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Rain Cell Ratio Technique in Path Attenuation for Terrestrial Radio Links

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Abstract**—**A rain cell ratio model is proposed that computes attenuation of the smallest rain cell which represents the maximum rain rate value i.e. the cell size when rainfall rate is exceeded 0.01% of the time, $R_{0.01}$ and predicts attenuation for other cells as the ratio with this maximum. This model incorporates the dependence of the path factor r on the ellipsoidal path variation of the Fresnel zone at different frequencies. In addition, the inhomogeneity of rainfall is modeled by a rain drop packing density factor. In order to derive the model, two empirical methods that can be used to find rain cell size distribution D_c are presented. Subsequently, attenuation measurements from different climatic zones for terrestrial radio links with frequencies F in the range 7-38 GHz are used to test the proposed model. Prediction results show that the path factor computed from the rain cell ratio technique has improved reliability when compared with other path factor and effective rain rate models, including the current ITU-R 530-15 model of 2013.

Keywords**—**Packing density of rain drops, prediction model, rain attenuation, rain cell ratio technique.

I. Introduction

RESEARCH in rain attenuation has been an active area due to the need for a more robust rain attenuation prediction model that may be recommended for universal application. The International Telecommunications Union group for Radio sector (ITU-R) has been updating rain attenuation prediction model in Recommendation ITU-R P.530 for terrestrial links, with the latest update being in 2013 [1]-[10]. Such prediction models are used in wireless communication systems during the phase of link planning process to provide for signal losses, especially for frequencies beyond 10 GHz. Indeed, the formulation of rain attenuation models requires complete knowledge of the characteristics of rainfall cell and rain attenuation models.

Presently, global telecommunication systems standards such as Global System for Mobile Communications (GSM) deploy wireless radio links in the backhaul, where rain attenuation has significant effect on degradation of the link's reliability. Line-of-Sight (LOS) links are normally deployed in GSM and factors such as the Fresnel zone radius are considered to ensure a clear LOS. Rain cell size in each geographical area is also vital in formulating a rain attenuation prediction model. Moreover, attenuation per kilometer, normally referred to as specific attenuation is vital in this process. Specific attenuation incorporates the physical parameters that measure the degree of interaction of waves with the rain drops. As such, wave scattering, absorption rate and polarization of the antenna are taken into account.

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In an effort to understand the characteristics of rain and provide rain attenuation prediction models, several early research outputs have been brought forward by many authors [1]-[8] and ITU-R standards [9], [10]. Furthermore, classical statistical analysis show that the growth in rain rates in a rain cell is non-uniform, inhomogeneous and characterized by spatial variation of rain drops depending on the geographical area. Rain rates depend on the number of rain drops, rain drop diameters (which can be measured directly from disdrometer) and the size of the spacing between rain drops [12]-[20].

II. Literature Review

A. Effective Path Length

Recent rain attenuation prediction models may be classified into empirical models, statistical models and hybrid models that employ both empirical and statistical methods. Nonetheless, extensive literature on rain attenuation shows similarity in theoretical approaches that have been adopted by different authors in developing rain attenuation prediction models [9]-[29]. In a nutshell, research in rain attenuation is concentrated on the studies of effective path length, frequency dependence, rain cell size, rain cell shape and rain drop packing density.

In the effective path length concept, specific attenuation is derived from point rain rate values and multiplied by the effective path length derived from the path factor. In this approach, the path factor is derived from a frequency dependence factor and the radio link length factor. On the other hand, path averaged-rain rate/effective rain rate concept utilizes the rain rate reduction factor to compute effective specific attenuation, thus incorporating frequency dependence as well. These two concepts have seen the emergence of many rain attenuation models for terrestrial links but the accuracy of many of these models can be improved and require improvements, especially for tropical sites. It is always feasible to estimate rain attenuation for terrestrial radio links based on the concept of path factor r. This forms the basis of the current model adopted by the ITU-R standard ITU-R P.530-15 which exploits the rain rate exceeded 0.01% of the time $R_{0.01}$ to compute path attenuation. Also, the concept of path-average/effective rain rate has shown reliable prediction results as well [2]-[10].

B. Frequency Dependence

The number of rain drops in a rain cell affects scattering and absorption mechanisms, which are accounted for by the computation of specific attenuation. Indeed, specific attenuation also accounts for the carrier frequency used in the radio link [9]. Furthermore, it is observed that the dependence of specific attenuation on carrier frequency is non-uniform along the radio link [21]-[23]. In addition, rain attenuation in the radio link depends on the Fresnel zone whose radial size is determined by the carrier frequency of the radio link [5]. As a result, frequency dependence of rain attenuation can be classified in two-fold.

