Probabilistic Model Development for Project Performance Forecasting

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Abstract—In this paper, based on the past project cost and time performance, a model for forecasting project cost performance is developed. This study presents a probabilistic project control concept to assure an acceptable forecast of project cost performance. In this concept project activities are classified into sub-groups entitled control accounts. Then obtain the Stochastic S-Curve (SS-Curve), for each sub-group and the project SS-Curve is obtained by summing sub-groups' SS-Curves. In this model, project cost uncertainties are considered through Beta distribution functions of the project activities costs required to complete the project at every selected time sections through project accomplishment, which are extracted from a variety of sources. Based on this model, after a percentage of the project progress, the project performance is measured via Earned Value Management to adjust the primary cost probability distribution functions. Then, accordingly the future project cost performance is predicted by using the Monte-Carlo simulation method.

Keywords—Monte Carlo method, Probabilistic model, Project forecasting, Stochastic S-curve

I. INTRODUCTION

FORECASTING is an essential element of project management throughout the life cycle of a project. Once a project gets started, reliable forecasts are critical because even with a detailed plan, there are inherent risk factors that may influence the actual performance of a project. As a result, the project manager constantly seeks leading indicators for potential problems so that appropriate actions can be taken in a timely manner. That is, a current deviation from the plan serves as an early indicator of potential deviation of the project duration and cost at completion from objectives of the project.

When controlling project performance, it is important not only to monitor cost and time variances for actual project progress, but also to properly establish the actual project status based on objective predictions (forecasts) of final project performance. Such forecasts are necessary for the project manager to determine if corrective actions are required to minimize the expected variances from planned performance.

Estimate of both final cost and duration values can be made through two different approaches: deterministic and probabilistic. The deterministic approach estimates final cost and project duration according to the most likely cost and duration values for each activity. The probabilistic approach estimates the planned cost and duration values based on the variability of cost and duration inherent in each of the project activities [1].

The ultimate goal of project performance forecasting is to provide decision makers with objective and refined forecasts in a timely manner. However, actual performance data, which are probably the most objective and reliable source of predictive performance information, are limited early in the project. Therefore, a major challenge in project performance forecasting is to make use of subjective judgment or prior knowledge to overcome the lack of measured performance data to work during the early phase of a project [2].

The purpose of this paper is to develop a method to use a probabilistic model to determine project performance and a probabilistic project control concept to assure an acceptable forecast of project performance in terms of not exceeding planned budget and schedule risk levels. An application of the developed method to an example project is also presented.

II. PROJECT PERFORMANCE MEASUREMENT APPROACHES

Performance monitoring methods are classified in two types: (1) Progress Based S-Curve; and (2) Time Based S-Curve.

In the Progress Based S-Curve, progress is most often measured in terms of the amount of work completed, rather than in the time expended to complete the work. The independent variable could be the planned percentage of accomplished work (progress), because it depends only on the project scope (same for actual and for planned performance); and project time and cost could be treated as progress-dependent variable - usually different for actual and planned performance [3].

The Integrated Cost/Schedule/Work and the Earned Value Management (EVM) are classified as time-based S-curves (TB-S-curves), because time is treated as an independent variable, and cost and progress are treated as time-dependent variables. In the integrated method both of the cumulative cost and the progress, as time progress, are represented in one graph. The accomplished work (progress) is measured based on the budget which is in turn measured as cost, labor-hours, or physical quantity of work [4].

Earned Value Management (EVM) is a methodology used to measure and communicate the real physical progress of a project and to integrate the three critical elements of project management (scope, time and cost management). It allows the calculation of cost and schedule variances and performance indices and forecasts of project cost and schedule duration. This method was originally developed for cost management

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and has not been used widely for forecasting project duration. However, recent research trends show an increase of interest in the use of performance indicators to predict total project duration [5].

