

# Probability-Based Damage Detection of Structures Using Kriging Surrogates and Enhanced Ideal Gas Molecular Movement Algorithm

M. R. Ghasemi, R. Ghiasi, H. Varaei

**Abstract**—Surrogate model has received increasing attention for use in detecting damage of structures based on vibration modal parameters. However, uncertainties existing in the measured vibration data may lead to false or unreliable output result from such model. In this study, an efficient approach based on Monte Carlo simulation is proposed to take into account the effect of uncertainties in developing a surrogate model. The probability of damage existence (PDE) is calculated based on the probability density function of the existence of undamaged and damaged states. The kriging technique allows one to genuinely quantify the surrogate error, therefore it is chosen as metamodeling technique. Enhanced version of ideal gas molecular movement (EIGMM) algorithm is used as main algorithm for model updating. The developed approach is applied to detect simulated damage in numerical models of 72-bar space truss and 120-bar dome truss. The simulation results show the proposed method can perform well in probability-based damage detection of structures with less computational effort compared to direct finite element model.

**Keywords**—Enhanced ideal gas molecular movement, Kriging, probability-based damage detection, probability of damage existence, surrogate modeling, uncertainty quantification.

## I. INTRODUCTION

IN recent years, significant efforts have been undertaken in the area of vibration-based damage detection methods. These methods are based on the fact that dynamic characteristics, i.e. natural frequencies, mode shapes and modal damping, are directly related to the stiffness of the structure. Therefore, a change in natural frequencies or a change in mode shapes may indicate a loss of stiffness. Some detailed literature reviews which describe the state of the art in the methods for damage detection, localization, and characterization, by examining changes in the dynamic response of a structure can be found in [1], [2].

The use of approximate models known as surrogate models with a much lower computational cost instead of expensive computer analysis codes (Finite Element Model) pervades much of today's engineering design and optimization. These approximations or meta-models are used to replace the actual expensive computer analyses and facilitated multidisciplinary, multi-objective optimization, reliability analysis, and concept exploration [3], [4].

Detection of damage severity is effectively the solution to the inverse problem [5]. However, it may be necessary in

many cases to solve the forward problem to generate data for the solution to the inverse problem. Generation of data is usually computationally expensive, and surrogate models are created to reduce the computational expense [6]. Simulation of efficient surrogate model of finite element (FE) as a response of updating damaged structure which is employed in the optimization loop through an inverse process to ascertain the damage parameters (damage severity and location), can replace expensive numerical simulations while enhancing computation efficiency. Fathnejat et al. [5] proposed solution procedure based on artificial neural network (ANN) to reduce the computational time of model updating during the process of damage severity detection.

Studies shows that ANNs are capable of providing correct damage identification, especially when the structural damage and the associated changes in vibration properties are simulated numerically and are error-free [1], [3]. However, in practice, uncertainties in the FE model parameters and modelling errors are inevitable. The existence of modelling error in the FE model due to the inaccuracy of physical parameters, non-ideal boundary conditions, FE discretization, and nonlinear structural properties may result in the vibration parameters generated from such a FE model which did not exactly represent the relationship between the modal parameters and the damage parameters of the real structure [7]. On the other hand, the existence of measurement error in the measured data that are normally used as testing data in a surrogate model is also unavoidable. Since the efficiency of a surrogate model prediction relies on the accuracy of both components, the existence of these uncertainties may result in false and inaccurate predictions. Therefore, the impact of uncertainties on the reliability of surrogate models for structural damage detection needs to be analyzed.

Hence, the objective of this paper is to study the influence of uncertainty on damage identification using a combination of frequency and mode shape as the input variables. To consider the uncertainties in the FE modelling and the measurement data, an approach introduced by Papadopoulos and Garcia [8] is applied. Using this method, the probability of damage existence (PDE) can be estimated by comparing the probability distribution of the undamaged and damaged models. To consider the effect of FE modelling error, a statistical surrogate model is trained with vibration data generated from the FE model, but smeared with random variations. To include the effect of noise in the measurement data, the testing data used as input to the statistical kriging

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model for damage identification are also smeared with random noises. Hence, in this paper, a methodology is presented for the damage detection of skeletal structure based on kriging metamodeling technique and enhanced version of ideal gas molecular movement (EIGMM) algorithm [9], [10]. The computational cost of model updating during the optimization process of damage detection is reduced efficiently by evaluating multiple damage location assurance criterion (MDLAC) index [11] based on the frequency change vector of structures, using kriging surrogate. To validate proposed probability-based damage detection method, two examples are presented.