C. Rain Cell Size

Physical interpretation of this rain cell ratio concept means that the minimum rain cell size represents the highest attainable rain rate in any rain event. Since rain rate information is available for rain rate R values exceeded 0.01% of the time $R_{0.01}$ and because terrestrial radio link planning is usually done at this level, the following discussion involves the use of the minimum rain cell size derived from $R_{0.01}$. It therefore becomes imperative to provide the relationship between the rain cell size and rain rate. Two methods can be used to describe rain cell size distributions with measurement data obtained from radar or point measurements from rain gauge or disdrometer equipment [12]-[20]. In this paper, only point measurements are used since radar data are not easily accessible. Fortunately, it has been shown that rain cell sizes can be derived from complementary cumulative distribution function (CCDF) of rain rates based on "*Synthetic Storm Technique*" [17]. As a supplement to existing literature, we will show that rain cell diameter distribution (RCDD) can be derived from point rain rate measurements with similar results for the rain cell size obtained from the CCDF analysis. In general, a power law relationship for rain cell size is used to describe the cell diameter D_c in the form $D_c = \lambda R^{-\beta}$ and used to develop the proposed model [24]-[27].

In this paper, we use the latter approach to monitor the maximum attenuation along a radio link and use the path factor concept to develop the proposed model. The unique feature in this model is that only the size of the smallest rain cell is changed per geographical area. When this rain cell size is known, then path attenuation at any other rain rate is computed from the ratio with this minimum rain cell size.

D. Shape of the Rain Cell

Broadly, spatial inhomogeneity of rain can be estimated by the concept of effective path length which is the product of the actual length of a given radio link and a path factor. As a result, rain induced attenuation is usually estimated from known rain rate distribution in a given climatic region. Reduction factor modeling involves the use of a reduction factor either for the point rain rate which results in path averaged-rain rate/effective rain rate concept [28] or a reduction factor for the length of the radio link which results in effective path length [10]. Depending on the approach taken, these reduction factors can be determined through empirical data measurements and fitting mixed with some scientific interpretation of the physical characteristics of wave propagation in different drop sizes of liquid water. The main characteristics of the radio waves are the wavelength and frequency. Depending on the size of the wavelength, absorption and scattering mechanisms encountered within the rain drops lead to attenuation. Through ITU-R, specific attenuation can be described as the attenuation in rain path per kilometer with the assumption of uniform attenuation and a non-uniformity factor [10].

Actually, ITU-R continues to refine and improve on rain attenuation models by allowing recommendations to the preceding models [9], [10]. In addition, new approaches to rain attenuation are encouraged so as to improve prediction results for attenuation in rain. In essence, research on rain attenuation has been in the modeling of the path factor or the rain rate reduction factor. Notably, the difference in these models depend on whether large data banks are used for fitting purposes or whether more physical analysis is infused with limited empirical information.

E. Rain Drop Packing Density

Theoretically, the spacing between the rain drops may be estimated by `listening' to the time intervals between the drops in measuring equipment. It can be said that if the drops coalesce, then the packing density of rain drops is unity which means that any wave that traverses this unity drop density will always be intercepted for all the cell length. Such rain forms have been reported to be of convective nature [1]-[8]. Stratiform rain drops do not coalesce and therefore the rain drop packing density can be assumed to be less than unity. The general inference of this rain drop packing density is that within a rain event, a radio wave does not always undergo attenuation in the whole length of the rain cell. This is because, depending on the rain structure, there are spaces between rain drops and therefore the effective length that is responsible for attenuating the waves will always be less than the observed rain cell diameter [12]-[16]. This may be referred to as rainy cell diameter. As a result, if the size of the rainy cell diameter is known, then attenuation due to rain in a link is determined from the size of this cell diameter only. The *rainy* cell diameter is estimated as the measured *rain* cell diameter (which includes spatial variation of rain drops) minus the spacing between rain drops. However, there is inherent difficulty in directly measuring the spacing between rain drops in a rain event especially for point rain measurements or even in rain field measurements from radar observations. Indeed, advection velocity of rain cells differs from the wind velocity [16], [17], [23].

The rest of the paper is organized as follows: Section III presents the derivation steps for the rain cell ratio model based on model parameters introduced in Section II. Section IV follows with a discussion of observations of rain attenuation parameters. Comparison of prediction results from various well-known attenuation models is presented in Section V. Finally, the paper is concluded in Section VI.

III. Derivation of Rain Cell Attenuation Model

A. Summary of Derived Model Parameters

The rain cell model has been derived and the main results have been provided in the following summary.

(a) Specific attenuation

$$A_s = kR^{\alpha} \quad (1)$$

(b) Rain cell diameter size,

$$D_c = \lambda R^{-\beta} \quad (2)$$

(c) Rain drop packing density,

$$\delta_c = \exp\left[-0.5\left(\frac{R-R_b}{R_{0.01}}\right)^2\right] \quad (3)$$

(d) Link factor,

$$\delta_l = \exp\left[-0.5\left(\sqrt{\frac{L-D_{0.01}}{D_{0.01}}}\right)^2\right] \quad (4)$$

(e) Fresnel zone factor,

$$\delta_{FL} = \sqrt{n\frac{L}{F}} \quad (5)$$

(f) Rain cell reduction factor,

$$r_c = \delta_c \quad (6)$$

(g) Rainy cell diameter,

$$D_r = \lambda R^{-\beta}.\delta_c \quad (7)$$

(h) Path factor

$$\delta_r = \left(\frac{L}{F}\right)^{0.1505}\left(\frac{R_{0.01}}{R}\right)^{\beta}\exp\left[-0.5\left\{\left(\frac{R}{R_{0.01}}-0.85\right)^2+\left(\frac{L}{D_{0.01}}-1\right)\right\}\right] \quad (8)$$

(i) Path attenuation for a link of length $l = L$, $A = kR^{\alpha}\delta_r.l$, with $R_{0.01}$ and break-point rain rate R_b.

