pth Moment Exponential Synchronization of a Class of Chaotic Neural Networks with Mixed Delays

Zixin Liu, Shu Lü, Shouming Zhong, and Mao Ye

Abstract—This paper studies the pth moment exponential synchronization of a class of stochastic neural networks with mixed delays. Based on Lyapunov stability theory, by establishing a new integro-differential inequality with mixed delays, several sufficient conditions have been derived to ensure the pth moment exponential stability for the error system. The criteria extend and improve some earlier results. One numerical example is presented to illustrate the validity of the main results.

Keywords—pth Moment Exponential synchronization; Stochastic; Neural networks; Mixed time delays.

I. INTRODUCTION

SINCE the seminal works of Pecora and Carroll [1], [2], chaos synchronization has been intensively researched because of its potential applications in various fields such as secure communication, biological systems, information science, and so on. On the other hand, delayed neural networks, as a kind of special complex dynamical systems, have also been found to exhibit unpredictable behaviors such as periodic oscillations, bifurcation and attractors. The study on chaos synchronization of delayed neural networks have also been proposed (see [3]-[13]).

The idea of synchronization is to construct a response system and feedback controller for a given chaotic system such that the error system can be stable in the trivial solution. As a generalized form of mean square exponential stability, pth moment exponential stability has been received considerable attention in recent years. In Ref [14], the authors investigated the exponential stability in pth mean of solutions, and of convergent Euler-type solutions, of stochastic delay differential equations with constant time delay. In Ref [15], Luo improved the corresponding results. By establishing an L-operator inequality and using the property of M-cone, Yang and Xu [16] derived some pth moment exponentially stable criteria for a class of general impulsive stochastic differential equations with constant delay. Based on the results of Mao [17], [18], Randjelović, and Janković [19] obtained some pth moment exponential stability criteria of neutral stochastic functional differential equations. By the method of variation parameter and inequality technique, Sun and Cao [20] generalized the results obtained in [21] from mean square

exponential stability to pth moment exponential stability for a class of stochastic recurrent neural networks with time-varying delays. For discarding the strict constraint of time delay, Huang and He [22] established an improved criteria via the technique of Halanay-type inequality. On the other hand, the study on p-Moment stability of stochastic differential equations with Markovian jumps are also received attention in recent years (see [23]-[25]).

However, to the best of our knowledge, few authors have considered the problem of pth moment exponential synchronization of stochastic recurrent neural networks (RNNs) with mixed delays. Motivated by the above discussions, the main aim of this paper is to study pth moment exponential synchronization of a class of stochastic RNNs with discrete and distributed time delays. Based on Lyapunov stable theorem, by establishing a new integro-differential inequality, and using stochastic analysis technique, some sufficient conditions are derived to guarantee pth moment exponential synchronization of the given derive-response systems. These results obtained in our paper generalize and improve some existing results, which will be shown by one simulation example later.

The rest of the paper is arranged as follows. In Section 2, related dynamical systems with discrete and distribute time delays will be presented, then some necessary notations, assumptions, lemma and definition will be given. The pth moment exponential synchronization conditions will be given in Section 3. One simulation example will be provided in Section 4 to demonstrate the validity of our results. Conclusions are drawn in section 5.

II. PRELIMINARIES

Notations. The notations are used in our paper except where otherwise specified. $E(\cdot)$ stands for the mathematical expectation operator; $|\cdot|$ denotes the Euclidean norm; $\|\cdot\|$ denotes a vector or a matrix norm; The notation $\|\cdot\|^p$ is used to denote a vector norm defined by $\|\cdot\|^p = \sum_{i=1}^{n}|x_i|^p$; $\|\cdot\|_\Delta^p = \sup_{-\infty \leq t \leq 0}|\cdot|^p$, R, R^+, R^n are real, positive real and n-dimension real number sets respectively, I is identity matrix, L denotes the well-known L-operator given by the Itô's formula.

In this paper, we consider the following chaotic neural networks

$$
\begin{cases}
dx_i(t) = [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t - \tau(t))) \\
\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_{ij}(t - s) f_j(x_j(s)) ds] dt, \ t > 0 \\
x_i(t) = \phi_i(t), i, j = 1, 2, \cdots, n \ t \leq 0.
\end{cases}
$$

$$(1)$$

Z. Liu, S. Lü, and S. Zhong are with the School of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu 610054, People's Republic of China e-mail: xinxin905@163.com. And Z. Liu is also with School of Mathematics and Statistics, Guizhou College of Finance and Economics Guiyang 550004, People's Republic of China

M. Ye is with School of Applied Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 610054, People's Republic of China.

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state vector associated with the neurons; $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and the external stochastic perturbations; a_{ij}, b_{ij} and d_{ij} represent the connection weight and the delayed connection weight, respectively; f_i is activation function, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$, $\tau(t) > 0$ is transmission delay, which is bounded and the up-bound is τ, $k_{ij}(s) \geq 0$ is the kernel function, $\phi_i(t)$ is continuous initial function.