The paper is organized as follows. The brief introduction about Kriging surrogate is presented in Section II. Section III then presents the analytical formulation of MDLAC. Ideal gas molecular movement and enhanced version is described in Sections IV and V. Probability based model updating and proposed damage detection procedure is described in Sections VI and VII. Numerical examples are studied in Section VIII. Finally, Section IX presents conclusions of the work.

## II. THE KRIGING SURROGATE

Kriging, which is widely used, is a combination of polynomial regression and Gaussian stochastic processes [10]. The regression model fits the samples according to the rule of least-squares estimation. The correlation model adjusts the prediction error by using maximum likelihood estimation. Kriging is formulated as [12]:

$$\hat{y} = Y(x) + Z(x) = \underbrace{\sum_{i=0}^n \beta_i f_i(x)}_{\text{regression}} + \underbrace{Z(x)}_{\text{correlation}} \quad (1)$$

where  $Y(x) = \sum_{i=0}^n \beta_i f_i(x)$  is the regression model, which usually adopts polynomials up to the second order to represent the global trend of the sample points.  $Z(x)$  is the correlation model, which is a Gaussian process with mean value 0 and covariance  $\sigma$ . The correlation model is given by

$$\text{cov}(Z(x_j), Z(x_k)) = \sigma^2 R_{jk}(\theta, x_j, x_k) \quad (2)$$

where  $R_{jk}$  is the Gaussian correlation function on the  $p$ -dimensional design space:

$$R_{jk}(\theta, x_j, x_k) = \prod_{i=1}^p e^{-\theta_i |x_{ji} - x_{ki}|^2} \quad (3)$$

In (2) and (3),  $\theta$  is the coefficient vector of the correlation model. The optimal coefficients  $\theta^*$  are found for maximum likelihood estimation. It is important to choose proper formulations of the regression functions which represent the system behavior as precisely as possible. Higher-order

polynomial regressions have the capability of approximating more complex responses, but they require more sample points to determine the polynomial coefficients. By properly selecting the polynomial orders and mixed terms of design variables using knowledge of a system, the number of coefficients in regression functions can be greatly reduced.

## III. MULTIPLE DAMAGE LOCATION ASSURANCE CRITERION (MDLAC)

Structural damage detection techniques are generally classified into two main categories. They include the dynamic and static identification methods requiring the dynamic and static test data, respectively. Furthermore, the dynamic identification methods have shown their advantages in comparison with the static ones. Among the dynamic data, the modal analysis information of a structure such as the natural frequencies and mode shapes were widely used for damage detection [13]-[15]. Determination of the level of correlation between the measured and predicted natural frequencies or mode shapes can provide a simple tool for identifying the location and severity of structural damages. When the natural frequencies are employed to identify the damage, two parameter vectors may be determined. One parameter vector consists of the ratios of the first  $n_f$  natural frequency changes  $\Delta F$  due to structural damage, i.e.:

$$\Delta F = \frac{F_h - F_d}{F_h} \quad (4)$$

where  $F_h$  and  $F_d$  indicate the natural frequency vectors of the healthy and damaged structure, respectively. Another parameter vector can be similarly defined as:

$$\delta F(ESV) = \frac{F_h - F(ESV)}{F_h} \quad (5)$$

where  $F(ESV)$  is a natural frequency vector that can be extracted from an analytic model and elemental stiffness vector (ESVs)  $ESV^T = [E_1, \dots, E_i, \dots, E_n]$  which represents a damage variable vector containing the elasticity modulus of structural elements ( $E_i, i = 1, \dots, n$ ) of all  $n$  structural elements.

Given the pair of parameter vectors, one can estimate the level of correlation in several ways. An efficient way is to evaluate a correlation index called the MDLAC which is expressed in the following form [11]:

$$MDLAC(ESV) = \frac{|\Delta F^T \delta F(ESV)|^2}{(\Delta F^T \Delta F)(\delta F^T(ESV) \delta F(ESV))} \quad (6)$$

The *MDLAC* compares two frequency change vectors, one of which is obtained from the examined structure, and the other from is an analytical model of the structure. The *MDLAC* varies from a minimum value 0 to a maximum value 1. It will be maximal when the vector of analytical frequencies equates to the frequency vector of damaged structure, i.e.

$$F(ESV) = F_d \quad (7)$$

#### IV. IDEAL GAS MOLECULAR MOVEMENT

The behavior of gas molecules in an isolated medium shows that they disperse rapidly in different directions and cover all the space inside. The essence of such manner lies on two factors; the high speed of ideal gas molecules and their collisions. Recently, the conventional IGMM was introduced by the authors, and its application in solving engineering problems was assessed then [9], [16]. The algorithm utilizes the governing equations for speed and collision of molecules in order to determine their new location. The speed of molecules thus is proportional to the temperature and inversely proportional to its mass. Besides, they collide with each other with a certain probability, increasing gradually with their motions. Ideal gas molecules have fully elastically collisions, and elastic collision governing equations can be used to determine the new position of gas molecules after collision [9]. Different steps of the IGMM algorithm can be summarized as follows [9]:

Step1. Generate the initial population of gas molecules with a uniform distribution from the allowable range of design variables.

