

Probability-Based Damage Detection of Structures Using Model Updating with Enhanced Ideal Gas Molecular Movement Algorithm

M. R. Ghasemi, R. Ghiasi, H. Varacee

Abstract—Model updating method has received increasing attention in damage detection structures based on measured modal parameters. Therefore, a probability-based damage detection (PBDD) procedure based on a model updating procedure is presented in this paper, in which a one-stage model-based damage identification technique based on the dynamic features of a structure is investigated. The presented framework uses a finite element updating method with a Monte Carlo simulation that considers the uncertainty caused by measurement noise. Enhanced ideal gas molecular movement (EIGMM) is used as the main algorithm for model updating. Ideal gas molecular movement (IGMM) is a multi-agent algorithm based on the ideal gas molecular movement. Ideal gas molecules disperse rapidly in different directions and cover all the space inside. This is embedded in the high speed of molecules, collisions between them and with the surrounding barriers. In IGMM algorithm to accomplish the optimal solutions, the initial population of gas molecules is randomly generated and the governing equations related to the velocity of gas molecules and collisions between those are utilized. In this paper, an enhanced version of IGMM, which removes unchanged variables after specified iterations, is developed. The proposed method is implemented on two numerical examples in the field of structural damage detection. The results show that the proposed method can perform well and competitive in PBDD of structures.

Keywords—Enhanced ideal gas molecular movement, ideal gas molecular movement, model updating method, probability-based damage detection, uncertainty quantification.

I. INTRODUCTION

STRUCTURAL systems in civil engineering are subjected to deterioration and damage during their service life. Damage is characterized as a weakening of the structure which may cause undesirable displacements, stresses, strain or vibrations to the structure leading to sudden and disastrous results. Damage can severely affect the safety and functionality of the structure and identification of it at early stage can increase safety and extend its serviceability. Thus, identification of damage is one of the most important factors in maintaining the safety and integrity of structures [1].

The structural damages are usually detected by the modal parameters of the structure [2], because not only are modal parameters (modal frequencies and mode shapes) functions of the physical parameters (mass and stiffness) and the existence

of damage may lead to changes in the modal properties of the structure, but also modal parameters can be measured conveniently and accurately. In most applications of model updating on damage detection, the experimentally measured modal parameters are considered to be exact and deterministic [3]. In reality, however, there are always uncertainties in the measured modal parameters, which may lead to unreliable and false prediction of structural damage, and, as a result, it is necessary to consider the uncertainties in the damage detection of structures [4], [5]. In general, there are two sources for the uncertainties in measured modal parameters. First, uncertainties are introduced in modal parameters by the inherent variability or randomness in structural parameters (physical material properties, geometric parameters), which cannot be reduced or eliminated by the knowledge or techniques available [6].

Second, uncertainties in modal parameters arise from the measurement noise and modal identification techniques. These kinds of uncertainties in modal parameters can be hopefully reduced, not certainly eliminated though, even by more precise measurement instrumentation and more appropriate modal identification techniques. In the case of uncertain measured modal parameters, the probabilistic approaches are frequently used for incorporating structural uncertainties and measurement noise in the damage detection by generally describing the uncertainties as random variables characterized by mean values and standard deviations [7].

The objective of this paper is to study the influence of uncertainty on damage identification using a combination of frequency and mode shape as the input variables. To consider the uncertainties in the measurement data, an approach introduced by Wang et al. [4] is applied. Using this method, the uncertainties in the measured modal data are assumed as independent normally distributed random variables with zero means and particular covariance. The statistics (mean value and standard deviation) of the elemental stiffness parameter (ESP) will be calculated by the Monte Carlo simulation.

Recently, an efficient and simple continuous optimization algorithm, so-called IGMM, for optimization problems, is developed by the present authors [8], [9]. In this study, the performance of IGMM has been enhanced by removing unchanged variables after specified iterations. This feature makes the approach more versatile for a wider range of practical applications, while preserving the attractive characteristics of the basic IGMM.

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A numerical steel frame model and 31-bar planar truss are used to demonstrate the robustness of the developed procedure. Some parametric calculations are also performed to investigate the influences of using different noise levels and damage severities on the damage identification results. In this study, only random errors are considered. The systematic errors which may also exist, especially in the FE model is not considered in the present study. Therefore, the main contribution of this study is to introduce an effective approach based on modal updating method and EIGMM to implement a damage detection procedure on structure considering uncertainty.