III. PROBABILISTIC COST BASELINE APPROACHES

A simulation approach is used for generating the stochastic S-curves (SS-curves), which is based on the defined variability in duration and cost of the individual activities within the process. SS-curves provide probability distributions for the budget and time values required to complete the project at every selected point of intermediate completion [6]. The general framework of the budget and time distribution determining method consists of three steps: (1) generating prior distributions of model parameters; (2) updating model parameters based on reported data; and (3) using the updated model for forecasting [7].

Monte Carlo simulation approach forecasts the project duration and cost values based on the variability of duration and cost inherent in each of the project activities. After estimating probability distributions of costs and activity durations, through the application of Monte Carlo simulation the probability distribution of project's total cost and schedule are provided. For each simulation, the Monte Carlo simulation engine randomly chooses one value for each variable within its range of possible values in accordance with their likelihood. This process is repeated a number of times (typically 1,000 iterations), and a range of equally likely potential outcomes is produced [8].

Recently, commercial computer programs have been developed with the specific purpose of probabilistic estimating [e.g., @Risk, Crystal Ball]. At first, a distribution function is assigned to each activity cost while activities durations are deterministic and defined as the most likely values. Then, in each iteration one value for cost is generated and these iterations are accomplished at time intervals during the project. Therefore, cumulative project cost is simulated. According to Central Limit Theorem, distribution of total cost is normal distribution.

In this case, since in probabilistic methods the term ''variance'' has a specific statistical meaning, the term ''variation'' will be used to represent measures of probabilistic project performance. The cost variation (CV) is evaluated as the difference between the expected budgeted cost of work performed (μ_{BCWP}) and actual cost of wok performed (ACWP); At-completion cost variation (ACV) is evaluated as the difference between expected budget at completion (μ_{BAC}) and expected estimate at completion (μ_{EAC}), as shown in Fig. 1.

Although SS-curves do not solve the problems associated with fundamentally poor estimates, they are preferred to deterministic S-curves because they provide information relative to the range of likely outcomes for the project at any percent of its progress [9].
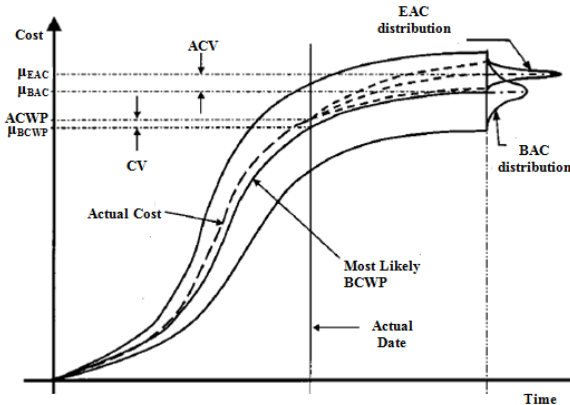

Fig. 1 Project Stochastic S-Curve

IV. PROBABILISTIC PROJECT PERFORMANCE MODEL DEVELOPMENT

A. The Activity Cost Distribution Function

In this model, the Beta distribution is used for the probability distribution functions. The Beta distribution has a long history of application in engineering and project management [10]. The primary advantage of applying the Beta distribution is the fact that the Beta distribution can generate a wide range of shapes with only two parameters [11]. The Beta distribution is a continuous probability distribution on a finite interval A to B with two shape parameters α and β. specifically, it was recommended that the Beta distribution be used in modelling random input processes of construction durations for simulation studies [12]. The Beta probability density function and the Beta function are obtained through (1) and (2) respectively:

$$f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} \frac{(x-A)^{\alpha-1}(B-x)^{\beta-1}}{(B-A)^{\alpha+\beta-1}} \quad A \le x \le B \qquad (1)$$

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt \qquad (2)$$

A practical method for modelling Beta distributions has been developed that would enable the engineer or the analyst to fit Beta distributions with minimal required background in statistics and programming [12]. Reference [10] has presented a visual interactive procedure to determine the shape parameters of a unique Beta probability distribution function, Visual Interactive Beta Estimation System (VIBES). This procedure uses a combination of four activity–duration characteristics, two of which must be the maximum and minimum duration. The possible characteristic combinations are the maximum and minimum activity durations and either: (1) the mean and standard deviation; (2) the mean and a selected percentile; (3) the mode and a selected percentile; or (4) two selected percentiles.