In order to synchronize system (1) via feedback controller, we introduce the respond system from the directional nonlinear coupling approach as follows

$$
\begin{cases}
dy_i(t) = [-c_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(y_j(t - \tau(t))) \\
\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_{ij}(t-s) f_j(y_j(s)) ds] dt \\
\quad + \sum_{j=1}^{n} \sigma_{ij}(t, e_j(t), e_j(t - \tau(t))) d\omega_j(t), \; t > 0, \\
y_i(t) = \varphi_i(t), i, j = 1, 2, \cdots, n,, \; t \leq 0.
\end{cases}
$$
(2)

where $\omega_j(t)$ is a standard Brown motion defined on a complete probability space (Ω, F, P) with a natural filtration $F_{t \geq 0}$(i.e., $F_t = \sigma\{\omega(x(s)) : -\infty \leq s \leq t\}$), and σ_{ij} is the diffusion coefficient, $u_i(t) = \sum_{i=1}^{n} m_{ij}(f_j(y_j(t)) - f_j(x_j(t))) + \sum_{i=1}^{n} n_{ij}(f_j(y_j(t - \tau(t))) - f_j(x_j(t - \tau(t))))$. , $e_j(t) = y_j(t) - x_j(t)$, $\varphi_i(t)$ is continuous initial function.

From Eq (1) and (2), we can get the following error system

$$
\begin{cases}
de_i(t) = [-c_i e_i(t) + \sum_{j=1}^{n} (a_{ij} + m_{ij}) g_j(e_j(t)) \\
\quad + \sum_{j=1}^{n} (b_{ij} + n_{ij}) g_j(e_j(t - \tau(t))) \\
\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_{ij}(t-s) g_j(e_j(s)) ds] dt \\
\quad + \sum_{j=1}^{n} \sigma_{ij}(t, e_j(t), e_j(t - \tau(t))) d\omega_j(t), t > 0, \\
y_i(t) = \varphi_i(t), i, j = 1, 2, \cdots, n,, \; t \leq 0.
\end{cases}
$$
(3)

where $g_j(e_j(t)) = f_j(y_j(t)) - f_j(x_j(t))$, $g_j(e_j(t - \tau(t))) = f_j(y_j(t - \tau(t))) - f_j(x_j(t - \tau(t)))$, $\psi_i(t) = \varphi_i(t) - \phi_i(t)$.

Throughout this paper, the following standard hypothesis are needed

(A_1) $\quad |f_i(u) - f_i(v)| \leq l_i |u - v|, l_i > 0, \forall u, v \in R.$

(A_2) $\quad \sum_{j=1}^{n} \sigma_{ij}^2(t, e_j(t), e_j(t - \tau(t))) \leq \sum_{j=1}^{n} \mu_j e_j^2(t) + \sum_{j=1}^{n} \nu_j e_j^2(t - \tau), \mu_j, \nu_j \in R^+.$

(A_3) $\quad \int_0^\infty e^{\varepsilon s} k_{ij}(s) ds < \infty, \int_0^\infty k_{ij}(s) ds = 1$ where $\varepsilon > 0$ is a constant scalar.

Remark 1. In previous publications, the variable time delay $\tau(t)$ is always required to be not only positive and bounded but also differentiable and derivative bounded. In this study, we only require it to be positive and bounded. It can be derivative unbounded, and even can be non-differentiable.

For further discussion, we introduce the following definition and lemmas.

Definition 1. System (1) and (2) are said to be pth moment exponentially synchronized if for a suitably designed feedback controller, there exist a pair of positive constants λ and α such that

$$
E\|e(t)\|^p \leq \alpha E\|\psi\|_\Delta^p e^{-\lambda(t - t_0)}, t \geq 0,
$$

hold for $\psi \in L_{F_{t_0}}^p([-\infty, 0], R^n)$, where $e(t) = (e_1(t), e_2(t), \cdots, e_n(t))^T$, $\psi(t) = (\psi_1(t), \psi_2(t), \cdots, \psi_n(t))^T$. Especially when $p = 2$ they are called to be exponentially synchronized in mean square.

Lemma 1.(Mao ([26])) Let $p \geq 2$ and $a > 0, b > 0$, then

$$
a^{p-1} b \leq \frac{(p-1)a^p}{p} + \frac{b^p}{p},
$$

and

$$
a^{p-2} b^2 \leq \frac{(p-2)a^p}{p} + \frac{2b^p}{p}.
$$

III. MAIN RESULTS

In this section, firstly, we will generalize a integro-differential inequality established by Xu and Wei (see [27]) from distribute delay to mixed delays, then some sufficient conditions will be derived to ensure the pth moment exponential stability for the error system.