The article is structured as follows. Section II presents the statement of damage detection problem for a truss structure. Probability based model updating is described in Section III. The fundamental basis of IGMM and proposed enhanced version will be described in Section IV and V. Section VI presents the main steps for the proposed damage detection method. In Section VII, the merits of the algorithms are assessed by solving the Damage Detection (DD) problems. Some concluding remarks are provided in Section VIII.

II. MULTIPLE DAMAGE LOCATION ASSURANCE CRITERION (MDLAC)

Structural DD techniques are generally classified into two main categories. They include the dynamic and static identification methods requiring the dynamic and static test data, respectively. Furthermore, the dynamic identification methods have shown their advantages in comparison with the static ones. Among the dynamic data, the modal analysis information of a structure such as the natural frequencies and mode shapes were widely used for DD [10]-[12]. Determination of the level of correlation between the measured and predicted natural frequencies or mode shapes can provide a simple tool for identifying the location and severity of structural damages. When the natural frequencies are employed to identify the damage, two parameter vectors may be determined. One parameter vector consists of the ratios of the first n_f natural frequency changes ΔF due to structural damage, i.e.:

$$\Delta F = \frac{F_h - F_d}{F_h} \quad (1)$$

where F_h and F_d denote the natural frequency vectors of the healthy and damaged structure, respectively. Another parameter vector can be similarly defined as:

$$\delta F(ESV) = \frac{F_h - F(ESV)}{F_h} \quad (2)$$

where $F(ESV)$ is a natural frequency vector that can be predicted from an analytic model and elemental stiffness vector (ESVs) $ESV^T = [E_1, \dots, E_i, \dots, E_n]$ which represents

a damage variable vector containing the elasticity modulus of structural elements $(E_i, i = 1, \dots, n)$ of all n structural elements.

Given the pair of parameter vectors, one can estimate the level of correlation in several ways. An efficient way is to evaluate a correlation index called the MDLAC, which is expressed in the following form [12]:

$$MDLAC(ESV) = \frac{|\Delta F^T \delta F(ESV)|^2}{(\Delta F^T \Delta F)(\delta F^T(ESV) \delta F(ESV))} \quad (3)$$

The MDLAC compares two frequency change vectors, one of which is obtained from the examined structure and the other from an analytical model of the structure. The MDLAC varies from a minimum value 0 to a maximum value 1. It will be maximal when the vector of analytical frequencies equates to the frequency vector of damaged structure, i.e.:

$$F(ESV) = F_d \quad (4)$$

III. MONTE CARLO SIMULATION FOR PBDD

Since that uncertainties (noises) inevitably exist in the measured vibration data, the updated ESV (E) is subjected to uncertainty as well. As mentioned before, the uncertainties in the measured modal data are assumed as independent normally distributed random variables with zero means and particular covariance. Accordingly, the eigenvalues and mode shapes can be expressed as [4]:

$$\lambda_i^E = \lambda_{i,0}^E (1 + X_{\lambda i}), \quad i = 1, 2, \dots, n_m \quad (5)$$

$$\phi_i^E = \phi_{i,0}^E (1 + X_{\phi i}), \quad i = 1, 2, \dots, n_m \quad (6)$$

where 0 represents the true values, $X_{\lambda i}$ and $X_{\phi i}$ indicate relative random noises in the measured frequencies and mode shapes, respectively. The mean value of vector X is zero and the standard deviation indicates the noise level.

The statistics (mean value and standard deviation) of E can then be calculated by the perturbation method [13] or Monte Carlo simulation. The second method can also give statistical samples of the updated ESVs, from which the statistical distribution can be obtained. Studies have demonstrated that the statistical distribution of the ESVs in the updated model is also normal [14], verified by the goodness-of-fit test [15]. Again, when the measured modal data in both undamaged and damaged states are available and the two-step model updating [14] is employed, the statistics of ESVs in both states (E^h and E^d) can be respectively computed.