Shape parameters are defined through data gathering from

previous projects based on Central Limit theorem [10]. The mean, variance and mode of the Beta distribution are respectively given by:

$$\mu = \frac{\alpha B + \beta A}{\alpha + \beta} \tag{3}$$

$$\sigma^2 = \frac{(A - B)^2 \alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} \tag{4}$$

$$m = \frac{(\alpha - 1)B + (\beta - 1)A}{\alpha + \beta - 2} \tag{5}$$

B. Performance Monitor and Control Functions

The earned value method provides early indications of project performance to highlight the need for eventual corrective action. For example, the Schedule Performance Index (SPI) indicates how efficiently the project team is using its time, and the Cost Performance Index (CPI) gauges how efficiently the team is using its resources. These indices are obtained through (6) and (7) respectively [13]:

$$SPI = \frac{BCWP}{BCWS} \tag{6}$$

$$CPI = \frac{BCWP}{ACWP} \tag{7}$$

Where, BCWS: Budgeted Cost of Wok Scheduled; BCWP: Budgeted Cost of Wok Performed; ACWP: Actual Cost of Wok Performed.

Variance thresholds should be established in the planning phase and should be used to guide the examination of performance. Project managers and others must decide where the problems lie and what actions to take or recommend. There are four major responses to a variance report: (1) Ignoring it; (2) Functional modification; (3) Replanning; and (4) System redesign [14].

Therefore, there are two basic reasons to change the plan and system redesign: (1) If the work scope is changed, then the estimated cost and possibly the schedule will change, and all of these changes need to be reflected in a revised baseline; (2) If poor performance in the past is rendering the baseline worthless as a tool for measuring present performance, then a revised baseline may be justified [5].

Since probabilistic methods predict activity performance using cost probability distributions, performance corrections and forecast estimates at completion (EAC) may be treated by considering two situations: (1) Future performance is independent of previous performance when current variance is seen as atypical and the project management team expectations are that similar variance will not occur in future, refer to (8); (2) Future performance is dependent on previous performance when current variance are seen as typical of

future variance, refer to (9) and (10) [13].

$$EAC = ACWP + (BAC - BCWP) \tag{8}$$

$$EAC = \frac{BAC}{CPI} \tag{9}$$

$$EAC = ACWP + [(BAC - BCWP)/(a_1 CPI + a_2 SPI)] \tag{10}$$

The EAC method based on (8), accepts the actual project performance to date as represented by the actual costs and predicts that all future work will be accomplished at the budgeted rate. The EAC method based on (9), assumes what the project has experienced to date can be continued in the future. The EAC method based on (10), forecasts remaining work will be performed at an efficiency rate that considers both the cost and schedule performance indices. It assumes both a negative cost performance to date, and a requirement to meet a firm schedule commitment by project. This method is most useful when the project schedule is a factor impacting the estimate to complete effort. Coefficients a_1 and a_2 are weighting coefficients of CPI and SPI at different values (e.g., 80/20, 50/50 or some other ratio) according to the project manager's judgment [13].

Another useful index is the To-Complete Performance Index (TCPI), which helps the team determine the efficiency that must be achieved on the remaining work for a project to meet a specified endpoint, such as the Budget at Completion (BAC) or the team's revised Estimate at Completion (EAC). The TCPI for achieving the BAC is calculated by dividing the work remaining by the budget remaining as follows [13].

$$TCPI = (BAC - BCWP)/(BAC - ACWP) \tag{11}$$

Finally, assigned distributions are adjusted through above approaches and future project performance is forecasted through the new distribution.

C. Control Accounts and Control Limits Assignment

Plotting a unique S-Curve for the entire project, and determining only one global performance index for whole of the project, involves some issues. This approach causes probable performance failures of some project work packages or sub-projects to be generalized to the whole project. For solving this issue project should be divided into sub-groups as control accounts, and performance of each control account is evaluated and controlled based on its assigned control limit, individually.