Step2. Evaluate each molecule and assign them a mass according to its fitness using the following relation:

$$m_i = \frac{1}{fit(i)} / \frac{1}{\sqrt{\sum (fit^2(i))}} \quad (8)$$

where  $m_i$  shows the mass of the  $i$ -th molecule and  $fit(i)$  reflects the fitness of the  $i$ -th molecule with regard to the objective function for the problem.

Step3. Pairing molecules without repetition. In this stage, based on the governing equations of the ideal gases, it was assumed that there are no simultaneous molecular collisions.

Step4. Determine collision probability (CP) based on (8).

$$CP = 1 - \exp(-0.63 \times iter) \quad (9)$$

Step5. Generate a random number between 0 and 1 and compare it with CP to determine whether collision occurs or not. According to this phenomenon, the following steps will proceed to calculate the new velocity and position of each molecule.

Step6. In collision phase, new post-collision velocities are obtained using (10) and (11). In using these equations, the molecule with a larger mass is assumed stationary, and the lighter molecule moves according to the hypotheses about the elastic collision between gas molecules. The initial velocity of the moving molecule is obtained using relation  $\Delta x = v\Delta t$  (for  $\Delta t = 1$ ) by subtracting the positions of the two molecules.

$$(v_1^d)' = \frac{(m_1 - Em_2)}{m_1 + m_2} \times v_1^d \quad (10)$$

$$(v_2^d)' = \frac{(1+E)m_1}{m_1 + m_2} \times v_1^d \quad (11)$$

where  $d$  indicates the dimension of the optimization problem. As stated, in the event of elastic collisions, parameter  $E$  is equal to 1, but in (10) and (11), this parameter is defined as a variable to guarantee the convergence in the algorithm. Therefore, in the first few steps of the optimization process, this variable has a value near 1, but with an increase in the number of optimization cycles, its value declines dynamically based on the following linear equation.

$$E = 1 - \left( \frac{iter}{\max it} \right) \quad (12)$$

where  $iter$  and  $\max it$  indicate current and the maximum iterations of optimization procedure, respectively. Having computed the new velocity of each molecule, its new position can be computed using (13) and (14):

$$(x_1^d)' = x_2^d + rand \times (v_1^d)' \quad (13)$$

$$(x_2^d)' = x_2^d + rand \times (v_2^d)' \quad (14)$$

where  $x_2^d$  shows the position of stationary molecule before the impact, and accordingly,  $(x_1^d)'$  and  $(x_2^d)'$  indicate the new positions after the impact, respectively.  $rand$  represents a random normal distributed value in the range [0,1].

Step7. In no collision phase, the new velocity of the  $i$ -th molecule is determined using (15).

$$v_i^d = 1.7 \sqrt{\frac{kT_i}{m_i}} \quad (15)$$

The Boltzmann constant value ( $k$ ) in (15) is assumed opposite to the number of molecules in the optimization process. Velocity of each molecule accords with the mass and temperature of that molecule. Hence, in this phase, it is necessary to calculate the new temperature of each molecule. To this end, a subtractive equation is defined as follows. The initial temperature is set to 1000 in original IGMM.

$$T_i' = T_i - 1/m_i \tag{16}$$

Finally, after determining the new velocity of each molecule using (15), the new position of each molecule is obtained as given

$$(x_i^d)' = x_2^d + rand \times (v_i^d)' \tag{17}$$

Step8. The convergence criterion will be checked, and if the algorithm does not converge, Steps 2-4 will be repeated.

V. ENHANCED IDEAL GAS MOLECULAR MOVEMENT

Enhanced version of IGMM is proposed by authors in [17]. In some optimization problems, number of variables that must be considered is very large. For example, in optimization based damage detection problem, damaged elements and damage extends are searched through an optimization process until the response of hypothesized damaged structure equals those of a real damaged structure. When real structure is large scale structure, number of elements (variables) will be increased [18]. Hence, when optimization method tries to minimize, objective function must handle with huge bunch of variable, and this decreases convergence speed of algorithm.

