G_4(s)$ and $G_2(s)$, respectively.
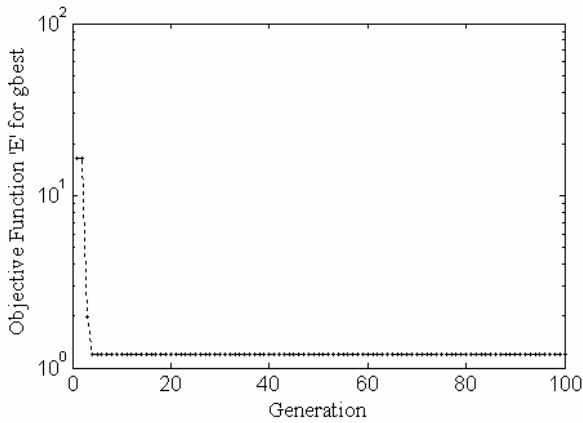
(a)
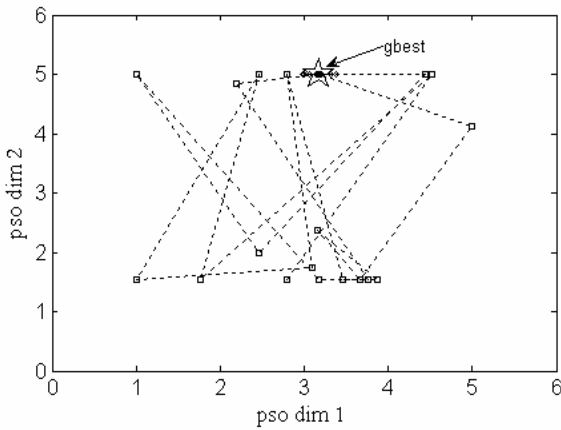
(b)

(c)

Fig. 4 (a) Convergence of objective function 'E' for gbest. (b) Movement of the particles in the PSO algorithm. (c) Step responses of $G_4(s)$ and $G_2(s)$.

TABLE II
COMPARISON OF REDUCED ORDER MODELS

Method of order reduction	Reduced Models	I	J
Proposed Algorithm	$\dfrac{0.6349\,s\ +\ 4}{s^2 + 5s\ +\ 4}$	1.85206×10^{-2}	4.19238×10^{-4}
Shamash [9]	$\dfrac{0.8334\,s + 2}{s^2\ +\ 3s\ +\ 2}$	6.96747×10^{-3}	5.80219×10^{-4}
Pal [10]	$\dfrac{16.0008\,s + 24}{30\,s^2\ +\ 42\,s\ +\ 24}$	5.65090×10^{-2}	2.04971×10^{-2}
Chen *et al.* [11]	$\dfrac{0.6997\,(s + 1)}{s^2\ +\ 1.45771s\ +\ 0.6997}$	1.45853×10^{-2}	4.67444×10^{-3}
Prasad and Pal [12]	$\dfrac{s + 34.2465}{s^2\ +\ 239.8082\,s\ +\ 34.2465}$	5.65229×10^{-1}	2.690595
Mukherjee and Mishra [16]	$\dfrac{0.80000033\,s + 2}{s^2\ +\ 3s\ +\ 2}$	6.71044×10^{-3}	4.17119×10^{-4}
Mittal *et al.* [21]	$\dfrac{0.799803\,s + 2}{s^2\ +\ 3s\ +\ 2}$	6.71125×10^{-3}	4.17124×10^{-4}
Hutton and Friedland [27]	$\dfrac{0.7947\,(s + 1)}{s^2\ +\ 1.65563s\ +\ 0.7947}$	5.84858×10^{-3}	6.29239×10^{-4}
Krishnamur-thy and Seshadri [28]	$\dfrac{20.5714\,s + 24}{30\,s^2\ +\ 42\,s\ +\ 24}$	2.01033×10^{-2}	1.68161×10^{-2}
Gutman *et al.* [29]	$\dfrac{2[48\,s + 144]}{70\,s^2 + 300\,s + 288}$	1.65857×10^{-1}	7.99545×10^{-2}
Lucas [30]	$\dfrac{0.833\,s + 2}{s^2\ +\ 3s\ +\ 2}$	6.95984×10^{-3}	5.76337×10^{-4}
Moore [31]	$\dfrac{0.8217\,s + 0.4543}{s^2\ +\ 1.268\,s\ +\ 0.4663}$	5.37477×10^{-3}	2.75246×10^{-4}
Safonov and Chiang [32]	$\dfrac{0.8213\,s + 0.4545}{s^2\ +\ 1.268\,s\ +\ 0.4664}$	5.37629×10^{-3}	2.77375×10^{-4}
Safonov *et al.* [33]	$\dfrac{0.7431\,s + 1.057}{s^2\ +\ 1.879\,s\ +\ 1.084}$	7.78347×10^{-3}	1.85358×10^{-3}