Theorem 1 Let $P = (p_{ij})_{n \times n}$ and $p_{ij} \geq 0$ for $i \neq j$, $H^{(r)} = (h_{ij}^{(r)})_{n \times n}, r = 1, 2, \cdots, n$, $h_{ij}^{(r)} \geq 0$, $\tau_{(r)}(t) \geq 0$, $\tau' = \max_r \tau_{(r)}(t)$ is a bounded constant, $K(t) = (k_{ij}(t))_{n \times n}$ are piecewise continuous and satisfy

$$
(A_3') \qquad \int_0^\infty e^{\varepsilon s} k_{ij}(s) ds < \infty.
$$

where $\varepsilon > 0$ is a constant scalar. Denote $K = (k_{ij})_{n \times n} = (\int_0^\infty k_{ij}(s) ds)_{n \times n}$ and let $D = -(P + K + \sum_{r=1}^{n} H^{(r)})$ be a nonsingular M-matrix, $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in C([0, +\infty], R^n)$ be a solution of the following integro-differential inequality with the initial condition $u(t), t < 0$

$$
D^+ u(t) \leq Pu(t) + \sum_{r=1}^{n} H^{(r)} u(t - \tau_{(r)}(t)) + \int_0^\infty K(s) u(t-s) ds.
$$
(4)

Then

$$
u(t) \leq z e^{-\delta t}, t \geq 0,
$$
(5)

if the initial conditions satisfy

$$
u(t) \leq z e^{-\delta t}, t \leq 0,
$$

where $z = (z_1, z_2, \cdots, z_n)^T \in \Omega_M(D) = \{z \in R^n | Dz > 0, z > 0\}$, and the constant δ satisfies $0 < \delta \leq \varepsilon$ which is determined by the following inequality

$$
(\delta I + P + \sum_{r=1}^{n} H^{(r)} e^{\delta \tau'} + \int_0^\infty K(s) e^{\delta s} ds) z < 0. \quad (6)
$$

Proof. Since matrix D is a nonsingular M-matrix, in views of the property of M-matrix, there exists a vector $z \in \Omega_M(D)$ such that

$$(P + K + \sum_{r=1}^{n} H^{(r)})z < 0.$$

By using continuity and condition (A_3'), we see that inequality (6) has at least one positive solution $\delta \leq \varepsilon$.

In what follows, we first prove that for any given positive scalar $\xi > 0$ we have

$$u_i(t) < (1+\xi)z_i e^{-\delta t} = y_i(t), t \geq 0, i = 1, 2, \cdots, n, \quad (7)$$

if $u(s) \leq z e^{-\delta s}, -\infty < s \leq 0$.

If (7) not true, then there must exist $t^* > 0$ and some integer m such that

$$u_m(t^*) = y_m(t^*), \qquad D^+ u_m(t^*) \geq y_m'(t^*), \quad (8)$$
$$u_i(t) \leq y_i(t), -\infty < t \leq t^*, i = 1, 2, \cdots, n. \quad (9)$$

From (4), we can get

$$
\begin{aligned}
D^+ u_m(t^*) &\leq \sum_{j=1}^{n}[p_{mj}u_j(t^*) + \sum_{r=1}^{n} h_{mj}^{(r)} u_j(t^* - \tau_{(r)}(t)) \\
&\quad + \int_0^\infty k_{mj}(s) u_j(t^* - s)ds] \\
&\leq \sum_{j=1}^{n}[p_{mj}(1+\xi)z_j e^{-\delta t^*} \\
&\quad + \sum_{r=1}^{n} h_{mj}^{(r)}(1+\xi)z_j e^{-\delta(t^* - \tau_{(r)}(t))} \\
&\quad + \int_0^\infty k_{mj}(s)(1+\xi)z_j e^{-\delta(t^* - s)}ds] \\
&= \sum_{j=1}^{n}[p_{mj} + \sum_{r=1}^{n} h_{mj}^{(r)} e^{\delta \tau_{(r)}(t)} \\
&\quad + \int_0^\infty k_{mj}(s)e^{\delta s}ds](1+\xi)z_j e^{-\delta t^*} \\
&\leq \sum_{j=1}^{n}[p_{mj} + \sum_{r=1}^{n} h_{mj}^{(r)} e^{\delta \tau'} \\
&\quad + \int_0^\infty k_{mj}(s)e^{\delta s}ds](1+\xi)z_j e^{-\delta t^*} \\
&\leq -\delta(1+\xi)z_m e^{-\delta t^*} = y_m'(t^*), \quad (10)
\end{aligned}
$$

which contradicts with inequality (8), namely inequality (7) holds for all $t \geq 0$. On the other hand, in views of the arbitrary of ξ, we can see that inequality (5) holds, which complete the proof.

Theorem 2. Under the assumptions $(A_1) - (A_3)$, system (1) and system (2) are pth moment exponentially synchronized $(p \geq 2)$, if the following conditions are satisfied

(1) $S' = -(P' + H' + K')$ is a nonsingular M-matrix, where $P' = (p_{ij}')_{n \times n}$, $H' = (h_{ij}')_{n \times n}$, $K' = (k_{ij}')_{n \times n}$,
$p_{ij}' = |(a_{ij} + m_{ij})l_j| + \mu_j(p-1), i \neq j$,
$p_{ii}' = |(a_{ii} + m_{ii})l_i| + \mu_i(p-1) - c_i p + \sum_{j=1}^{n}(p-1)|(a_{ij} + m_{ij})l_j| + \sum_{j=1}^{n}(p-1)|(b_{ij} + n_{ij})l_j| + \sum_{j=1}^{n}(p-1)|d_{ij}| + \sum_{j=1}^{n}\mu_j \frac{(p-1)(p-2)}{2} + \sum_{j=1}^{n}\nu_j \frac{(p-1)(p-2)}{2}$,
$h_{ij}' = |(b_{ij} + m_{ij})l_j| + \nu_j(p-1), k_{ij}' = |d_{ij}||l_j|^p$.