IV. IDEAL GAS MOLECULAR MOVEMENT

The behavior of gas molecules in an isolated medium shows that they disperse rapidly in different directions and cover all the space inside. The essence of such manner lies on two factors; the high speed of ideal gas molecules and their collisions. Recently the conventional IGMM was introduced by the authors and its application in solving engineering problems was assessed then [8], [9]. The algorithm utilizes the governing equations for speed and collision of molecules in order to determine their new location. The speed of molecules thus is proportional to the temperature and inversely proportional to its mass. Besides they collide with each other with a certain probability, increasing gradually with their motions. Ideal gas molecules have fully elastically collisions and elastic collision governing equations can be used to determine the new position of gas molecules after collision [9]. The different steps of the IGMM algorithm can be summarized as follows [9]:

Step1. Generate the initial population of gas molecules with a uniform distribution from the allowable range of design variables.

Step2. Evaluate each molecule and assign them a mass according to its fitness using the following relation:

$$m_i = \frac{1}{fit(i)} / \sqrt{\sum (fit^2(i))} \quad (7)$$

where m_i shows the mass of the i -th molecule and $fit(i)$ reflects the fitness of the i -th molecule with regard to the objective function for the problem.

Step3. Pairing molecules without repetition. In this stage, based on the governing equations of the ideal gases assumed that there are no simultaneous molecular collisions.

Step4. Determine collision probability (CP) based on (8):

$$CP = 1 - \exp(-0.63 \times iter) \quad (8)$$

Step5. Generate a random number between 0 and 1 and compare it with CP to determine whether collision occurs or not. According to this phenomenon, the following steps will proceed to calculate the new velocity and position of each molecule.

Step5.1 In the collision phase, new post-collision velocities are obtained using (9) and (10). In using these equations, the molecule with a larger mass is assumed stationary and the lighter molecule moves according to the hypotheses about the elastic collision between gas molecules. The initial velocity of the moving molecule is obtained using relation $\Delta x = v \Delta t$ (for $\Delta t = 1$) by subtracting the positions of the two molecules.

$$(v_1^d)' = \frac{(m_1 - Em_2)}{m_1 + m_2} \times v_1^d \quad (9)$$

$$(v_2^d)' = \frac{(1+E)m_1}{m_1 + m_2} \times v_1^d \quad (10)$$

where, d indicates the dimension of the optimization problem. As stated, in the event of elastic collisions parameter E is equal to 1, but in (9) and (10) this parameter is defined as a variable to guarantee the convergence in the algorithm. Therefore, in the first few steps of the optimization process, this variable has a value near 1 but with an increase in the number of optimization cycles its value declines dynamically based on the following linear equation.

$$E = 1 - \left(\frac{iter}{\max it} \right) \quad (11)$$

where $iter$ and $\max it$ indicate current and the maximum iterations of optimization procedure, respectively. Having computed the new velocity of each molecule, its new position can be computed using (12) and (13):

$$(x_1^d)' = x_2^d + rand \times (v_1^d)' \quad (12)$$

$$(x_2^d)' = x_2^d + rand \times (v_2^d)' \quad (13)$$

where x_2^d shows the position of a stationary molecule before impact, and accordingly, $(x_1^d)'$ and $(x_2^d)'$ indicate the new positions after the impact, respectively. $rand$ represents a random normal distributed value in the range [0,1].

Step5.2 In a no collision phase, the new velocity of the i -th molecule is determined using (14).

$$v_i^d = 1.7 \sqrt{\frac{kT_i}{m_i}} \quad (14)$$

The Boltzmann constant value (k) in (14) is assumed opposite to the number of molecules in the optimization process. The velocity of each molecule accords with the mass and temperature of that molecule. Hence, in this phase it is necessary to calculate the new temperature of each molecule. To this end, a subtractive equation is defined as follows. The initial temperature is set to 1000 in the original IGMM.

$$T_i' = T_i - 1 / m_i \quad (15)$$

Finally, after determining the new velocity of each molecule using (14), the new position of each molecule is obtained and given as:

$$(x_i^d)' = x_2^d + rand \times (v_i^d)' \quad (16)$$

Step6. The convergence criterion will be checked and if the algorithm does not converge, steps 2 to 4 will be repeated.