The major work components should be identified in the form of major cost packages and their related subcategories, which can be restricted to the major items that affect the total cost bottom line by a certain percentage. These sub-groups or control accounts determined through project manager are possible to be obtained according to MasterFormat system divisions [15]. Then obtain SS-Curve for each control account

and project S-Curve is obtained by summing control accounts' S-Curve. Likewise, control limits are assigned for each control account to control project performance. In this paper CPI and SPI are considered as control limits for project performance control accounts.

D. Specification Limits

Specification limits, the area on either side of the expected line, meet the customer's requirement for a product or service [13].

An important reason for quantifying uncertainty at some stage is that doing so helps to force all members of an organization's management to appreciate the significance of differences between 'targets' that people can aspire to, 'expected values' used to provide an unbiased predictor of outcomes, and 'commitments' that provide some level of contingency allowance. Targets, expected values, and commitments need to be distinguished in terms of cost, time, and all other relevant measures of performance.

Targets need to be realistic to be credible, but they also need to be lean. If targets that are optimistic are not aimed for, expected costs will not be achieved on average and contingency funds will be used more often than anticipated. If expected costs together with contingency funds are treated as targets, following a version of Parkinson's Law, work will expand to fill the time available for its completion, leaving insufficient margin when anything goes wrong. Targets are sometimes referred to as 'stretch targets' to reflect this and might be set at a level that has less than a 20% chance of being achieved [16].

Commitments usually involve 'asymmetric penalties' if they are not met or exceeded, with respect to costs, durations, and other performance measures (e.g., the implications of being over cost are not the same as being under cost). Determining this level of commitment ought to involve an assessment of perceived threats and the extent to which these may be covered by a contingency fund, together with an assessment of the opportunities and the implications of both over- and underachievement in relation to the commitment. High penalties associated with being over cost relative to the penalties associated with being under cost can justify setting commitment levels that have a higher probability of being met than the 50–60% chance an expected value might provide. Setting commitment levels that have an 80 or 90% chance of not being exceeded are common and we assume 80% for commitment [16].

V. NUMERICAL EXAMPLE

In this section, the developed model applied on a nineteen-activities construction project as a numerical example. The scope of this project is constructing a 900-meters square, 6-meters high, side and roof sheeting warehouse. The project duration is 85 days which its network diagram is shown in Fig. 2. Two shape parameters α and β are defined through data gathering from previous projects and fitting distribution to data (most distributions are negatively skewed because of incremental cost inherence). We use Crystal Ball's distribution fitting to match our data against each continuous probability distribution. Project management assigns maximum and minimum of activities cost and project performance control accounts and limits according to project and contract condition, depicted in Table I, II.
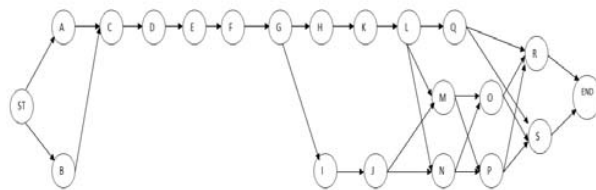