Example-2. Consider a eighth-order system [9] described by the transfer function:

$$G_8(s) = \frac{a(s)}{b(s)} \qquad (22)$$

where,

$$a(s) = 18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3$$
$$+ 222088s^2 + 185760s + 40320$$
$$b(s) = s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3$$
$$+ 118124s^2 + 109584s + 40320$$

By using the proposed algorithm, the following reduced second-order approximant is obtained:

$$G_2(s) = \frac{22.8360\,s + 8}{s^2 + 9s + 8} \qquad (23)$$

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-

order reduced model is given in Table III. Fig. 5(a)–(c) presents diagrams of convergence of the objective function 'E', for gbest, movement of the particles in the PSO algorithm, step responses of $G_8(s)$ and $G_2(s)$, respectively.

TABLE III
COMPARISON OF REDUCED ORDER MODELS

| Method of order reduction	Reduced Models	I	J
Proposed Algorithm	$\dfrac{22.8360\,s + 8}{s^2 + 9s + 8}$	4.63051 x 10^{-2}	2.50868 x 10^{-2}
Mukherjee et al. [7]	$\dfrac{11.3909\,s + 4.4357}{s^2 + 4.2122\,s + 4.4357}$	8.99334 x 10^{-2}	3.88109 x 10^{-2}
Shamash [9]	$\dfrac{6.7786\,s + 2}{s^2 + 3s + 2}$	3.02978 x 10^{-1}	1.90469 x 10^{-1}
Pal [10]	$\dfrac{151776.576\,s + 40320}{65520\,s^2 + 75600\,s + 40320}$	7.29677 x 10^{-1}	1.126099
Chen et al. [11]	$\dfrac{0.72046\,s + 0.36669}{s^2 + 0.02768\,s + 0.36669}$	1.031795	4.918133
Prasad and Pal [12]	$\dfrac{17.98561\,s + 500}{s^2 + 13.24571\,s + 500}$	7.88491 x 10^{-1}	9.94796 x 10^{-1}
Mukherjee and Mishra [16]	$\dfrac{7.0903\,s + 1.9907}{s^2 + 3s + 2}$	2.86389 x 10^{-1}	1.83434 x 10^{-1}
Mittal et al. [21]	$\dfrac{7.0908\,s + 1.9906}{s^2 + 3s + 2}$	2.86362 x 10^{-1}	1.83413 x 10^{-1}
Hutton and Friedland [27]	$\dfrac{1.98955\,s + 0.43184}{s^2 + 1.17368\,s + 0.43184}$	7.59574 x 10^{-1}	1.307654
Krishnamur-thy and Seshadri [28]	$\dfrac{155658.6152\,s + 40320}{65520\,s^2 + 75600\,s + 40320}$	7.24657 x 10^{-1}	1.127673
Gutman et al. [29]	$\dfrac{4[133747200\,s + 203212800]}{85049280\,s^2 + 552303360\,s + 812851200}$	3.64418 x 10^{-1}	9.38578 x 10^{-1}
Lucas [30]	$\dfrac{6.7786\,s + 2}{s^2 + 3s + 2}$	3.02978 x 10^{-1}	1.90469 x 10^{-1}

(b)