B. Derived Parameters from Measurements

Here, attenuation analysis due to a rain event is presented. Conventionally, total path attenuation is given as the sum of the product of the specific attenuation and the distance covered by each rain rate threshold in a given rain event. As a result path attenuation A is written as [23]

$$A = A_s(R,F,l)\int_{D_{min}}^{D_{max}} N(D_c)\,dD_c \quad (9)$$

$$A = N(D_c)A_s(R,F,l)D_c \quad (10)$$

$$N(D_c) = N_0(D_c)\exp-\left(\Phi(L,R)\right) \quad (11)$$

$$N_0(D_c) = \frac{\lambda}{D_c} \quad (12)$$

where $N(D_c)$ is the distribution of rain cell diameters in the rain event, $N_0(D_c)$ represents the possible number of cells of diameter D_c that can occupy the rain event of total length $\lambda = D_{max}$ [km], $A_s(R,F,l)$ is the specific attenuation, which depends on the rain rate R, carrier frequency F and the link length l.

It is evident that the maximum number of rain cells that can occur within a rain event belongs to those cells with minimum cell diameter i.e. $D_c = D_{min}$. For telecommunication applications D_{min} may be expressed as a function of $R_{0.01}$. Therefore, maximizing attenuation in the rain event results in

$$A_{max} = \frac{\lambda}{D_{min}}A_s(R,F,L)D_c(R)\exp-\left(\Phi(l,R)\right) \quad (13)$$

It follows that specific event attenuation per km is given as

$$\frac{A_{max}}{\lambda} = \frac{D_c}{D_{min}}A_s(R,F,L)\exp-\left(\Phi(l,R)\right) \quad (14)$$

In terms of the Fresnel zone, rain attenuation is affected by the radius d_e of the Fresnel zone such that $d_e = 8.657\sqrt{n\frac{L}{F}}$, where the integer n refers to the radius of the n^{th} zone, L denotes the length per kilometer inside the rain event thus $L = l/1$ and F denotes the new carrier frequency that causes the changes in the ellipsoidal radius thus $F = f\sqrt{1+10^{-4}f^2}$ according to ITU-R P.530-15 [9].

For simplicity, the changes in the radius of the Fresnel zone can be modeled by a power law formula thus $n\frac{L}{F} = \left(\frac{L}{F}\right)^{\mu}$ and

$$\mu = \log_{10}\left(n\frac{L}{F}\right)/\log_{10}\left(\frac{L}{F}\right) \quad (15)$$

In order to determine μ for the model, the following parameters are chosen: $L = 1$ km and $F = 10$ GHz. This is because rain attenuation is significant above 10 GHz and μ coefficient measures the effect of frequency per kilometre.

By re-writing (15) as $\mu = \log_{10}(0.1n)/\log_{10}(0.1)$, it is evident that:

$$\begin{matrix} \mu = 1 & \text{if} & n = 1 \\ \mu = 0 & \text{if} & n = 10 \end{matrix} \quad (16)$$

As a result, we choose the median value of $n = 5$, which results in $\mu = \log_{10} 2 = 0.3010$ and

$$\delta_{FL} = \sqrt{\left(L/F\right)^{\mu}} \quad (17)$$

Furthermore, information on specific attenuation is obtained from the dependency of attenuation on the rain rate and the carrier frequency of the link alone. In addition, the rain cell diameter can be expressed as $\lambda R^{-\beta}$. Therefore, we can separate $A_s(R,F,L)$ into two forms: the conventional specific attenuation $A_s(R,f)$ and Fresnel zone radius factor δ_{FL}. δ_{FL} is given as $\delta_{FL} = \sqrt{\left(L/F\right)^{\mu}}$. Then, the following are deduced

$$\frac{A_{max}}{\lambda} = A_s(R,f)\sqrt{\left(\frac{L}{F}\right)^{\mu}}\left(\frac{R_{0.01}}{R}\right)^{\beta}\exp-\left(\Phi(l,R)\right) \quad (18)$$

Consequently, since $A_s(R,f) = kR^{\alpha}$, the path attenuation and r for a path length l is given as

$$A(l) = kR^{\alpha}\sqrt{\left(\frac{L}{F}\right)^{\mu}}\left(\frac{R_{0.01}}{R}\right)^{\beta}\exp-\left(\Phi(l,R)\right).l \quad (19)$$

$$r = \sqrt{\left(\frac{L}{F}\right)^{\mu}}\left(\frac{R_{0.01}}{R}\right)^{\beta}\exp-\left(\Phi(l,R)\right) \quad (20)$$

where r is the path factor, $F = f\sqrt{1+10^{-4}f^2}$ and $L = l$.

It can be seen that (20) allows for the computation of a rain cell ratio r_D, which can be written as

$$r_D = \left(\frac{R_i}{R_{0.01}}\right)^{-\beta} \quad (21)$$

where R_i is any rain rate, and $R_i < R_{0.01}$.