(2) Initial conditions satisfy

$$E|e_i(t)|^p \leq z_i e^{-\delta t}, i = 1, 2, \cdots, n, t \in (-\infty, 0],$$

where $z = (z_1, z_2, \cdots, z_n)^T \in \Omega_M(S')$ and the positive scalar δ satisfies the following inequality
$[\delta I + P' + H'e^{\delta \tau} + \int_0^\infty K'(s)e^{\delta s}ds]z < 0$,
$K'(s) = (|d_{ij}||l_j|^p k_{ij}(s))_{n \times n}$.

Proof. Constructing Lyapunov functional for error system (3) as $V(e(t)) = (V_1(e_1(t)), V_2(e_2(t)), \cdots, V_n(e_n(t)))^T$, where $V_i(e_i(t)) = |e_i(t)|^p$.

By Itô's formula and lemma 1, we have

$$
\begin{aligned}
LV_i(t, e_i(t)) &= p|e_i(t)|^{p-1} sgn\{e_i(t)\}[-c_i e_i(t) \\
&\quad + \sum_{j=1}^{n}(a_{ij} + m_{ij})g_j(e_j(t)) \\
&\quad + \sum_{j=1}^{n}(b_{ij} + n_{ij})g_j(e_j(t - \tau(t))) \\
&\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_{ij}(t-s)g_j(e_j(s))ds] \\
&\quad + \frac{1}{2}p(p-1)|e_i(t)|^{p-2} \sum_{j=1}^{n} \sigma_{ij}^2 sgn\{e_i(t)\} \\
&= -c_i p|e_i(t)|^p + \sum_{j=1}^{n} p|e_i(t)|^{p-1} sgn\{e_i(t)\}(a_{ij} + m_{ij})g_j(e_j(t)) \\
&\quad + \sum_{j=1}^{n} p|e_i(t)|^{p-1} sgn\{e_i(t)\}(b_{ij} + n_{ij})g_j(e_j(t - \tau(t))) \\
&\quad + \sum_{j=1}^{n} p|e_i(t)|^{p-1} sgn\{e_i(t)\}d_{ij} \int_{-\infty}^{t} k_{ij}(t-s)g_j(e_j(s))ds \\
&\quad + \frac{1}{2}p(p-1)|e_i(t)|^{p-2} \sum_{j=1}^{n} \sigma_{ij}^2 sgn\{e_i(t)\} \\
&\leq -c_i p|e_i(t)|^p + \sum_{j=1}^{n} p|(a_{ij} + m_{ij})l_j||e_i(t)|^{p-1}|e_j(t)| \\
&\quad + \sum_{j=1}^{n} p|(b_{ij} + n_{ij})l_j||e_i(t)|^{p-1}|e_j(t - \tau(t))| \\
&\quad + \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)p|e_i(t)|^{p-1}|g_j(e_j(s))|ds \\
&\quad + \frac{1}{2}p(p-1)|e_i(t)|^{p-2} \sum_{j=1}^{n}(\mu_j e_j^2(t) + \nu_j e_j^2(t - \tau(t))) \\
&= -c_i p|e_i(t)|^p + \sum_{j=1}^{n} p|(a_{ij} + m_{ij})l_j||e_i(t)|^{p-1}|e_j(t)| \\
&\quad + \sum_{j=1}^{n} p|(b_{ij} + n_{ij})l_j||e_i(t)|^{p-1}|e_j(t - \tau(t))| \\
&\quad + \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)p|e_i(t)|^{p-1}|g_j(e_j(s))|ds \\
&\quad + \frac{p(p-1)}{2} \sum_{j=1}^{n} \mu_j|e_i(t)|^{p-2}e_j^2(t) \\
&\quad + \frac{p(p-1)}{2} \sum_{j=1}^{n} \nu_j|e_i(t)|^{p-2}e_j^2(t - \tau(t))
\end{aligned}
$$

$$\leq -c_i p |e_i(t)|^p + \sum_{j=1}^{n} |(a_{ij} + m_{ij})l_j|[(p-1)|e_i(t)|^p$$

$$+ |e_j(t)|^p] + \sum_{j=1}^{n} |(b_{ij} + n_{ij})l_j|[(p-1)|e_i(t)|^p$$

$$+ |e_j(t - \tau(t))|^p] + \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)[(p-1)|e_i(t)|^p ds$$

$$+ |g_j(e_j(s))|^p] + \frac{(p-1)}{2} \sum_{j=1}^{n} \mu_j[(p-2)|e_i(t)|^p$$

$$+ 2|e_j(t)|^p] + \frac{(p-1)}{2} \sum_{j=1}^{n} \nu_j[(p-2)|e_i(t)|^p + 2|e_j(t-\tau(t))|^p]$$