Therefore, an approach introduced by Ghasemi et al. [17] is applied in this paper to resolve this problem. In damage detection problem, in the first stage when initial population generated, each molecule has a velocity vector that represents its speed in an n-dimensional space. Each variable of this vector represents elasticity modulus of structural elements. In the proposed method, first, number of variables in each stage of IGMM algorithm is considered as the total number of elements. Then, all the intact elements are eliminated in each stage, and the algorithm converges to the exact locations and severity of damages. Zero values for the variables signify that the *i*-th element of structure is intact, and a non-zero value refers to the damaged element. If the variables with near zero values ( $SRF_i \leq 0.05$ ) do not alter for 10 iteration, this variable will be eliminated.

As far as the objective function is concerned, it is defined here as an unconstrained optimization problem as follows:

$$\begin{aligned} \text{Find : } &ESV_i = \{E_1, E_2, E_3, \dots, E_n\} \\ \text{Minimize : } &F(ESV) = \|1 - MDLAC\|^2 \tag{18} \\ \text{Where : } &E_{\min} \leq E \leq E_{\max} \end{aligned}$$

where  $F(ESV)$  is that minimization problem, and  $E_{\min}$  and  $E_{\max}$  are the lower and upper bounds of the damage vector, respectively. It is necessary that the bounds represent the physical behavior of the structure. By using an optimization algorithm and solving (18), the damage variables are determined.

VI. MONTE CARLO SIMULATION FOR PROBABILITY BASED MODEL UPDATING

Since the uncertainties (noises) inevitably exist in the measured vibration data, the updated ESV ( $E$ ) is subjected to uncertainty as well. As mentioned before, the uncertainties in the measured modal data are assumed as independent normally distributed random variables with zero means and particular covariance. In this regard, the eigenvalues and mode shapes can be expressed as [19]:

$$\begin{aligned} \lambda_i^E &= \lambda_{i,0}^E (1 + X_{\lambda_i}), \\ i &= 1, 2, \dots, n_m \end{aligned} \tag{19}$$

$$\begin{aligned} \phi_i^E &= \phi_{i,0}^E (1 + X_{\phi_i}), \\ i &= 1, 2, \dots, n_m \end{aligned} \tag{20}$$

where  $0$  represents the true values,  $X_{\lambda_i}$  and  $X_{\phi_i}$  denote relative random noises in the measured frequencies and mode shapes, respectively. The mean value of vector  $X$  is zero, and the standard deviation represents the noise level.

The statistics (mean value and standard deviation) of  $E$  can then be calculated by the perturbation method [20] or Monte Carlo simulation (MCS). The latter method can also give statistical samples of the updated ESVs, from which the statistical distribution can be obtained. Studies have demonstrated that the statistical distribution of the ESVs in the updated model is also normal [21], verified by the goodness-of-fit test [22]. Again, when the measured modal data in both undamaged and damaged states are available and the model updating method [21] is employed, the statistics of ESVs in both states ( $E^h$  and  $E^d$ ) can be respectively calculated.

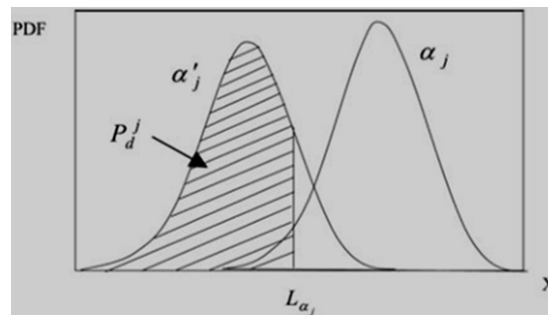


Fig. 1 Probability density functions for  $\alpha_j$  and  $\alpha'_j$  and probability of damage existence  $P_d^j$

The PDE can be estimated from statistical distributions of the stiffness parameters of the undamaged and damaged models. For example, if the stiffness parameter ( $\alpha_j$ ) of the undamaged segment  $j$  is normally distributed with mean  $E(\alpha_j)$  and standard deviation  $\sigma(\alpha_j)$ , the probability density

function can be obtained as illustrated in Fig. 1, where  $L_{\alpha_j}$  is the lower bound of the healthy parameter.