C. Horizontal Packing Density of Rain Drops

The distribution of the rain drops show variation in the drop diameters whose rate of rain drop coalescence is dependent on the climate of a given geographical area. The spatial occurrence of rain drops is illustrated in Fig. 1. It is observed that the rain drops coalesce more closely at the rain rate exceeded 0.01% of the time, just approximately the break point, where a second cell is assumed to occur. The growth in coalescence leads to high attenuation because high number of rain drops lead to the high scattering and absorption of waves. By assigning a unity value at the break point for the packing density δ_c, we propose to model rain drop packing density as a Gaussian growth, which is written as

$$\delta_c = \exp\left[-0.5\left(\frac{R-R_b}{R_{0.01}}\right)^2\right] \quad (22)$$

We noted that the break point rain rate is about 85% of the value exceeded 0.01% of the time and (22) may be re-written as

$$\delta_c = \exp\left[-0.5\left(\frac{R}{R_{0.01}} - 0.85\right)^2\right] \quad (23)$$

The rain cell model computes attenuation of a specific cell bound by a given rain rate threshold. This means that any link shorter than the rain cell diameter will experience a fraction of the cell attenuation. The smallest cell that can occur on the link 0.01% of the time is determined by $D_{0.01}$.

Noting that the ellipsoid presents changes in $\sqrt{L}$ per unit length ($D_0 = 1$) of the cell diameter D_c then similarly, a Gaussian packing density relationship from the minimum diameter may be represented as

$$\delta_l = \exp\left[-0.5\sqrt{\left(\frac{l-d_{0.01}}{d_{0.01}}\right)^2}\right] \quad (17)$$

Finally, in general, path attenuation is given as

$$A_C = (kR^{\alpha})\left(\frac{R}{R_{0.01}}\right)^{-\beta}(\delta_{FL}\delta_c\delta_l).L = (kR^{\alpha})rL \quad (25)$$

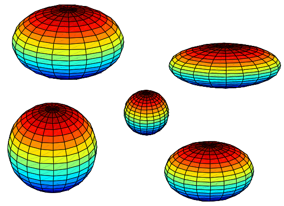

Fig. 1 An example of spatial occurrence of rain drops per unit volume

Since R_b is taken to be 85% of $R_{0.01}$, the general path factor is given as

$$r = \left(\frac{L}{F}\right)^{0.1505}\left(\frac{R_{0.01}}{R}\right)^{\beta}\exp\left[-0.5\left\{\left(\frac{R}{R_{0.01}} - 0.85\right)^2 + \left(\frac{L}{d_{0.01}} - 1\right)\right\}\right] \quad (26)$$

Also, the path factor $r_{0.01}(F,R,L)$ at $R_{0.01}$ is simply written as

$$r_{0.01} = \left(\frac{L}{F}\right)^{0.1505}\exp\left[-0.5\left\{\left(\frac{L}{32R_{0.01}^{-0.46}} - 0.9375\right)\right\}\right] \quad (27)$$

where $F = f\sqrt{1+10^{-4}f^2}$ and the expression in (27) implies that the path factor will most often be less than unity for any $f \geq$10 [GHz].

IV. Observations from Measurements

The results that follow are based on measurement information of rainfall rate [21] and attenuation [23]-[27] collected beyond two years in Durban, South Africa. A disdrometer was used to collect rain data at an integration time of one minute. ITU-R classifies Durban area as a subtropical site, and rain data from the site are useful in describing a more universal attenuation model for both temperate and tropical sites. Furthermore, most of the prior studies by the author were based on the same site [23]-[27].

A. Rain Cell Diameter Size, $d_{0.01} = \lambda R_{0.01}^{-\beta}$

During point measurements of rain rates, only rain drops from a section of the rain cell will be collected over the measuring equipment. This is the same way a rain cell may intercept a radio link at different cell diameter sections. As a result, rain cell sizes may vary according to the location of the point measurements and it is useful to exploit the mean value at a desired percentage of time e.g. 0.01%. This has the implication that a point rain rate measured at the transmitter may be different to that observed at the receiver at the same instance of a rain event. If this is the case, then specific attenuation within the cell will vary. In order to account for this, *equivalent* path rain rate will have to be computed, especially for low rain rates. High rain rates dominate specific attenuation and the equivalent rain rate remains the same. This is one of the reasons why the path factor for short path lengths should be greater than unity. If the specific attenuation is computed for the conventional rain rate, then the non-uniformity in specific attenuation may be accounted for in the cumulative *rainy* cell diameter, especially for radio links longer than the rain cell diameter. This argument has a further implication that the rain cell size distribution for radio links shorter than r*ainy* cell diameter is different from longer links [8], but that discussion is beyond the scope of this research work.

Fig. 2 illustrates the number of rain events or cells that account for rain forms that have different peak rain rates. These plots are deduced from measurements studied in [23], where rain field area is discussed. It shows that rain events with low rain rates cover a larger area than the convective types. For example, there is a case, where a rain event with a peak of 3 mm/hr covered a diameter of 12 km. Fig. 2 also shows that the maximum diameter occupied by rain rate above 20 mm/hr is about 6 km. However, for the computation of path attenuation, the rain cell size is computed as $d_{0.01} = \lambda R_{0.01}^{-\beta}$, where the values for λ and β can be developed.