(c)
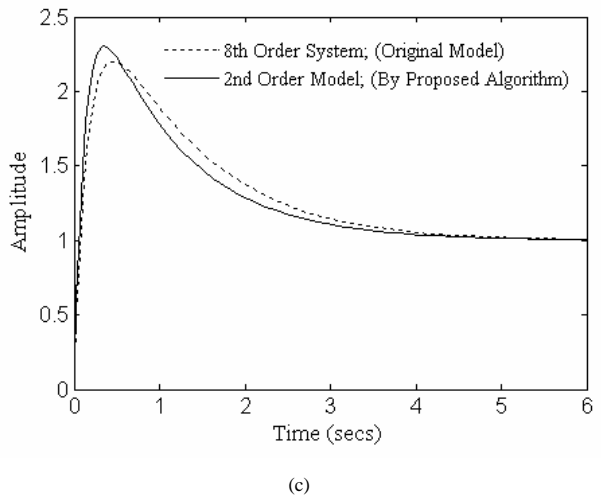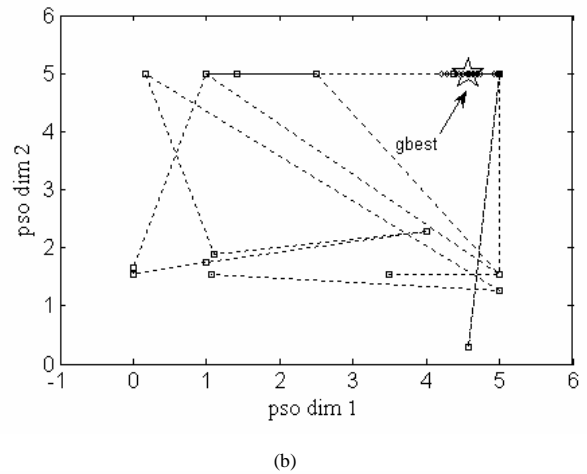
Fig. 5 (a) Convergence of objective function, 'E' for gbest. (b) Movement of the particles in the PSO algorithm. (c) Step response of $G_8(s)$ and $G_2(s)$.
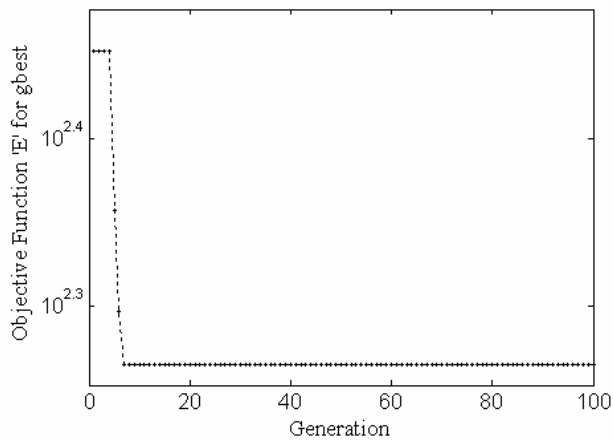
IV. STABILITY OF THE LOS

The proposed algorithm leads to a stable low order system (LOS), if the original high order system is stable. The algorithm has been successfully applied to several examples from the literature. Due to the following inbuilt feature of the method, the resulting low order systems are found to be stable as shown in Numerical Examples 1 and 2.

In this algorithm, the denominator of the LOS is found by Eigen spectrum analysis. For a stable system, all the poles should lie in the left half of the complex s-plane. So, if the original high order system is stable with all its poles lying in the left-hand side of the complex s-plane, the poles lying in the ESZ zone will always give a stable reduced order model as shown in Fig.1.

(a)

V. CONCLUSIONS

An algorithm for the order reduction of linear dynamic systems has been given based on the eigen spectrum analysis and the error minimization by particle swarm optimization technique. In this algorithm, the poles are synthesized by eigen spectrum analysis while the zeros are determined by minimizing the integral square error between the transient responses of original and low order systems using particle swarm optimization technique, pertaining to a unit step input. The algorithm is simple, rugged and computer oriented. It has been implemented in Matlab 7.0.1 on a Pentium-IV processor and the computation time is negligible being less than 1 minute.

The algorithm has been illustrated with the help of two numerical examples having real poles only. It is being extended for the systems with imaginary poles and also for multi-input multi-output (MIMO) systems. The numerical examples are chosen from the literature in such a way, so that a comparison of the proposed algorithm with some well known existing order reduction techniques can be made as shown in Tables II and III. It can be seen in Tables II and III that the proposed algorithm is comparable in quality with the other existing techniques. The algorithm preserves model stability and avoids any error in between the initial or final values of the responses of original and reduced order models.