In this study, the confidence level is set to 95%, thus the lower bound is  $L_{\alpha_j} = E(\alpha_j) - 1.645\sigma(\alpha_j)$ , which indicates that there is a probability of 95% that the healthy stiffness parameter falls in the range of  $[E(\alpha_j) - 1.645\sigma(\alpha_j), \infty]$ . Similarly, for the stiffness parameter of segment  $j$  in the damaged state ( $\alpha'_j$ ), the distribution is again assumed as normal with mean  $E(\alpha'_j)$  and standard deviation  $\sigma(\alpha'_j)$ , and the corresponding probability density function is also plotted in Fig. 1. The PDE is defined as the probability of not being  $\alpha'_j$  within the 95% confidence healthy interval. Thus, the PDE of segment  $j$  is

$$P_d^j = 1 - \text{prob}(L_{\alpha_j} \leq x_{\alpha_j} \leq \infty) \\ = \text{prob}(-\infty \leq x_{\alpha_j} \leq L_{\alpha_j}) \quad (21)$$

PDE is a value between 0 and 1, and if the PDE of a segment is close to 1, then it is most likely the element which is damaged. If the PDE is close to 0, damage existing in the element is very unlikely [23]. It should be noted again that the stiffness parameters of the undamaged and damaged state have normal distributions because the random variations in (20) are assumed as zero mean normally distributed random variables.

In most surrogate applications for damage detection, the training data are obtained from FE analysis, which involved generating large number of damage cases based on an initial baseline FE model. Once the surrogate model is well-trained, the testing data are then applied to the model to obtain the locations and severities of any damages. In most of the previous studies, both training and testing data are assumed to be free from modelling and measurement error. In practice, however, modelling error and measurement noise are inevitable.

According to Xia et al. [24], the inaccuracy due to modelling and measurement error can be overcome by taking into account the uncertainties through a statistical method. In this study, modelling error and measurement noise are assumed to be normally distributed with zero means and specific variance. The noise is applied in terms of coefficient of variations (COV). The statistical properties of E value for each segment are obtained by using MCS. This is followed by calculation of the PDE of E values for each segment.

#### VII. MAIN STEPS FOR PROPOSED DAMAGE DETECTION METHOD

The main steps for the proposed damage detection method using kriging surrogate and EIGMM algorithm are summarized as follows:

Step1. Generating failure scenarios with the damage severity range between 0.05 and 0.35 with the pace of 0.05.

Step2. Developing FE model which computes the natural frequencies of the structure and finally the MDLAC corresponding to the failure scenarios that have been defined in the previous step.

Step3. Using the FE model of the structure in order to generate training and testing datasets for development of surrogate model that is used in the optimization process of damage detection.

Step4. Setting the initial number of design variables equal to total number of elements.

Step5. Engaging directly the surrogate model by the optimizer (EIGMM) to evaluate the objective function to be minimized to determine the damage of elements. (Applying the surrogate model).

Step6. Finding  $i$  as  $X_i = 0$  for all components of damage vector and determining the total number of intact elements.

Step7. Removing the intact elements from the damage vector and thus reducing number of variables from the optimization problem.

Step8. Performing EIGMM once again based on the new optimization size from Step 7.

Step9. Checking the convergence by computing  $1 - MDLAC$  from (18). If two response vectors are almost indifferent, save the results and terminate the optimization process, otherwise, go to the Step 6.

In this study, in order to generate failure scenarios which completely span the design space, Latin Hypercube Sampling (LHS) method has been applied. LHS generates a sample of plausible collections of parameter values from a multidimensional distribution. The LHS was presented by McKay in 1979 [25].

#### VIII. NUMERICAL RESULTS OF DAMAGE DETECTION

In this study, two structures are selected as the numerical examples to reveal the robustness and the degree of accuracy of the proposed damage detection method. These structures are:

1. 72-bar space truss
2. 120-bar Dome Truss

The mass matrix is assumed to be constant, and damage in the structure is simulated as a relative reduction in the elasticity modulus of individual element. Stiffness reduction ratio (SRF) is defined as:

$$SRF_i = \frac{E - E_i}{E}, i = 1, \dots, n \quad (22)$$

where  $E$  is the original modulus of elasticity and  $E_i$  is the final modulus of elasticity of the  $i$ -th element. For the optimization process, the number of molecules for EIGMM was fixed to 50 for each run along a maximum of 200 iterations

A. The 72-Bar Space Truss

A 72-bar spatial truss is considered as the first numerical example as shown in Fig. 2. Four non-structural masses of 2270 kg are attached to the nodes 1–4. This structure has also been investigated as an example in the field of structural optimization with frequency constraints by different researchers [26], [15]. As it can be seen from Fig. 2, the structure has 48 degrees of freedom. Table I represents the properties of this example.

Two cases of damage are assumed for this structure:  
 Damage case 1: 15% of damage in element 55; (15% of damage in each of the vertical members of the first story will

result in the same set of natural frequencies).

Damage case 2: 10% of damage in element 4 and 15% of damage in element 58; (90, 180, and 270-degree rotation along the z axis will result in the same set of natural frequencies).