$$= -c_i p |e_i(t)|^p + \sum_{j=1}^{n}(p-1)|(a_{ij}+m_{ij})l_j||e_i(t)|^p$$

$$+ \sum_{j=1}^{n} |(a_{ij}+m_{ij})l_j||e_j(t)|^p + \sum_{j=1}^{n}(p-1)|(b_{ij}+n_{ij})l_j||e_i(t)|^p$$

$$+ \sum_{j=1}^{n} |(b_{ij}+n_{ij})l_j||e_j(t-\tau(t))|^p$$

$$+ \sum_{j=1}^{n}(p-1)|d_{ij}||e_i(t)|^p \int_{-\infty}^{t} k_{ij}(t-s)ds$$

$$+ \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)|g_j(e_j(s))|^p ds$$

$$+ \sum_{j=1}^{n} \mu_j \frac{(p-1)(p-2)}{2} |e_i(t)|^p + \sum_{j=1}^{n} \mu_j(p-1)|e_j(t)|^p$$

$$+ \sum_{j=1}^{n} \nu_j \frac{(p-1)(p-2)}{2} |e_i(t)|^p + \sum_{j=1}^{n} \nu_j(p-1)|e_j(t-\tau(t))|^p\}$$

$$= [-c_i p + \sum_{j=1}^{n}(p-1)|(a_{ij}+m_{ij})l_j| + \sum_{j=1}^{n}(p-1)|(b_{ij}+n_{ij})l_j|$$

$$+ \sum_{j=1}^{n}(p-1)|d_{ij}| + \sum_{j=1}^{n} \mu_j \frac{(p-1)(p-2)}{2}$$

$$+ \sum_{j=1}^{n} \nu_j \frac{(p-1)(p-2)}{2}] \cdot |e_i(t)|^p + \sum_{j=1}^{n}[|(a_{ij}+m_{ij})l_j|$$

$$+ \mu_j(p-1)] \cdot |e_j(t)|^p + \sum_{j=1}^{n}[|(b_{ij}+n_{ij})l_j|$$

$$+ \nu_j(p-1)] \cdot |e_j(t-\tau(t))|^p$$

$$+ \sum_{j=1}^{n} |d_{ij}||l_j|^p \int_{-\infty}^{t} k_{ij}(t-s)|(e_j(s))|^p ds. \tag{11}$$

That is

$$LV(e(t)) \leq P'V(e(t)) + H'V(e(t-\tau(t))) + \int_{0}^{\infty} K'(s)V(e(t-s))ds. \tag{12}$$

On the other hand, by Itô's formula, for all $t > 0$, we have

$$dV(e(t)) = LV(e(t))dt + \frac{\partial V(e(t))}{\partial e(t)} \sigma(t, e(t), e(t-\tau))d\omega(t). \tag{13}$$

Then for $t > 0$, integrate both side of equation (13) from t to $t + \Delta t$, we can obtain

$$V(e(t+\Delta t)) = V(e(t)) + \int_{t}^{t+\Delta t} LV(e(s))ds$$

$$+ \int_{t}^{t+\Delta t} \frac{\partial V(e(s))}{\partial e(s)} \sigma(s, e(s), e(s-\tau))d\omega(s). \tag{14}$$

Taking mathematical expectation of the both side of equation (14), we have

$$EV(e(t+\Delta t)) = EV(e(t)) + \int_{t}^{t+\Delta t} ELV(e(s))ds. \tag{15}$$

From inequality (12), we get

$$EV(e(t+\Delta t)) - EV(e(t)) = \int_{t}^{t+\Delta t} ELV(e(s))ds$$

$$\leq \int_{t}^{t+\Delta t} [P'EV(e(s)) + H'EV(e(s-\tau(s)))$$

$$+ \int_{0}^{+\infty} K'(\xi)EV(e(s-\xi))d\xi]ds,$$

thus $D^+ EV(e(t)) \leq P'EV(e(t)) + H'EV(e(t-\tau(t))) + \int_{0}^{+\infty} K'(s)EV(e(t-s))ds$. By Theorem 1, we get

$$EV(e(t)) \leq ze^{-\delta t} \leq EV(e(0))e^{-\delta t}, t > 0,$$

which implies that

$$EV_i(e_i(t)) \leq z_i e^{-\delta t} \leq EV_i(e_i(0))e^{-\delta t}, i = 1, 2, \cdots, n, t > 0.$$

Namely,

$$E|e_i(t)|^p \leq z_i e^{-\delta t} \leq E|e_i(0)|^p e^{-\delta t}, i = 1, 2, \cdots, n, t > 0.$$

Thus we have

$$E\|e(t)\|^p \leq E\|\psi(0)\|_\Delta^p e^{-\delta t},$$

which complete the proof.