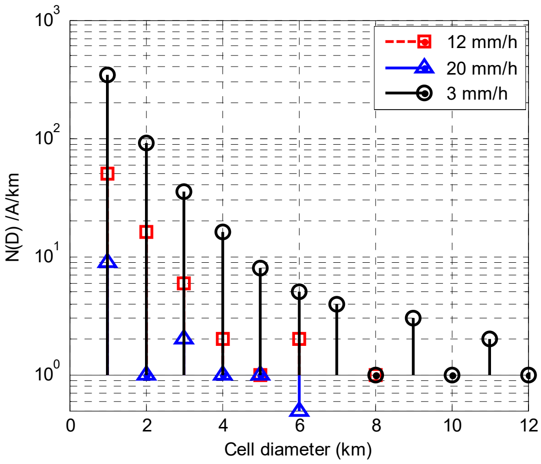

Fig. 2 Rain cell number distributions from point rain rate

B. Rain Drop Packing Density

It is well-known that propagation losses increase with the path length of a radio link between a transmitter and a receiver. Since we have fixed the median cell diameter, any geographical differences in the path factor will be as a result of the differences in the median cell diameter. By applying (22), Fig. 3 shows how close the rain drops are spaced from one another within a rain event for various peak rain rate thresholds. The graph implies that at the peak rain rate for any rain event, the rain form becomes more of a sheet of water throughout the cell diameter, thus a unity packing density.

The packing density varies from one climatic region to another. It suffices to state that a stratiform rain in temperate and tropical regions have different spatial characteristics. For example, in the tropics ($R_{0.01}$ > 60 mm/h), low rain rates in the range of 1-20 mm/h have lower packing densities than those in the subtropical and temperate regions. As a result, this model will be able to track these climatic variations and their effects on the scale of attenuation.

Fig. 4 shows the variation of rainy cell diameter with rain rates for different rain rate thresholds. It is shown that the largest cell that can occur in any rain event in Durban is slightly less than 35 km. In addition, rain rates above 100 mm/hr extend for less than 3 km. However, in terms of rain attenuation, it is desirable to compute the rain cell diameter that adversely affects the signals. It has been observed that there are time differences between adjacent rain drops, which implies that rain does not occur as a sheet of water, but has spaces between drops. The length of the sheet of water that contributes to absorption or scattering of the signals is referred to as rainy cell diameter. It is therefore evident from Fig. 4 that the maximum cell diameter is 20 km instead of 33 km. In addition, it is observed that rain forms that portray various break points take unique *rainy* cell size distributions. Nevertheless, the lower the rain rates, the larger the *rainy* cell diameter. The rainy cell diameter does not go beyond 20 km. As a result, the path factor for any radio link more than 20 km will be equal to r at 20 km. This result is similar to observation made by Crane's two component model.

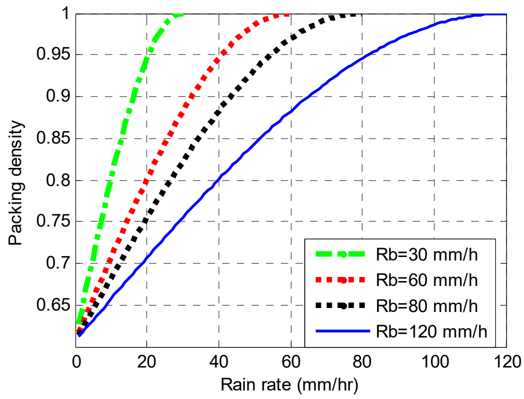

Fig. 3 Variation of rain drop packing density with rain rate of various R_b

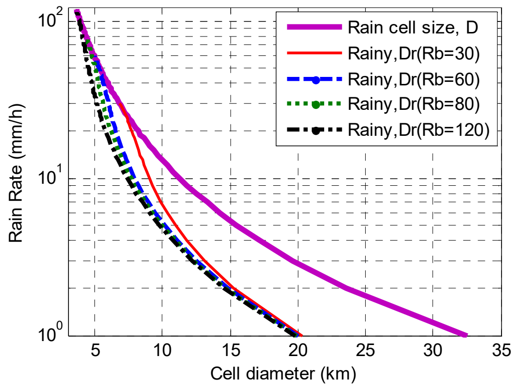

Fig. 4 Variation of rainy cell diameter with rain rates of various R_b

C. Path Factor

In view of the previous observations, the following remarks can be deduced from the modified path factor:

1. Shorter path lengths may be dominated by high rain rates

at the center of the rain cell.

2. Both the transmit and receive antennae terminals could be wet for short links since the rain cell goes beyond it, thus more signals are absorbed at the two terminals.
3. A second rain cell may occur on a link if the path length is longer than the maximum rain cell diameter. This implies a factor that decreases to unity at the edge of the first cell and then increases thereafter, where a second rain cell is assumed in the horizontal path.