V. ENHANCED IDEAL GAS MOLECULAR MOVEMENT

In this paper, an enhanced version of IGMM is proposed. In some optimization problems, the number of variables that must be considered is very large. For example, in an optimization based DD problem, damaged elements and damage extents are searched through an optimization process until the response of hypothesized damaged structure equals those of a real damaged structure. When a real structure is a large-scale structure, the number of elements (variables) will increase [16]. Hence, when the optimization method tries to minimize the objective function, it must handle a huge bunch of variables which decreases the convergence speed of the algorithm. Therefore, a method is presented in this paper to resolve this problem. In the DD problem, in first stage when the initial population is generated, each molecule has a velocity vector that represents its speed in an n -dimensional space. Each variable of this vector represents the elasticity modulus of the structural elements. In the proposed method, first, the number of variables in each stage of the IGMM algorithm is considered as the total number of elements. Then, all the intact elements are eliminated in each stage and the algorithm converges to the exact locations and severity of damages. Zero values for the variables signifies that the i -th element of the structure is intact and a non-zero value refers to the damaged element. If the variables with near zero values ($SRF_i \leq 0.05$) do not alter for 10 iterations, this variable will be eliminated.

As far as the objective function is concerned, it is defined here as an unconstrained optimization problem as:

$$\begin{aligned} \text{Find : } &ESV_i = \{E_1, E_2, E_3, \dots, E_n\} \\ \text{Minimize : } &F(ESV) = \|1 - MDLAC\|^2 \\ \text{Where : } &E_{\min} \leq E \leq E_{\max} \end{aligned} \quad (17)$$

where $F(ESV)$ is that minimization problem and E_{\min} and E_{\max} are the lower and upper bounds of the damage vector, respectively. The bounds are required to meet the physical behavior of the structure. By using an optimization algorithm and solving (17), the damage variables are determined.

VI. MAIN STEPS FOR PROPOSED DD METHOD

The main steps for the proposed DD method using the EIGMM algorithm are summarized as follows:

- Setting the initial number of design variables equal to the total number of elements.
- Employing the EIGMM to find the optimal solution.

- Finding i as $X_i = 0$ for all components of the damage vector and determining the total number of intact elements.
- Removing the intact elements from the damage vector and thus reducing the number of variables from the optimization problem.
- Performing EIGMM once again based on the new optimization size from d).
- Checking the convergence by computing $1 - MDLAC$ from (17). If two response vectors are almost indifferent, save the results and terminate the optimization process, otherwise, go to the step c).

VII. NUMERICAL RESULTS OF DD

In this study, two structures are selected as the numerical examples to reveal the robustness and the degree of accuracy of the proposed DD method. These structures are:

- 31-Bar planer truss, and
- Five-story and four-span frame.

The mass matrix is assumed to be constant and damage in the structure is simulated as a relative reduction in the elasticity modulus of an individual element. The stiffness reduction ratio (SRF) is defined as:

$$SRF_i = \frac{E - E_i}{E}, i = 1, \dots, n \quad (18)$$

where E is the original modulus of elasticity and E_i is the final modulus of elasticity of the i -th element. For the optimization process, the number of molecules for EIGMM was fixed to 50 for each run along a maximum of 200 iterations.

A. Thirty One-Bar Planar Truss

The 31-bar planar truss shown in Fig. 1 is modeled using the conventional finite element method without internal nodes leading to 25 degrees of freedom [12]. In this example, the first five vibrating modes are utilized for DD. The material density and elasticity modulus are 2770 kg/m and 70 GPa, respectively. Two different damage scenarios given in Table I are induced in the structure and the proposed method is tested for each case.

In this section, the influence of the noise in the accuracy of structural DD based on modal data is investigated. Generally, the uncertainties of the measured mode shapes are larger than those of the frequencies in the modal testing. In this study, the uncertain frequencies are considered as the normal distributed random variables in (5). The mean values of the relative random noises $X_{\lambda i}$ are zeros, and standard deviations ξ_{λ} indicate the noise level. The mean values and standard deviations of SRFs are calculated from 500 samples, based on the Monte Carlo simulation framework.

Figs. 2 and 3 show the mean value and standard deviation of SRFs for damage scenario 1, respectively. The noise level

is set as $\xi_{\lambda} = 1\%$. Figs. 4 and 5 show the statistics of SRFs for damage scenario 2.