Fig. 2 Project network diagram

TABLE I
MAGNET PROJECT ACTIVITIES DATA

Activity	Predecessor	Description	Duration	Cost ($)		α	β
				Minimum	Maximum		
A	-	Mobilization	4	5000	6100	3	2
B	-	Site clear	2	2000	2700	3.2	2.1
C	A,B	Foundations excavation	2	5500	6200	3.7	2.5
D	C	Lean concrete	1	3000	4000	3.3	2.1
E	D	Form foundation	11	5000	5900	4	2.7
F	E	Reinforce foundation	8	7500	8400	4	3.1
G	F	Foundations concrete	7	7000	7800	4.4	3.2
H	G	Steel erection	12	20517	21551	3.4	2.5
I	G	Rebar lay	13	11000	13000	3.6	2.2
J	I	Floor slab concrete	3	2500	3100	3.9	2.3
K	H	Roof sheet install	6	17500	18100	3.3	2.7
L	K	Sides sheet install	5	17900	18300	4.2	3.1
M	J,L	Mechanical work	12	8900	9400	3.7	2.9
N	J,L	Electrical work	11	10000	10400	4.8	3.7
O	M,N	Floor covering	13	19000	21500	3.8	2.6
P	M,N	Painting	5	12300	12800	3.6	2.1
Q	L	Landscaping	10	19000	19600	4.5	3.3
R	O,P,Q	Clean up	4	1000	1400	3.2	2.5
S	O,P,Q	Demobilization	4	3000	3400	4.3	3.5

TABLE II
PERFORMANCE CONTROL ACCOUNTS AND LIMITS

Control account		Concrete	Metal	Electrical	Mechanical	Finishes	Earth works	General requirements
Activity		D,E,F,G,I,J	H,K,L	N	M	O,P	B,C,Q	A,R,S
Control limit	CPI	0.86-1.14	0.95-1.05	0.92-1.08	0.91-1.09	0.94-1.06	0.92-1.08	0.89-1.11
	SPI	0.92-1.08	0.92-1.08	0.92-1.08	0.92-1.08	0.92-1.08	0.92-1.08	0.92-1.08

Crystal Ball 7.3.1 is used for Monte Carlo simulation in this example. After running 1,000 trials, the expected estimations of project cost were obtained from the simulated SS-curve values and from the correspondent cumulative distribution functions at project completion. The predicted total project cost is 187,126$ and its standard deviation is 787 $, as shown in Fig. 3. Also Fig. 4 illustrates sensitivity analysis that represents which control account has more effect on the project total cost.
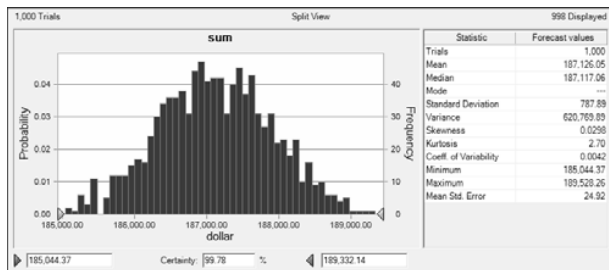

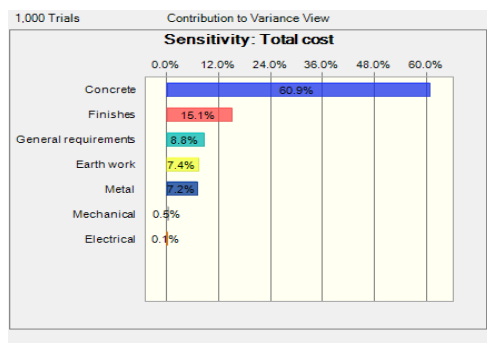Fig. 3 Monte Carlo simulation output of project cost

Fig. 4 Project control accounts costs sensitivity

The results of the project performance monitoring at an intermediate point, day 40, by application of the developed model, are depicted in Tables III and IV.

As it is depicted in Table IV, "Earth works" is outside of the acceptable time and cost control limits (SPI, and CPI). Therefore, this control account should be adjusted and re-planned. If it is desirable to preserve initial BAC, according to (11), subsequent "Earth works" productivity must increase about 4%. On the other hand (8), (9), and (10) can be used to forecast new "Earth works" cost estimation. Since "Earth works" is outside of the acceptable time limit so the use of (10) with 50-50 coefficient is an appropriate way to forecast new prediction because time performance effect is considered too. Once the project data have been updated with a corrective action, a new project performance forecast could be run in

order to evaluate effects in the probabilistic schedule and the revised at-completion performance forecast. If revised at-completion cost and duration variances were improved with respect to the previous forecasted values, the proposed corrective action could be considered as acceptable. The corrective performance productivity and the estimate cost forecasts for "Earth works" are shown in Table V.