Evidently, rain attenuation prediction models require the knowledge of specific attenuation A_s, rain cell diameter D_0, fixed path length L of a radio link and path factor, r. Under some conditions, r is normally greater than unity, a fact that surprisingly implies an increase in L. However, it is argued that r actually modifies the rain cell diameters within a rain event and can take values above unity. As such, specific rain cell (SC) attenuation A_c is computed from the product of specific attenuation and rain cell adjustment factor δ_r which is independent of L. The case where $r > 1$ may also imply that the rain drops coalesce and thus result in heavier penalty in absorption and scattering processes.

In order to compute δ_r, a method describing packing density δ of rain drops within the rain cell has been introduced. Therefore, total path attenuation is the product of SC attenuation A_c and $r = \delta_r$. Immediate results validate the observation that r can be greater than unity for low rain rates, which have larger cell diameters and moreover, δ_r may be adapted for different rain regions.

The following can be further deduced from the proposed theoretical framework. The cell ratio between the smallest and largest rain cell is given as $r_D = \frac{D_{max}(1\ mm/h)}{D_{minx}(R_{0.01}\ mm/h)} = R_{0.01}^{\beta}$.

(a) For a tropical location with $R_{0.01} = 120$ mm/h, $r_D = 9.0453$, which is consistent with the large rain drop sizes observed in tropical locations with minimum cell size of 3.6118 km.

(b) For a sub-tropical location with $R_{0.01} = 60$ mm/h, $r_D = 6.5758$. The rain cell diameter at 60 mm/h is 4.9682 km i.e. 32.67/6.5758.

(c) For a temperate location with $R_{0.01} = 30$ mm/h, $r_D = 4.7805$

Fig. 5 shows the comparison between the ITU-R P. 530-15 model and the proposed model in terms of the variation of the path factor with path length at 15 GHz. Generally, Fig. 5 shows that the proposed model gives more attenuation for links between 3 km and 7 km in tropical and sub-tropical locations i.e. rain rates between 30 mm/hr and 120 mm/hr. Furthermore, the following observations can be made: The path factor should be approximately equal to unity. Except for rain drop packing density variations, if the link is shorter than $D_{0.01}$, then the effective path length is equal to the link length otherwise the effective path length should be approximately equal to $D_{0.01}$ for all link lengths. This models a single rain event. Beyond 20 km, another rain event may occur thus forming multiple cells in the horizontal path.

Fig. 6 illustrates the variation of the path factor i.e. the factor with carrier frequency at a given break point rain rate for different path lengths. The wavelengths at higher frequencies are shorter than those of low frequencies. Therefore, signals with shorter wavelength are prone to absorption. As a result, more attenuation will occur at higher carrier frequencies.

V. Comparison of Prediction Results

The following discussions illustrate the comparison of high performance models with the proposed cell ratio model. The models used for comparison purposes are derived from different theoretical approaches.

A. Selected Rain Attenuation Models

The following models are used in evaluating prediction results for rain attenuation in various regions. The selected models are known to be more accurate models for rain attenuation when compared with other existing models in open literature.

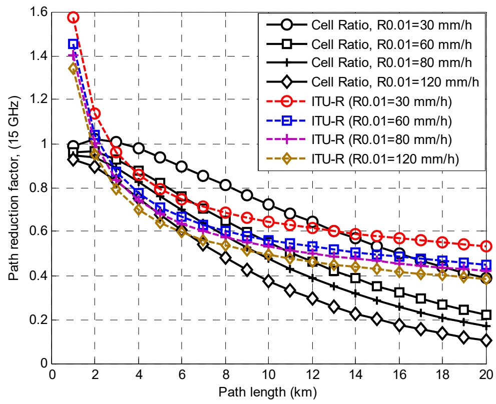

Fig. 5 Path factor comparison with ITUR model at 15 GHz

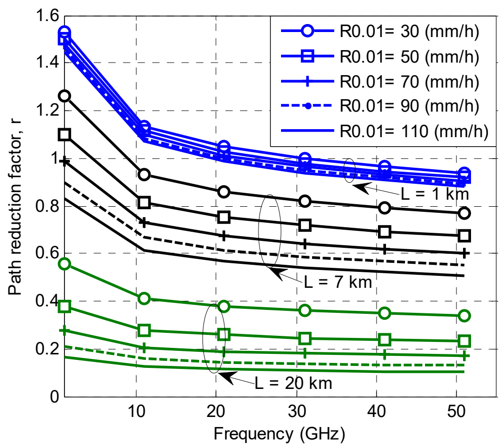

Fig. 6 Path factor for various frequencies at different rain rates exceeded 0.01% of the time

(1) Moupfouma model [7]

$$r = \frac{1}{\left(1+0.03L^{m}\left(\frac{P}{0.01}\right)^{-\beta}\right)} \tag{28}$$

(2) IT-R P.530-15 model [9]

$$r = \frac{1}{0.477d^{0.633}R_{0.01}^{0.073\alpha}f^{0.123}-10.579[1-\exp(-0.024d)]} \quad (29)$$

(3) Mello and Pontes's effective rain rate attenuation model [29]

$$A(p) = k\left(1.763R_p^{0.753+\frac{0.197}{d}}\right)^{\alpha} \cdot \frac{d}{1+d/(119R_p^{-0.244})} \quad (30)$$

B. Rain Attenuation Measurements Results

In order to evaluate prediction accuracies of the selected models, practical link measurement results are required. Firstly, the link that was set up in Durban, South Africa is used for evaluation purposes. The center frequency of the carrier was 19.5 GHz and the path was about 7 km. The rain in the location has a break point around 60 mm/hr, which depicts sub-tropical location. The link exploited vertical polarization.