TABLE I  
 PROPERTIES OF 72-BAR SPACE TRUSS

Property (unit)	Value
$E$ , modulus of elasticity (N/m <sup>2</sup> )	$6.98 \times 10^{10}$
$\rho$ , material density (kg/m <sup>3</sup> )	2770.0
Added mass (kg)	2270
$A$ , cross-sectional area of the members(m <sup>2</sup> )	0.0025

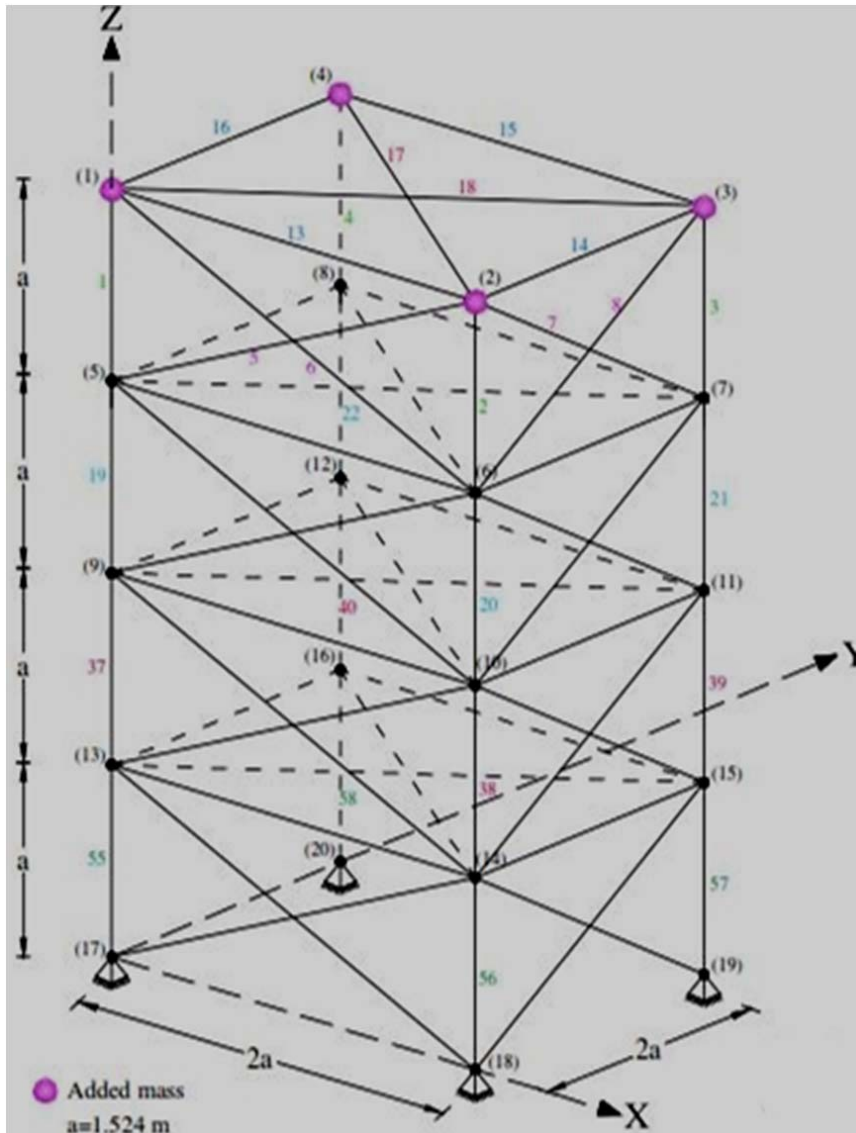


Fig. 2 A 72-bar spatial truss

Using the trained kriging model with 2% and 15% random errors (COV) in frequencies and mode shapes, and the testing data with the same level of noise, the mean values and

standard deviations of structural stiffness parameters corresponding to the two damage scenarios are estimated based on proposed procedure. From the normally distributed

probability density function of the damaged and undamaged states, the PDEs can be calculated. The PDEs for scenario 1 and scenario 2 are depicted in Figs. 3 and 4. From these figures, it is observed that, in scenario 1, the PDEs of element 55 are very high, and the PDEs of the other elements are low. This indicates that it is very likely that damage exists in segment 55 only. For scenario 2, the highest PDE occurred at segments 4 and 58, which are also the true damage locations. These results show that, using the proposed kriging surrogate model that coupled with EIGMM, the damages are detected with high confidence, and undamaged segments are less likely to be falsely identified. Moreover, in Figs. 3 and 4, the improved algorithm's capability of finding all of the global optimal solutions (damage states) is apparent. Furthermore, engaging EIGMM by efficient surrogate model, maintains the acceptable accuracy of damage detection.