TABLE III
PROJECT PERFORMANCE MONITORING DATA AT DAY 40

Control account	Activity	Expected values	Actual values	Earned values
General requirements	Mobilization	5,650 $	5,990 $	5,660 $
	Sum	5,650 $	5,990 $	5,660 $
Earth works	Site clear	2,430 $	2,480 $	2,150 $
	Foundations excavation	5,920 $	5,990 $	5,500 $
	Sum	8,350 $	8,470 $	7,650 $
Concrete	Lean concrete	3,600 $	3,630 $	3,500 $
	Form foundation	5,540 $	5,510 $	5,440 $
	Reinforce foundation	8,020 $	8,160 $	8,000 $
	Foundations concrete	7,460 $	7,600 $	7,100 $
	Rebar lay	7,310 $	7,350 $	7,100 $
	Sum	31,930$	32,250$	31,140$
Metal	Steel erection	12,240$	12,240$	12,300$
	Sum	12,240$	12,240$	12,300$

TABLE IV
PROJECT PERFORMANCE INDICES FOR CONTROL ACCOUNTS AT DAY 40

Control account		Concrete	Metal	Earth works	General requirements
Performance Index	CPI	0.96	1.005	0.9	0.94
	SPI	0.97	1.005	0.91	1.001

Based on performance reports on a project, the project manager updates probability distribution functions and decides whether the project performance is under control and within acceptable control limits so that intervention is not necessary. If the project or task is deemed not in control, the project manager needs to identify the causes of the variance and take necessary actions to get the project back under control and within the acceptable performance limits.

TABLE V
PRODUCTIVITY CORRECTION AND FORECASTING COST FOR "EARTH WORK" AT DAY 40

TCPI	EAC (Eq. 8)	EAC (Eq. 9)	EAC (Eq. 10)	
			80-20	50-50
1.04	29,628 $	31,895 $	31,828 $	31,728 $

The graphical representations of forecasting SS-curves for the project based on EACs which are depicted in Table V are shown in Fig. 5.
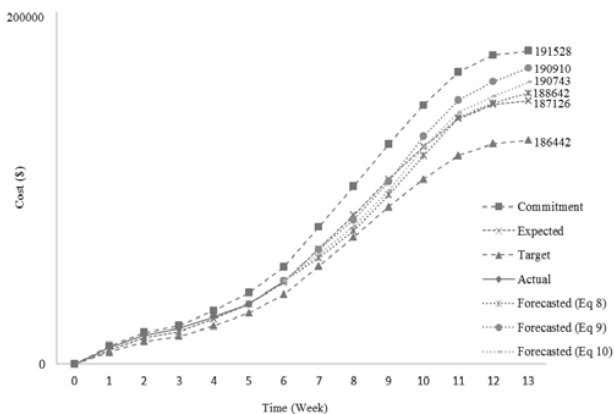

Fig. 5 The Project forecasting Stochastic S-Curves at day 40

VI. CONCLUSIONS

In this paper a new probabilistic cost forecasting method has been developed. The Beta S-curve method is a probabilistic method that provides confidence bounds on predictions. It is also an adaptive method that starts with the original estimation of project cost and adjusts the influence of prior performance information on prediction as actual performance data accrues.

In this method project activities are classified into sub-groups entitled control accounts. Then, Stochastic S-Curve is obtained for each each sub-group and project SS-Curve is obtained by summing sub-groups' SS-Curve. Thus, project is divided into sub-projects that cause easier and more accurate forecasting and monitoring. Moreover, control limit is determined for each control account to control project performance. If one sub-group needs modification, it just justifies and it does not affect other sub-groups.

In this model we assign one distribution function for each project activity cost. In future attempt we will contemplate cost variation along elapsed time. So we will assign two distribution functions for each activity cost; First distribution is determined based on productivity rate defined at time of tender submitting; Second distribution is determined based on cost variation along the time according to inflation rate. Ultimate cost of the activity is determined by multiplying these two functions.

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