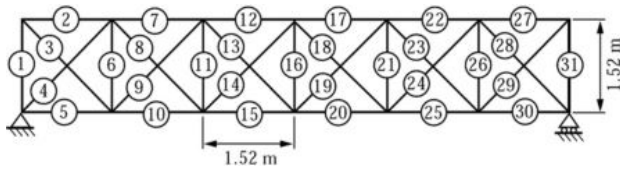


Fig. 1 The 31-bar planar truss

TABLE I
DIFFERENT DAMAGE SCENARIOS FOR PLANAR TRUSS

Case 1		Case 2	
Element Number	SRF	Element Number	SRF
11	0.25	1	0.30
25	0.15	2	0.20

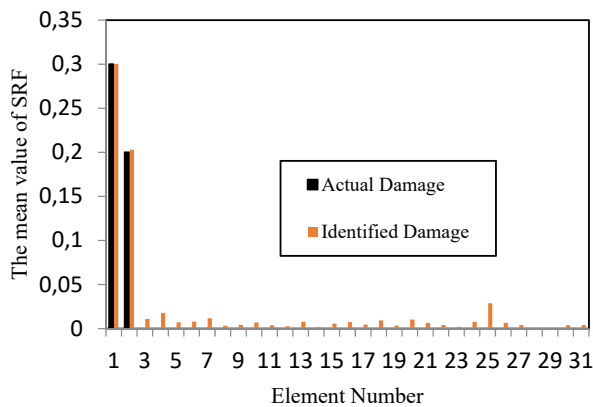


Fig. 2 Mean value of SRF for damage scenario 1 ($\xi_{\lambda} = 1\%$)

In both damage scenarios, the mean values of SRFs using the proposed method perform accurate results. Moreover, the standard deviations of SRFs using the MDLAC index are small, denoting that the damaged elements can be detected reliably. On the contrary, if the standard deviations of SRFs being significant it will show that the damage cannot be identified reliably.

For a larger noise level, that is $\xi_{\lambda} = 10\%$, the statistics of SRF for damage scenario 2 are drawn in Figs. 6 and 7. In comparison with the lower uncertainty level, the proposed method having larger errors, but can still acquire accurate mean values and small standard deviations of the SRFs. All these figures demonstrate that the one stage method for damage identification using EIGMM algorithm is robust to the measurement noise. Also, these figures represent that the proposed method is very efficient for multiple structural damages; even though the damage severity is low.

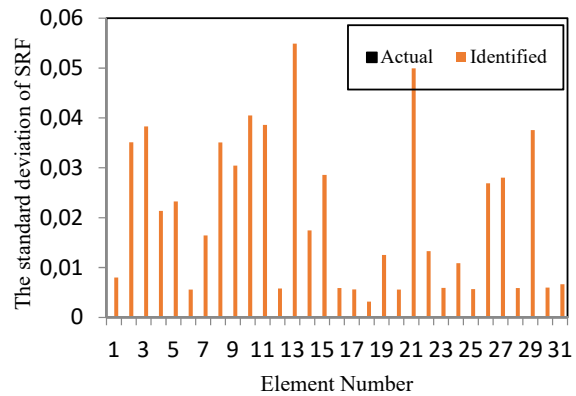


Fig. 3 Standard deviation of SRF for damage scenario 1 ($\xi_{\lambda} = 1\%$)

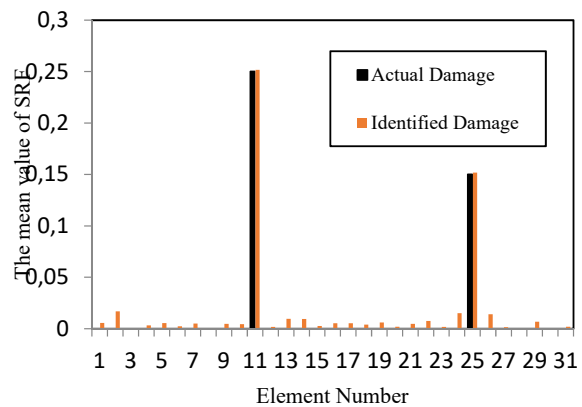


Fig. 4 Mean value of SRF for damage scenario 2 ($\xi_{\lambda} = 1\%$)