Corollary 1. Under the assumptions $(A_1) - (A_3)$, system (1) and system (2) are mean square exponentially synchronized, if the following conditions are satisfied

(1) $S^{(1)} = -(P^{(1)} + H^{(1)} + K^{(1)})$ is a nonsingular M-matrix, where $P^{(1)} = (p_{ij}^{(1)})_{n \times n}$, $H^{(1)} = (h_{ij}^{(1)})_{n \times n}$, $K^{(1)} = (k_{ij}^{(1)})_{n \times n}$,
$p_{ij}^{(1)} = |(a_{ij} + m_{ij})l_j| + \mu_j, i \neq j$,
$p_{ii}^{(1)} = |(a_{ii} + m_{ii})l_i| + \mu_i - 2c_i + \sum_{j=1}^{n}|(a_{ij} + m_{ij})l_j| + \sum_{j=1}^{n}|(b_{ij} + n_{ij})l_j| + \sum_{j=1}^{n}|d_{ij}|$,
$h_{ij}^{(1)} = |(b_{ij} + m_{ij})l_j| + \nu_j, k_{ij}^{(1)} = |d_{ij}||l_j|^2$.

(2) Initial conditions satisfy

$$E|e_i(t)|^2 \leq z_i e^{-\delta t}, i = 1, 2, \cdots, n, t \in (-\infty, 0]$$

where $z = (z_1, z_2, \cdots, z_n)^T \in \Omega_M(S^{(1)})$ and the positive scalar δ satisfies the following inequality
$[\delta I + P^{(1)} + H^{(1)}e^{\delta\tau} + \int_{0}^{\infty} K^{(1)}(s)e^{\delta s}ds]z < 0$,
$K^{(1)}(s) = (|d_{ij}||l_j|^2 k_{ij}(s))_{n \times n}$.

Remark 2. When matrix $D = (d_{ij})_{n \times n} = 0$, similar to the proof of Theorem 2, we can get the following result.

Corollary 2. Under the assumptions $(A_1) - (A_2)$, system (1) and system (2) are pth moment exponentially synchronized, if the following conditions are satisfied

(1) $S^{(2)} = -(P^{(2)} + H^{(2)})$ is a nonsingular M-matrix, where $P^{(2)} = (p_{ij}^{(2)})_{n \times n}$, $H^{(2)} = (h_{ij}^{(2)})_{n \times n}$,
$p_{ij}^{(2)} = |(a_{ij} + m_{ij})l_j| + \mu_j(p-1), i \neq j$,
$p_{ii}^{(2)} = |(a_{ii} + m_{ii})l_i| + \mu_i(p-1) - c_i p + \sum_{j=1}^{n}(p-1)|(a_{ij} + m_{ij})l_j| + \sum_{j=1}^{n}(p-1)|(b_{ij} + n_{ij})l_j| + \sum_{j=1}^{n}\mu_j\frac{(p-1)(p-2)}{2} + \sum_{j=1}^{n}\nu_j\frac{(p-1)(p-2)}{2}$,
$h_{ij}^{(2)} = |(b_{ij} + m_{ij})l_j| + \nu_j(p-1)$.

(2) Initial conditions satisfy

$$E|e_i(t)|^p \le z_i e^{-\delta t}, i = 1, 2, \cdots, n, t \in (-\infty, 0]$$

where $z = (z_1, z_2, \cdots, z_n)^T \in \Omega_M(S^{(2)})$ and the positive scalar δ satisfies the following inequality

$$[\delta I + P^{(2)} + H^{(2)} e^{\delta \tau}]z < 0.$$

Similarly, in all kinds of special cases such as matrix $B = (b_{ij})_{n \times n} = 0$, $p = 2$, $\tau(t)$ is a constant, et al., respectively, we can get the corresponding results, which are omitted here.

IV. SIMULATION EXAMPLE

In this section, one numerical example will be presented to show the validity of the main results derived above.

Example 1. Consider a chaotic neural networks(see figure 1) as system (1), the response system as system (2), and the error system as system (3), where $C = (c_{ij})_{2 \times 2}$, $A = (a_{ij})_{2 \times 2}$, $B = (b_{ij})_{2 \times 2}$, $D = (d_{ij})_{2 \times 2}$, $\sigma(t, e(t), e(t-\tau)) = (\sigma_{ij})_{2 \times 2}$, $K(t) = (k_{ij}(t))_{2 \times 2}$, and

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{bmatrix}, B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.001 & 0 \\ 0 & -0.001 \end{bmatrix}, K(t) = \begin{bmatrix} e^{-t} & e^{-t} \\ e^{-t} & e^{-t} \end{bmatrix}$$

$$\sigma(t, e(t), e(t-\tau)) = \begin{bmatrix} \sqrt{0.1}e_1(t) & \sqrt{0.1}e_2(t-\tau) \\ \sqrt{0.1}e_1(t-\tau) & \sqrt{0.1}e_2(t) \end{bmatrix},$$

$$f(x(t)) = \tanh(x(t)), \quad \tau(t) = 1.$$

Then we have $l_1 = l_2 = 1, \mu_1 = \mu_2 = 0.1, \nu_1 = \nu_2 = 0.1$. Set $p = 3$, and if we take the controller matrix as

$$M = (m_{ij})_{2 \times 2} = \begin{bmatrix} -2.0 & 0.1 \\ 5.0 & -3.0 \end{bmatrix}, N = (n_{ij})_{2 \times 2} = \begin{bmatrix} 1.5 & 0.1 \\ 0.2 & 2.5 \end{bmatrix}$$