Fig. 7 shows that the highly empirical model proposed by Moupfouma [5] closely predicts the measured rain attenuation, especially beyond rain rates of 20 mm/hr. The effective rain rate model proposed by Mello and Pontes [29] is seen to follow the model of Moupfouma in terms of accuracy. The proposed ITU-R model seems to be accurate at the break point rain rate, which is actually the point at which the model is optimized, i.e. around 60 mm/hr. The semi-empirical, direct frequency-dependent model uses data majorly collected from temperate regions, which is not consistent with rain data collected from sub-tropical regions. It is also worthy to note that since link availability may be set at 99.99% of the time, all three models seem to satisfy that criteria. However, the rain cell ratio model is seen to be accurate throughout the rain rate profile. This is expected because all signal dependency factors have been included in the cell ratio model for the site in Durban. Generally, it is shown that a provision of about 30 dB in the link budget is sufficient to achieve link availability of 99.99% during rainy season.

C. Frequency Scaling

It would be interesting to carry out comparisons at different frequencies. However, link deployments are very expensive. To solve this lack of robust or a variety of link data, ITUR P.530-15 provides a method of estimating attenuation at other frequencies. This is referred to as frequency scaling. Therefore, the measured attenuation results at 19.5 GHz can be scaled and used to obtain attenuation values at 15 GHz, 26 GHz and 38 GHz as follows [9]:

$$A_1(f_1)/A_2(f_2) = \left({}^{\Phi_1}/_{\Phi_2}\right)^{1-H(\Phi_1,\Phi_2,A_1)}$$

where, $\Phi(f) = {}^{f^2}/_{(1+10^{-4}f^2)}$ and $H(\Phi_1,\Phi_2,A_1) = 1.12 \times 10^{-3}\left({}^{\Phi_2}/_{\Phi_1}\right)^{0.6}(\Phi_2 A_1)^{0.55}$. $A_1(dB)$ and $A_2(dB)$ are the equivalent probable values of the excess rain attenuation at frequencies $f_1(GHz)$ and $f_2(GHz)$, respectively.

Fig. 8 illustrates prediction results for the four models at 15 GHz. A similar observation for all the models is made in terms of the prediction accuracies as in the case with Fig. 7, where the center frequency for the carrier is 19.5 GHz.

Fig. 9 illustrates prediction results for the four models at 26 GHz. It is observed that the models compare similar to the observations made in Figs. 7 and 8, where the center frequency for the carrier is 19.5 GHz and 15 GHz. Generally, it is shown that at 26 GHz, a provision of about 50 dB in the link budget is sufficient to achieve link availability of 99.99% during rainy season.

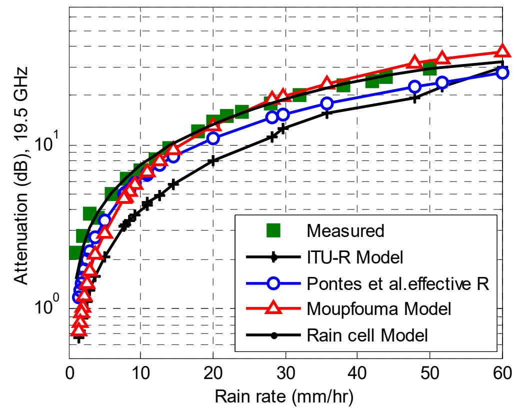

Fig. 7 Model comparison for prediction of rain attenuation at 19.5 GHz

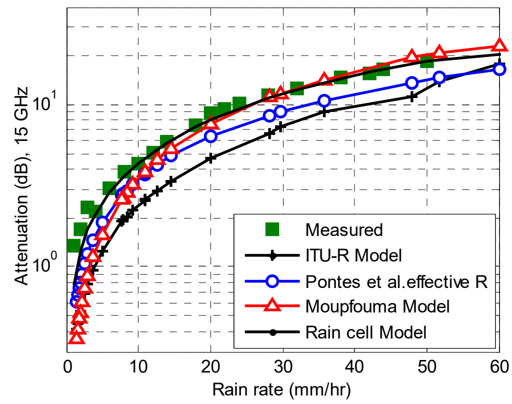

Fig. 8 Model comparison for prediction of rain attenuation at 15 GHz

Fig. 10 illustrates prediction results for the four models at 38 GHz. It is observed that the models compare similar to the observations made in Fig. 9. Generally, it is shown that at 38 GHz, a provision of about 70 dB in the link budget is sufficient to achieve link availability of 99.99% during rainy season.

Fig. 11 illustrates the percentage error deviations of the prediction results and the measured results for the two models with higher accuracy. It can be seen that as earlier discussed in the text, the cell ratio model offers a more accurate model for the measured attenuation data. Moreover, in order to perform more universal tests for the models, some link measurement results from a tropical zone in Malaysia are used. The results of the prediction tests are presented in Tables I-IV.