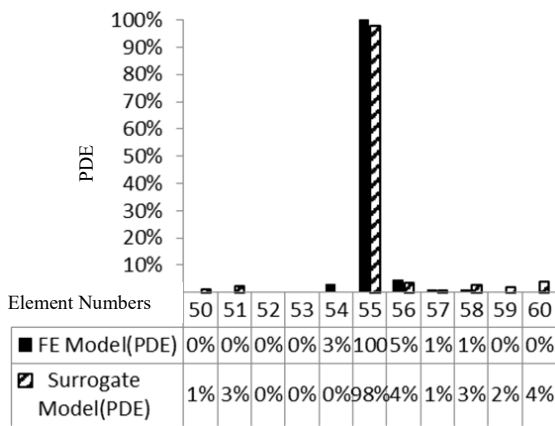


Fig. 3 Probability of damages existence for damage case 1 of bar truss

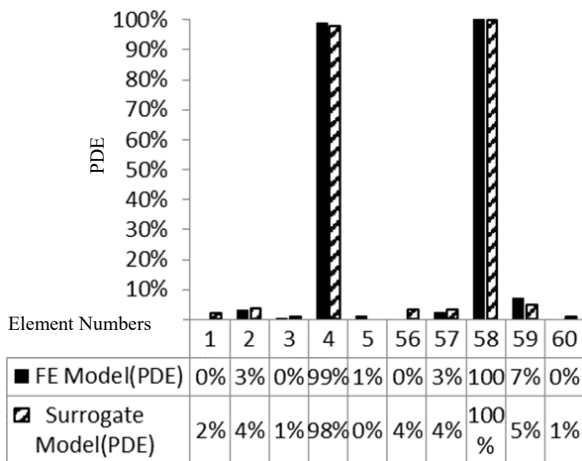


Fig. 4 Probability of damages existence for damage case 2 of bar truss

**B. 120-Bar Dome Truss**

A 120-bar dome truss, shown in Fig. 5 is considered as the second example [3], [27].

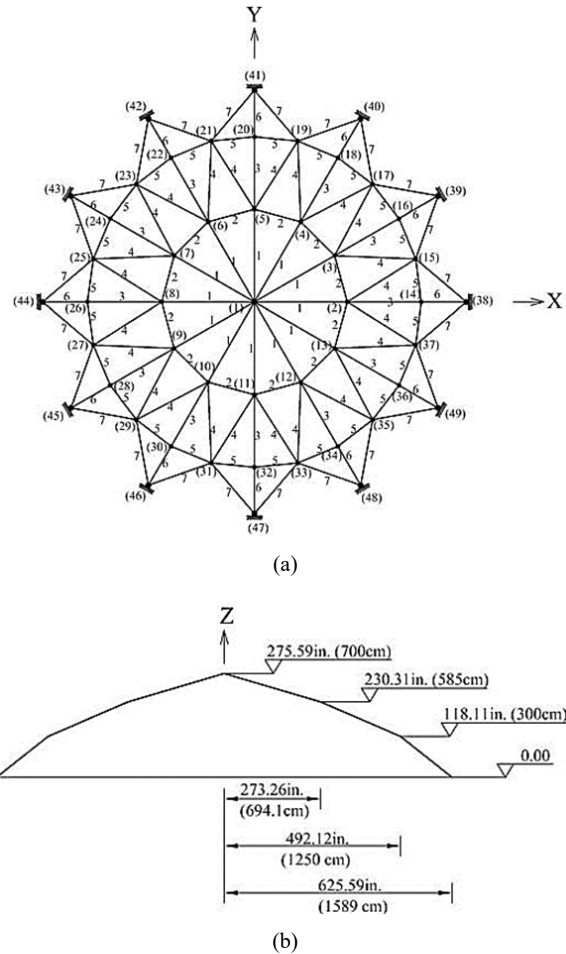


Fig. 5 120-bar dome truss (a) plan view, (b) section view

The diameter and the height of the dome are 31.78 m and 7 m, respectively. The material is a seamless steel pipe with a modulus of elasticity of 30,450 ksi (210,000 MPa), and the material density is 0.288 lb/in<sup>3</sup> (7971.810 kg/m<sup>3</sup>). The external diameter of the pipes is 0.2 m, and the thickness is 0.006 m. For generating training and testing datasets, FE program OpenSees [28] is used for structural analysis. Different damage scenarios are considered as shown in Table II. Figs. 6 and 7, Tables III and IV show the performance of the proposed method on this regard.