By direct calculation, we can get $S' = \begin{bmatrix} 1.599 & -1 \\ -1 & 1.599 \end{bmatrix}$ is a M-matrix, and if the given the initial condition satisfied (2) in theorem 2, then we can see that all of the conditions in theorem 2 are satisfied, so we can say that the given drive-response system are synchronized in 3th moment exponentially. Fig 2 shows that the trajectories of the error system converge to trivial solution exponentially. When $D = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$, we get $S' = \begin{bmatrix} 1,5 & -1 \\ -1 & 1.5 \end{bmatrix}$ is a M-matrix, the trajectory of the drive system become the form as shows in Fig 3, and the conditions in theorem 2 are also satisfied, thus we can say that the given drive-response system are synchronized in 3th moment exponentially. Fig 4 shows that the trajectories of the error system converge to trivial solution exponentially.

V. CONCLUSIONS

In this paper, we consider the pth moment exponential synchronization problems of a class of chaotic neural networks with mixed delays, by establishing a new integro-differential inequality, some sufficient synchronization conditions have been derived. These conditions extend and improve some earlier results cited therein. Simulation example shows that our results are valid.

ACKNOWLEDGMENT

REFERENCES

[1] T .L. Carroll, L .M. Pecora. *Synchronization in chaotic systems.* Phys. Rev. Lett, 1990, 64: 821-824.

[2] T .L. Carroll, L .M. Pecora. *Synchronizing chaotic circuits.* IEEE Trans. Circ. Syst, 1991, 38: 453-456.

[3] T .Li, S .M. Fei, K .J. Zhang. *Synchronization control of recurrent neural networks with distributed delays.* Physica A, 2008, 387: 982-996.

[4] T .Liu, G .M. Dimirovskib, J .Zhao. *Exponential synchronization of complex delayed dynamical networks with general topology.* Phys. Lett. A, 2008, 387: 643-652.

[5] X .Lou, B .Cui. *Synchronization of neural networks based on parameter identification and via output or state coupling*, Journal of Computational and Applied Mathematics (2007), doi:10.1016/j.cam.2007.11.015.

[6] S .Li. et al., *Adaptive exponential synchronization of delayed ...*, Chaos, Solitons, Fractals (2007), doi:10.1016/j.chaos.2007.08.047.

[7] T . Li, S .M. Fei, Q .Zhu, S .Song. *Exponential synchronization of chaotic neural networks with mixed delays*, Neurocomputing (2008), doi:10.1016/j.neucom.2007.12.029.

[8] J .Yan, J .Lin, M .Hung, T .Liao. *On the synchronization of neural networks containing time-varying delays and sector nonlinearity*, Phys. Lett. A, 2007, 361: 70-77.

[9] Y .Dai, Y .Z. Cai, X .M. Xu. *Synchronization and Exponential Estimates of Complex Networks with Mixed Time-varying Coupling Delays.* Int. Jour. Auto. Comput.,, 2007, 4(1): 100-106.

[10] Y .Q. Yang, J .D. Cao. *Exponential lag synchronization of a class of chaotic delayed neural networks with impulsive effects.* Physica A, 2007, 386: 492-502.

[11] W .Yu, J .D. Cao. *Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification.* Phys. Lett. A,, 2007, 375: 467-482

[12] Q .J. Zhang, J .A. Lu. *Chaos synchronization of a new chaotic system via nonlinear control.* Chaos, Solitons and Fractals, 2008, 37: 175-179

[13] H .G. Zhang, Y .H. Xie, Z .L. Wang. *Adaptive synchronization between two different chaotic neural networks with time delay.* IEEE Transaction on Neural Networks, 2007, 18: 1841-1845.

[14] C.T.H. Baker, E. Buckwar. *Exponential stability in pth mean of solutions, and of convergent Euler-type solutions, of stochastic delay differential equations.* J. Comput. Appl. Math, 2005, 184: 404-427.

[15] J .W. Luo. *A note on exponential stability in pth mean of solutions of stochastic delay differential equations.* J. Comput. Appl. Math, 2007, 198: 143-148.

[16] Z .G. Yang, D .Y. Xu, L .Xiang. *Exponential p-stability of impulsive stochastic differential equations with delays.* Phys. Lett. A, 2006, 359: 129-137.

[17] X .R. Mao. *Exponential stability in mean square of neutral stochastic differential functional equations.* Sys. Contr.Lett, 1995, 26: 245-251.

[18] X .R. Mao. *Razumikhin type theorems on exponential stability of neutral stochastic functional differential equations.* SIAM J. Math. Anal, 1997, 28(2): 389-401.

[19] J. Randjelović, S. Janković. *On the pth moment exponential stability criteria of neutral stochastic functional differential equations.* J. Math. Anal. Appl, 2007, 326: 266-280.

[20] Y .H. Sun, J .D. Cao. *pth moment exponential stability of stochastic recurrent neural networks with time-varying delays.* Nonlinear Anal: Real. world. Appl., 2007, 8: 1171-1185.