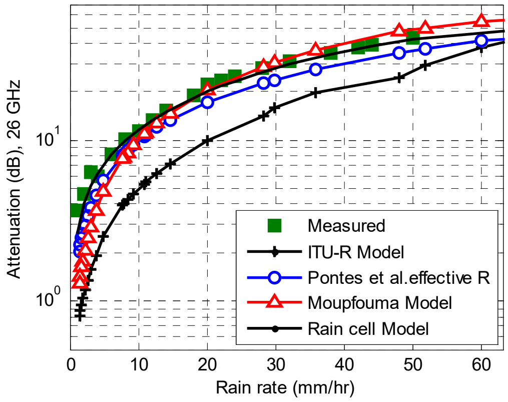

Fig. 9 Model comparison for prediction of rain attenuation at 26 GHz

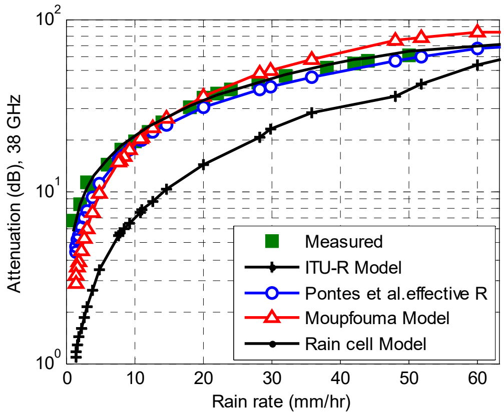

Fig. 10 Model comparison for prediction of rain attenuation at 38 GHz

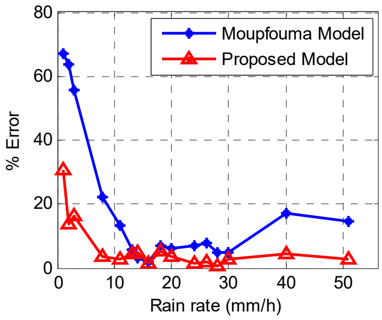

Fig. 11 Prediction errors for Moupfouma and proposed cell ratio model at 19.5 GHz

D. Attenuation Measurements in Malaysia [28]

The site in Malaysia exploited a carrier with center frequency of F = 15 GHz, where $R_{0.01}$ = 120 mm/h, and the link length was L = 5.83 km. The data used for specific attenuation are enumerated in ITU-R P 837-5.

Tables I and II show the path factor and attenuation prediction results for rain attenuation in Malaysia at a site whose rainfall break point is 120 mm/h. Attenuation measurements were performed at center frequency of 15 GHz. Tables III and IV show the prediction results for rain attenuation in Malaysia at a site whose rainfall break point is 48 mm/h. Performance tests are then carried out for the ITU-R P.530-15 model, Mello and Pontes model and the cell ratio model by computing the path factors and attenuation at 0.01% and 0.1% link unavailability. It is clear that the cell ratio model provides the best prediction results both for the path factor and the measured attenuation.

TABLE I
ATTENUATION MEASUREMENTS IN MALAYSIA AT 120 MM/H

F (GHz)	Path Factor, *r* (0.01%), 120 mm/h			
	Measured	ITU-R [9]	[29]	Ratio Model
15	**0.6430**	**0.6030**	**0.5115**	**0.6299**
26	**0.6025**	**0.5692**	**0.5359**	**0.5780**
38	**0.5191**	**0.5567**	**0.5603**	**0.5430**

TABLE II
ATTENUATION MEASUREMENTS IN MALAYSIA AT 120 MM/H

Attenuation (0.01%)			
Measured	ITU-R [9]	[29]	Ratio Model
34.5	**32.35**	**27.71**	**34.12**
73.07	**68.42**	**64.99**	**70.09**
92.55	**99.24**	**99.88**	**96.80**

TABLE III
ATTENUATION MEASUREMENTS IN MALAYSIA AT 48 MM/H

F (GHz)	Path factor, *r* (0.1%), 48 mm/h			
	Measured	ITU-R [9]	[29]	Ratio Model
15	**0.9083**	**0.6972**	**0.6590**	**0.8775**
26	**0.8605**	**0.6470**	**0.6767**	**0.8051**
38	**0.7801**	**0.6244**	**0.6940**	**0.7565**

TABLE IV
ATTENUATION MEASUREMENTS IN MALAYSIA AT 48 MM/H

Attenuation (0.1%)			
Measured	ITU-R [9]	[29]	Ratio Model
17.08	**12.60**	**12.39**	**16.49**
39.78	**28.58**	**31.28**	**37.22**
58.00	**43.50**	**51.60**	**56.25**

VI. CONCLUSION

The theoretical idea of rain cell ratio has been used to evaluate rain attenuation in terrestrial radio links through the concept of effective path length. Through this model, it is easy to explain why the effective path length may seem longer than the actual path. The observed maximum effective path length is 20 km, which is the maximum diameter of the rainy path in Durban. In addition, the path factor remains the same for radio links longer than 20 km. For further tests, rain cell shapes other than the Gaussian shape can be included.

The proposed approach for rain attenuation prediction is informative as it incorporates frequency scaling, the radius of the first Fresnel zone and rain cell size scaling.

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