REFERENCES

[1] R. Genesio, and M. Milanese, "A note on the derivation and use of reduced order models", *IEEE Trans. Automat. Control*, Vol. AC-21, No. 1, pp. 118-122, February 1976.

[2] M. S. Mahmoud, and M. G. Singh, *Large Scale Systems Modelling*, 1st Ed., International Series on Systems and Control, Vol. 3, Pergamon Press, 1981.

[3] M. Jamshidi, *Large Scale Systems Modelling and Control Series*, Vol. 9, North Holland, Amsterdam, Oxford, 1983.

[4] K. Jhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*, Prentice Hall, Upper Saddle River, New Jersey, 1996.

[5] A. Varga, *Model Reduction Software in the SLICOT Library*, in: B. Datta (Ed.), Applied and Computational Control, Signals and Circuits, Vol. 629, Kluwer Academic Publishers, Boston, 2001. http://www.robotic.dlr.de/~varga/.

[6] S. K. Nagar, and S. K. Singh, "An algorithmic approach for system decomposition and balanced realized model reduction", *Journal of Franklin Inst.*, Vol. 341, pp. 615-630, 2004.

[7] S. Mukherjee, Satakshi and R.C.Mittal, "Model order reduction using response-matching technique", *Journal of Franklin Inst.*, Vol. 342, pp. 503-519, 2005.

[8] G. Parmar, S. Mukherjee, and R. Prasad, "System reduction using factor division algorithm and eigen spectrum analysis (Accepted for publication)", *Applied Mathematical Modelling*, to be published. http://www.elsevier.com/locate/apm.

[9] Y. Shamash, "Linear system reduction using Pade approximation to allow retention of dominant modes", *Int. J. Control*, Vol. 21, No. 2, pp. 257-272, 1975.

[10] J. Pal, "Stable reduced order Pade approximants using the Routh Hurwitz array", *Electronic Letters*, Vol. 15, No. 8, pp.225-226, April 1979.

[11] T.C. Chen, C.Y. Chang, and K.W. Han, "Model reduction using the stability equation method and continued fraction method", *Int. J. Control*, Vol. 32, No. 1, pp. 81-94, 1980.

[12] R. Prasad, and J. Pal, "Stable reduction of linear systems by continued

fractions", *Journal of Institution of Engineers, India, IE(I) Journal – EL*, Vol. 72, pp. 113-116, October 1991.

[13] V. Singh, D. Chandra and H. Kar, "Improved Routh Pade approximants: A computer aided approach", *IEEE Trans. Automat. Control*, Vol. 49, No. 2, pp 292-296, February 2004.

[14] B. Salimbahrami, and B. Lohmann, "Order reduction of large scale second-order systems using Krylov subspace methods", *Linear Algebra Appl.*, Vol. 415, pp. 385-405, 2006.

[15] C. Hwang, "Mixed method of Routh and ISE criterion approaches for reduced order modelling of continuous time systems", *Trans. ASME, J. Dyn. Syst. Meas. Control*, Vol. 106, pp. 353-356, 1984.

[16] S. Mukherjee, and R.N. Mishra, "Order reduction of linear systems using an error minimization technique", *Journal of Franklin Inst.*, Vol. 323, No. 1, pp. 23-32, 1987.

[17] S. S. Lamba, R. Gorez, and B. Bandyopadhyay, "New reduction technique by step error minimization for multivariable systems", *Int. J. Systems Sci.*, Vol. 19, No. 6, pp. 999-1009, 1988.

[18] S. Mukherjee, and R.N. Mishra, "Reduced order modelling of linear multivariable systems using an error minimization technique", *Journal of Franklin Inst.*, Vol. 325, No. 2 , pp. 235-245, 1988.

[19] N.N. Puri, and D.P. Lan, "Stable model reduction by impulse response error minimization using Mihailov criterion and Pade's approximation", *Trans. ASME, J. Dyn. Syst. Meas. Control*, Vol. 110, pp. 389-394, 1988.

[20] P. Vilbe, and L.C. Calvez, "On order reduction of linear systems using an error minimization technique", *Journal of Franklin Inst.*, Vol. 327, pp. 513-514, 1990.