[21] L .Wan, J .Sun. *Mean square exponential stability of stochastic delayed Hopfield neural networks.* Phys. Lett. A, 2005, 343:306-318.

[22] C .X. Huang, et al. *pth moment stability analysis of stochastic recurrent neural networks with time-varying delays.* Inf.Sci., 2008, 178: 2194-2203.

[23] S .J. Wu, D .Han, X .Z. Meng. *p-Moment stability of stochastic differential equations with jumps.* J. Comput. Appl. Math, 2004, 152: 505-519.

[24] S .J. Wu, X .L. Guo, Y .Zhou. *p-moment stability of functional differential equations with random impulsive.* Comput. Math. Appl., 2006, 52: 1683-1694.

[25] H .J. Wu, J .T. Sun. *p-Moment stability of stochastic differential equations with impulsive jump and Markovian switching.* Automatica, 2006, 42: 1753-1759.

[26] X .R. Mao. *Stochastic Differential Equation and Application*, Horwood Publishing, Chichester, 1997.

[27] D .Y. Xu,Z .Wei, S. J. Long. *Global exponential stability of impulsive integro-differential equation.* Nonlinear Analysis, 2006, 64: 2805-2816.

ZiXin Liu was born in Sichuan Province, China, in 1977. He received the B.S. degree from China West Normal University, Sichuan, in 1999 and the M.S. degree from the University of Electronic Science and Technology of China (UESTC), Sichuan, in 2006, both in applied mathematics. He is currently pursuing the Ph.D. degree with UESTC. His research interests include neural networks, chaos synchronization and stochastic delay dynamic systems.
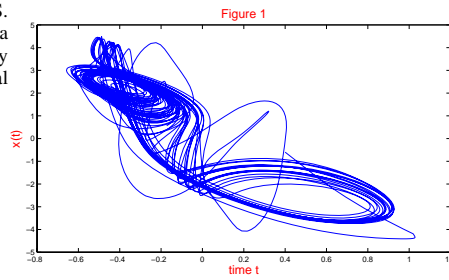
Fig. 1.   *State trajectories of drive system*

,

Shu Lü Shu L was born in 1963 in Jilin, China. She received B.S. degree in Mathematics Department of Northeast Normal University, Changchun, China, in 1984 and the M.S. degree from the School of mathematical statistics, China University of Technology. She is now a professor with UESTC. Her research interests include the theory and application of economics mathematics.
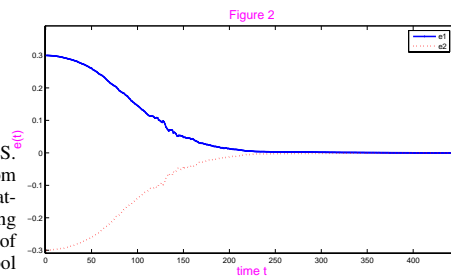
Shouming Zhong was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yat-sen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the In International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.
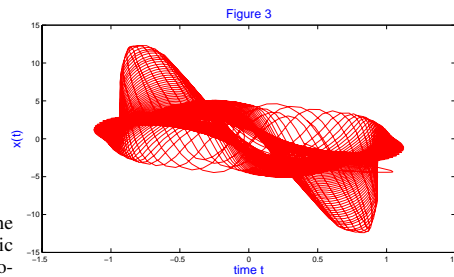
Fig. 2.   *State trajectories of error system*

Fig. 3.   *State trajectories of drive system*

Mao Ye was born in September1973,obtained his doctoral degree from the Chinese University of Hong Kong. He is serving the University of Electronic Science and Technology Computer Science and Engineering College as associate professor and has to his credit 30 papers,of which more than 15 have SCI, EI above 12. In 2006, he worked with the Ministry of Education to support the new century talents funded plans. Starting 2002.9 to date is teaching as associate professor at the electronic Department of Chengdu University of Science and Technology Computer Science and Engineering Institute. In 2005,he was engaged in blind source separation, data mining and mobile business functions of research University of Queensl and in Australia. During 2005.11-2006.8 he served in Information Science and Technology College of Engineering as visiting scholars and engaged in theoretical research on high dimensional data-mining. During 1999.8-2002.7 guided doctoral student of Chinese University of Hong Kong Institute on ordinary differential in theory as well as applied research. During 1998.4-1999.729 he served the Ministry of Information Industry Institute of electronic software group as senior programmer and engaged in information and data integration software research and preparation. During 1995.9-1998.4 he studied in University of Electronic Science and Technology Department of Applied Mathematics, Master's,in the neural network convergence of theoretical research. During 1991.9-1995.7 he studied in Sichuan Normal University Department of Mathematics as an undergraduate.
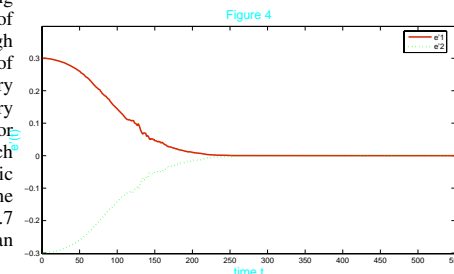
Fig. 4.   *State trajectories of error system*