Proton and Neutron Magnetic Moments Based On Bag Models

G. R. Boroun, R. Harami

Abstract—Using form factors of the proton and the neutron for different of Q^2 , bag radius of the proton and the neutron can be obtained based on bag models. Then using static bag radius, magnetic moments of the proton and the neutron can be obtained and compared with other results.

Keywords-MIT bag model, proton and neutron, magnetic moment.

I. INTRODUCTION

BAG model is one of the phenomological methods for studying of the light hadrons structures. Extensive studies on this model began from 1960s and continue to the present model revived mainly after QCD theory in 1970s that stated concepts such as asymptotic freedom in short distances and confinement barrier in long distances. MIT bag model [1]-[2] was proposed based on these concepts by Chodos et al. [3].

According to this model, a fix region of space which contains hadronian field is called bag. The pressure of the hadron components at the bag surface is constant and the vacuum around the bag causes external pressure of B on the bag surface. Here, B is the only parameter of the theory and any shape can be chosen for the bag. However the simplest shape is sphere. Based on MIT bag model the mass of the nucleon is presented according to this equation, as [4]:

$$M_N = \frac{4}{3}\pi \left(0.145\right)^4 R^3 - \frac{1.84}{R} + \frac{6.12}{R} - \frac{0.768}{R} \tag{1}$$

By replacing the static bag radius, the mass of nucleon is obtained .Also based on the model, magnetic moment (in units of the nuclear magneton) of the proton and the neutron can be defined as [5]:

$$\mu_0^p = 1.9R$$
 (2)

and

$$\mu_0^n = -1.284R \tag{3}$$

where R is the bag radius.

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Two key components of the model are center-of-mass corrections and running effective quark – gluon coupling constant. In this model, unlike in MIT bag model, zero point energy terms and the energy itself are omitted from the formula of the model [6] as we have:

II. THEORY

$$E_M = E_B + E_q + \Delta E \tag{4}$$

where E_B is the Energy related to the volume of the bag and defined according this equation, as:

$$E_B = \frac{4\pi B R^3}{3} \tag{5}$$

Also, E_q is the total energy of quarks, as:

$$E_q = \sum_i n_i \varepsilon_i \tag{6}$$

where \mathcal{E} follows the following equation

$$\tan(R\sqrt{\varepsilon^2 - m^2}) = \frac{R\sqrt{\varepsilon^2 - m^2}}{1 - mR - \varepsilon R}$$
(7)

So ΔE is the energy of the interaction among the quarks caused by exchange of the gluon, as:

$$\Delta E = E^{m} + E^{e} \tag{8}$$

In this relation, E^m and E^e are the color-magnetostatic and color-electrostatic pieces of the interaction energy.

Therefore, in the improved MIT bag model, the mass of the hadrons can be written as follows:

$$M = \left[E^2 - \eta \sum_i n_i p_i^2 \right]^{\frac{1}{2}}$$
(9)

where η and p_i are center of-mass corrections parameter and momentum of the i- th quark. Thus, magnetic moments of nucleon are defined as:

$$\mu = \frac{E}{M} \mu_0 \tag{10}$$

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where μ_0 is the same as magnetic moment in MIT bag model, and E, M defined by (4)-(9).

III. RESULT AND DISCUSSION

In MIT bag model, form factors of the proton and the neutron based on static spherical cavity approximation can be written, by these equations as [7], [8]:

$$G_{E}^{sph}(Q^{2}) = \int_{0}^{R} d^{3}r j_{0}(Qr) \Big[g^{2}(r) + f^{2}(r) \Big]$$
(11)

And

$$G_{E}^{sph}(Q^{2}) = 2m_{N} \int_{0}^{R} d^{3}r \frac{j_{1}(Qr)}{Q} [2g(r)f(r)]$$
(12)

where m_N is the mass of nucleon, and g(r) and f(r) are given as:

$$g(r) = Nj_0\left(\frac{\omega r}{R}\right)$$
, $f(r) = Nj_1\left(\frac{\omega r}{R}\right)$ (13)

In our calculations, in the lowest mode $\omega = 2.04$ and $N^2 = \frac{\omega}{8\pi R^3 j_0^2(\omega)(\omega - 1)}$. So with regard to corrections

from the Lorentz contraction [8], (11) and (12) are presented by these equations, as:

$$G_E(Q^2) = \left(\frac{m_N}{E}\right)^2 G_E^{sph}(Q^2 \frac{m_N^2}{E^2})$$
(14)

$$G_M\left(Q^2\right) = \left(\frac{m_N}{E}\right)^2 G_M^{sph}\left(Q^2 \frac{m_N^2}{E^2}\right) \tag{15}$$

In which E is energy on the nucleon shell in Berit frame and is defined as [9]:

$$E = \sqrt{m_N^2 + \frac{Q^2}{4}}$$
(16)

IV. CONCLUSIONS

Using the form factors data in [10], [11] and (14) and (15), bag radius for different Q^2 for proton and neutron can be calculated (see Tables I, II). In limit $Q^2 \rightarrow 0$ based on the fits of the graphs in Figs. 1, 2, static bag radius (R_0) can be obtained. So using (1), (9), (10), magnetic moments proton and neutron can be obtained as comparing our results with the experimental data and the results of others [12] show that this method can be have comparable results for the magnetic moments of the proton and the neutron (see Tables III, IV).

| TABLE I The Radius of the Bag Proton Is Shown Based On [10] dat | | | | | |
|--|---------|-----------------------|-----------|--|--|
| $Q^2 (GeV/c)^2$ | G_E^P | $\frac{G_M^P}{\mu_P}$ | $R_P(fm)$ | | |
| 0.65 | 0.2688 | 0.2713 | 0.9672 | | |
| 0.9 | 0.1941 | 0.1971 | 0.9228 | | |
| 2.2 | 0.0557 | 0.0627 | 0.8092 | | |
| 2.75 | 0.0382 | 0.0442 | 0.7834 | | |
| 4.25 | 0.0202 | 0.0210 | 0.7345 | | |

| TABLE II The Radius of the Bag Neutron Is Shown Based On [11] data | | | | |
|---|---------|-----------|--|--|
| $Q^2 (GeV/c)^2$ | G_M^n | $R_n(fm)$ | | |
| 1.75 | 0.1644 | 0.8943 | | |
| 2.5 | 0.10114 | 0.8404 | | |
| 3.25 | 0.6422 | 0.8162 | | |
| 4.0 | 0.0480 | 0.7896 | | |

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| Our results | EXP | [10] | [5] | [12] |

| | Our results | EXP | [10] | [5] | [12] |
|-----------|-------------|------|-------|-------|-------|
| $R_0(fm)$ | 1.045 | - | 1.011 | 1.005 | 1.084 |
| μ | 2.79 | 2.79 | - | 1.9 | 2.54 |
| | | | | | |

TABLE IV Our Results for the Neutron Compared with Those Obtained by Others

| | | 0.11111 | | | |
|-----------|-------------|---------|-------|-------|-------|
| | Our results | EXP | [11] | [5] | [12] |
| $R_0(fm)$ | 1.056 | - | 1.044 | - | 1.084 |
| μ | -1.87 | -1.91 | -1.34 | -1.27 | -1.65 |



Fig. 1 The bag radius proton vs of Q²



Fig. 2 The bag radius neutron vs of Q^2

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