[21] A.K. Mittal, R. Prasad, and S.P. Sharma, "Reduction of linear dynamic systems using an error minimization technique", *Journal of Institution of Engineers IE(I) Journal – EL*, Vol. 84, pp. 201-206, March 2004.

[22] G.D. Howitt, and R. Luus, "Model reduction by minimization of integral square error performance indices", *Journal of Franklin Inst.*, Vol. 327, pp. 343-357, 1990.

[23] J. Kennedy, and R. C. Eberhart, "Particle swarm optimization", *IEEE Int. Conf. on Neural Networks*, IV, 1942-1948, Piscataway, NJ, 1995.

[24] J. Kennedy, and R. C. Eberhart, *Swarm intelligence*, San Francisco: Morgan Kaufmann Publishers, 2001.

[25] R. C. Eberhart, and Y. Shi, "Particle swarm optimization: developments, applications and resources", *Congress on evolutionary computation*, pp.81-86, 2001, Seoul Korea.

[26] T.N. Lucas, "Further discussion on impulse energy approximation", *IEEE Trans. Automat. Control*, Vol. AC-32, No. 2, pp. 189-190, February 1987.

[27] M. F. Hutton, and B. Friedland, "Routh approximation for reducing order of linear, time invariant systems", *IEEE Trans. Automat. Control*, Vol. AC-20, No. 3, pp. 329-337, June 1975.

[28] V. Krishnamurthy, and V. Seshadri, "Model reduction using the Routh stability criterion", *IEEE Trans. Automat. Control*, Vol. AC-23, No. 4, pp. 729-731, August 1978.

[29] P.O. Gutman, C.F. Mannerfelt, and P. Molander, "Contributions to the model reduction problem", *IEEE Trans. Automat. Control*, Vol. AC-27, No. 2, pp. 454-455, April 1982.

[30] T. N. Lucas, "Factor division: a useful algorithm in model reduction", *IEE Proceedings*, Vol. 130, No. 6, pp. 362-364, November 1983.

[31] B. C. Moore, "Principal component analysis in linear systems: controllability, observability and model reduction", *IEEE Trans. Automat. Control*, Vol. AC-26, No. 1, pp. 17-32, February 1981.

[32] M. G. Safonov, and R. Y. Chiang, "A Schur method for balanced-truncation model reduction", *IEEE Trans. Automat. Control*, Vol. 34, No. 7, pp. 729-733, 1989.

[33] M. G. Safonov, R. Y. Chiang, and D J. N. Limebeer, "Optimal Hankel model reduction for nonminimal systems", *IEEE Trans. Automat. Control*, Vol. 35, No. 4, pp. 496-502, 1990.

Author's Biography

Girish Parmar was born in Bikaner (Raj.), India, in 1975. He received B.Tech. in Instrumentation and Control Engineering from Regional Engineering College, Jalandhar (Punjab), India in 1997 and M.E. (Gold Medalist) in Measurement and Instrumentation from University of Roorkee, Roorkee, India in 1999. Since then, he is working as a Lecturer in Government Engineering College at Kota (Rajasthan), India. Presently he is QIP Research Scholar in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). He is life member of Systems Society of India (LMSSI), Associate member of Institution of Engineers, India (AMIE) and member of ISTE.

Dr. Shaktidev Mukherjee was born in Patna, India, in 1948. He received B.Sc. (Engg.), Electrical from Patna University in 1968 and M.E., Ph.D. from the University of Roorkee in 1977 and 1989 respectively. After working in industries till 1973, he joined teaching and taught in different institutions. Presently he is Professor in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). His research interests are in the area of Model Order Reduction and Process Instrumentation and Control. He is Fellow of Institution of Engineers, India (FIE) and life member of Systems Society of India (LMSSI).

Dr. Rajendra Prasad was born in Hangawali (Saharanpur), India, in 1953. He received B.Sc. (Hons.) degree from Meerut University, India, in 1973. He received B.E., M.E. and Ph.D. degrees in Electrical Engineering from University of Roorkee, India, in 1977, 1979, and 1990 respectively. From 1983 to 1996, he was a Lecturer in the Electrical Engineering Department, University of Roorkee, Roorkee (India). Presently, he is an Associate Professor in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). His research interests include Control, Optimization, System Engineering and Model Order Reduction of large scale systems.