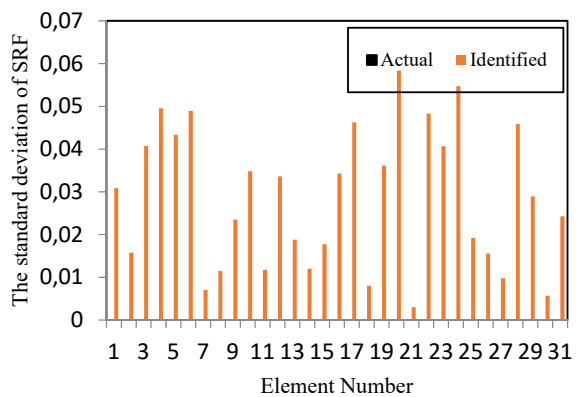
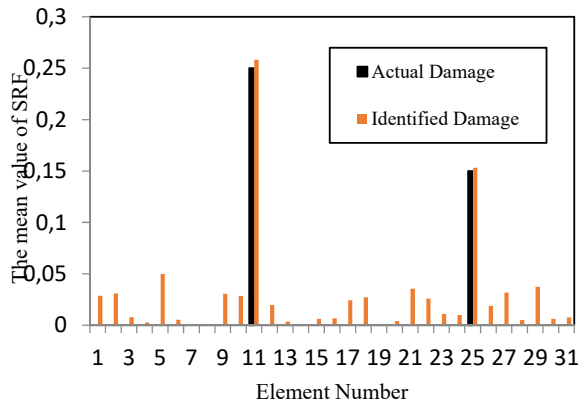
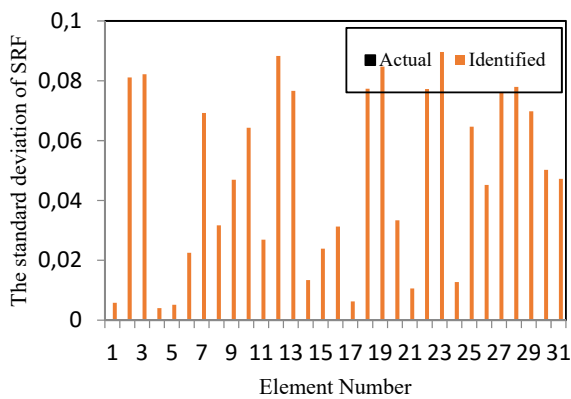


Fig. 5 Standard deviation of SRF for damage scenario 2 ($\xi_{\lambda} = 1\%$)

Fig. 6 Mean value of SRF for damage scenario 2 ($\xi_\lambda = 10\%$)Fig. 7 Standard deviation of SRF for damage scenario 2 ($\xi_\lambda = 10\%$)

B. Five-Story and Four-Span Frame

The second example is a five-story and four-span frame, as illustrated in Fig. 8 [17]. The sections used for the beams and columns are (W12×87) and (W14×145), respectively. The material density is 7780 kg/m³ and the modulus of elasticity is 210 GPa. Different damage scenarios are considered, as shown in Table II. Figs. 9 and 10 show the performance of the proposed method on this regard.

TABLE II
DIFFERENT DAMAGE SCENARIOS FOR PLANAR FRAME

Case 1		Case 2	
Element Number	SRF	Element Number	SRF
10	0.25	14	0.35
30	0.20	28	0.30
40	0.25	38	0.35

The results demonstrated the fact that EIGMM could effectively find the correct locations and severity of the damages.

For a larger noise level, that is $\xi_\lambda = 10\%$, the statistics of SRF for damage scenario 2 are drawn in Figs. 11 and 12.