TABLE II  
DIFFERENT DAMAGE SCENARIOS 120-BAR DOME TRUSS

Case 1		Case 2	
Element Number	SRF	Element Number	SRF
12	0.30	4	0.35
38	0.20	30	0.20
53	0.25	51	0.35
79	0.2	58	0.25
		89	0.2
		105	0.40

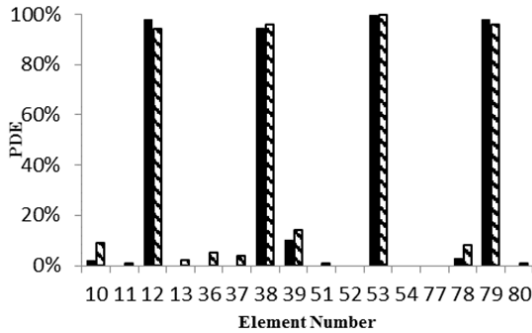


Fig. 6 Probability of damages existence for damage case 1 of dome truss

TABLE III  
PROBABILITY OF DAMAGES EXISTENCE FOR DAMAGE CASE 1 OF DOME TRUSS

Element Number	FE Model (PDE)	Surrogate Model (PDE)
10	2%	9%
11	0%	1%
12	98%	94%
13	0%	2%
36	0%	5%
37	0%	4%
38	95%	96%
39	10%	14%
51	0%	1%
52	0%	0%
53	100%	100%
54	0%	0%
77	0%	0%
78	3%	8%
79	98%	96%
80	0%	1%

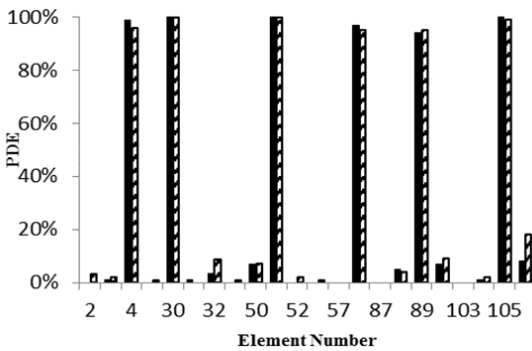


Fig. 7 Probability of damages existence for damage case 2 of dome truss

Results demonstrate that the statistical approach provides reliable prediction of damage occurrence by taking into consideration the uncertainties present in the real data when performing the training.

In this section, the proposed method of using appropriate kriging model instead of FE model as an updating model in optimization process of damage detection has been analyzed and compared. In both solution procedures, EIGMM specifications are the same. Table V shows the results of

comparing between two solution methods in terms of computational speed and accuracy. To compute process time when using a surrogate model, data generation time, training and testing time, and EIGMM implementation time are considered together (core™ i7 2.67 GHz CPU).

TABLE IV  
PROBABILITY OF DAMAGES EXISTENCE FOR DAMAGE CASE 2 OF DOME TRUSS

Element Number	FE Model (PDE)	Surrogate Model (PDE)
2	0%	3%
3	1%	2%
4	99%	96%
5	0%	1%
30	100%	100%
31	1%	0%
32	4%	9%
33	0%	1%
50	7%	7%
51	100%	100%
52	0%	2%
56	0%	1%
57	0%	0%
58	97%	95%
87	0%	0%
88	5%	4%
89	94%	95%
90	7%	9%
103	0%	0%
104	1%	2%
105	100%	99%
106	7%	18%

TABLE V  
COMPARISON THE RESULTS BETWEEN TWO SOLUTION METHODS IN TERMS OF COMPUTATIONAL SPEED AND ACCURACY

	Damage detection process time (sec)	RMSE of determined damage probability
FE model	3480	$5.02 \times 10^{-04}$
Kriging surrogate model	302	$1.29 \times 10^{-03}$

RMSE: root mean squared error.

It can be concluded from Table V that the idea of using a kriging model as a surrogate of FE model substantially reduces the computation time of damage severity detection. By this proposed solution method, computation time of the proposed procedure is reduced to one-tenth of the former one. Using kriging model in process of damage detection done by optimization algorithm accelerates this process besides maintaining the acceptable detection accuracy.

### IX. CONCLUDING REMARKS

This study presented a statistical surrogate method that accounts for the inevitable FE modelling error and measurement noise for structural damage detection. MCS method is used to derive the statistical surrogate model and to identify the structural condition. Both the modelling error and measurement noise are assumed to have normal distribution and zero means. Using this method, the probability of damage existence can be estimated. The numerical and experimental results demonstrated that the computational time of damage



detection using EIGMM engaged by kriging model as a surrogate of FE model is significantly reduced compared to using direct FE model based EIGMM (about one-tenth). Using this solution procedure contributes to a substantial reduction in the number of FE structural analysis which is further highlighted in damage detection of large-scale structures. However, further investigation needs to be conducted in order to see the sensitivity of the proposed method to different damage levels and different uncertainties levels.

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