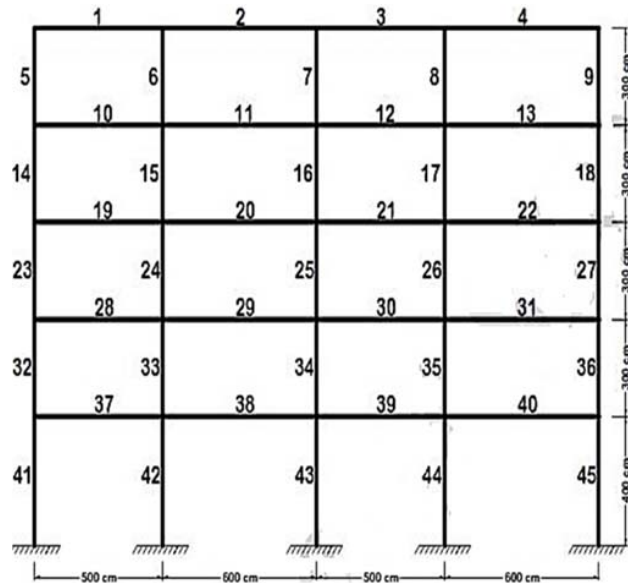
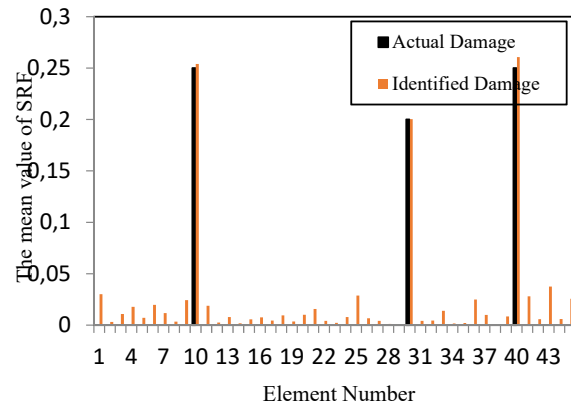
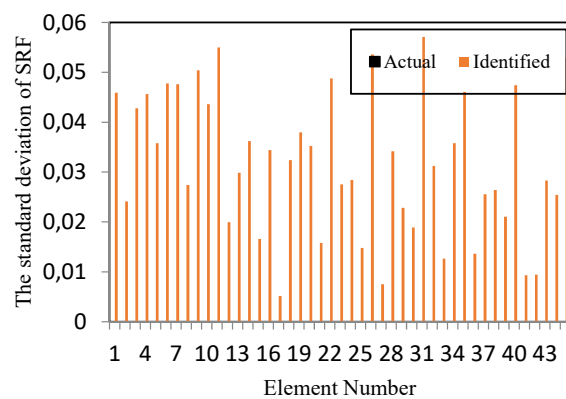
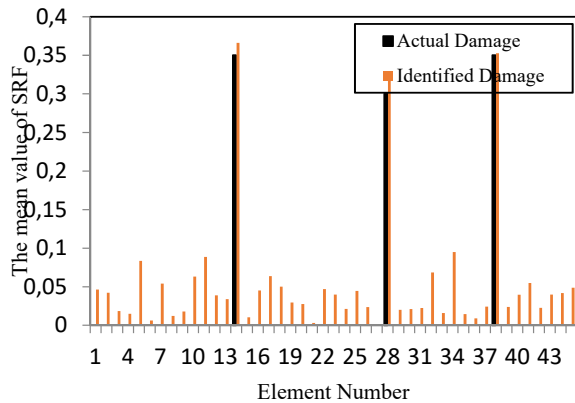
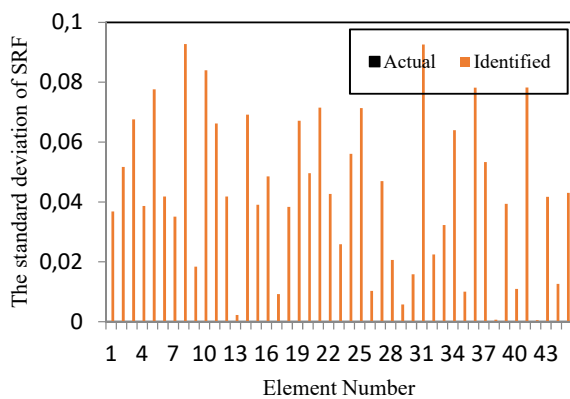


Fig. 8 A four-span five-story frame

Fig. 9 Mean value of SRF for damage scenario 2 ($\xi_\lambda = 1\%$)Fig. 10 Standard deviation of SRF for damage scenario 2 ($\xi_\lambda = 1\%$)

Fig. 11 Mean value of SRF for damage scenario 2 ($\xi_d = 10\%$)Fig. 12 Standard deviation of SRF for damage scenario 2 ($\xi_d = 10\%$)

The numerical results reveal the high performance of the proposed method for an exact detecting of the location and severity of various damage scenarios. The PBDD methods give the higher SRF values at the damaged elements and give lower SRF values at the undamaged elements.

VIII. CONCLUDING REMARKS

In this paper, a novel method was developed for DD at a solely one-stage procedure, using the most recently defined optimization algorithm, introduced by the authors. In real problems, there are always uncertainties in the measured modal parameters, which may lead to unreliable and false prediction of structural damage. Hence, uncertainties are also considered in the DD process. For this purpose, an IGMM algorithm is modified to improve its performance in handling the problem with a large array of variables. The numerical results reveal the high performance of the proposed method for an exact detection of the location and severity of various damage scenarios. It shows that the proposed PBDD procedure can obtain accurate mean values and small standard deviations of the ESPs and can provide robust damage identification results.

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