Probabilistic Approach as a Method Used in the Solution of Engineering Design for Biomechanics and Mining

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***Abstract*—**This paper focuses on the probabilistic numerical solution of the problems in biomechanics and mining. Applications of Simulation-Based Reliability Assessment (SBRA) Method are presented in the solution of designing of the external fixators applied in traumatology and orthopaedics (these fixators can be applied for the treatment of open and unstable fractures etc.) and in the solution of a hard rock (ore) disintegration process (i.e. the bit moves into the ore and subsequently disintegrates it, the results are compared with experiments, new design of excavation tool is proposed.

***Keywords*—**probabilistic approach, engineering design, traumatology, rock mechanics

I. INTRODUCTION

LET us consider the Simulation-Based Reliability Assessment (SBRA) Method, a probabilistic direct Monte Carlo approach, in which all inputs are given by bounded histograms. Bounded histograms include the real variability of the inputs, see references [1], [2], [4], [5], [7], [10] and [11].

Application of the SBRA Method is a modern and innovative trend in mechanics. Using SBRA Method, the probability of failure (i.e. the probability of an undesirable situation) is obtained mainly by analyzing the reliability function "RF":

$$\mathrm{RF} = \mathrm{RF} - \mathrm{S}\,, \tag{1}$$

see Fig. 1. Where "RV" is the reference (allowable) value and "S" is a variable representing the load effect combination. The probability of failure is the probability that "S" exceeds "RV", i.e.:

$$P(\mathrm{RF} \le 0). \tag{2}$$

The probability of failure is a relative value depending on the definition of "RV" and it usually does not reflect an absolute value of the risk of failure (for example, it usually does not correspond to a "total" collapse).

Hence, this paper focuses on the probabilistic numerical solution of the problems mainly in biomechanics (i.e. designing of the external fixators applied in traumatology and orthopaedics) and also in geomechanics (i.e. solution of a hard rock disintegration process in mining). Application of the SBRA Method connected with the Finite Element Method (FEM) in these areas is a new and innovative trend in engineering.

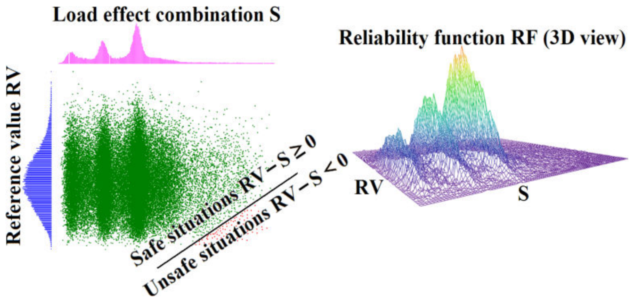

Fig. 1 Reliability function RF (SBRA Method)

II. SBRA METHOD IN BIOMECHANICS – REPORT ABOUT THE DESIGN OF EXTERNAL FIXATORS APPLIED TO THE TREATMENT OF OPEN AND UNSTABLE FRACTURES

Fig. 2 Design of external fixators a) based on metals - current design, heavier, expensive etc. b) based on polymers reinforced by carbon nanotubes - new design, lighter, x-ray invisible - leads to shortening the operating time and reducing the radiation exposure of patients and surgeons, with antibacterial protection, cheap, overall more friendly etc.)

According to the current studies and research, performed at VŠB – Technical University of Ostrava and Traumatology Centre of the University Hospital of Ostrava (Ostrava, Czech Republic), for example see references [3], [6], [9], [12], [13],

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[14] and [15], the current design of external fixators must be modified, see Fig. 2 and 3.

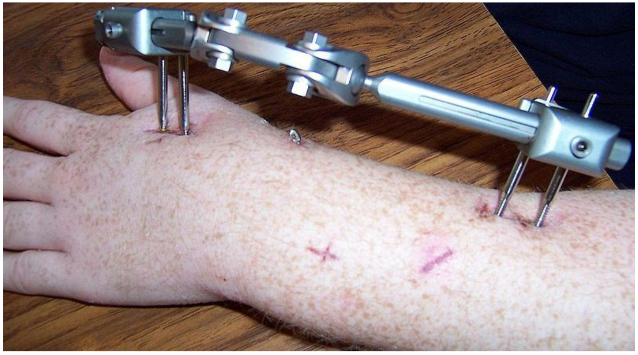


Fig. 3 Current design of external fixators

Fixators can be applied in traumatology, surgery and orthopaedics for treatments such us: open and unstable (complicated) fractures, limb lengthening, deformity correction, consequences of poliomyelitis, foot deformities, hip reconstructions, for treatment of humans and animals etc. However, there are real needs to make a modern design of fixators which satisfy new trends and demands in medicine, see references [3] and [6]. These demands, which are mutually connected, are solving by:

A. Applications of new smart materials

The outer parts of fixators must be x-ray invisible - which leads to shortening the operating time and reducing the radiation exposure of patients and surgeons. Antibacterial protection - application of nanotechnologies on the surface of the outer parts of the fixators to prevent or reduce possible infection (see references [14] and [15]) and weight optimalization, see Fig. 4. It is possible to satisfy all these demands by new composite materials using proper polymers reinforced by the carbon nanotubes (CNT) or carbon fibres. Some current solutions based on light metals are heavy and visible in x-ray diagnostic, see Fig. 5.

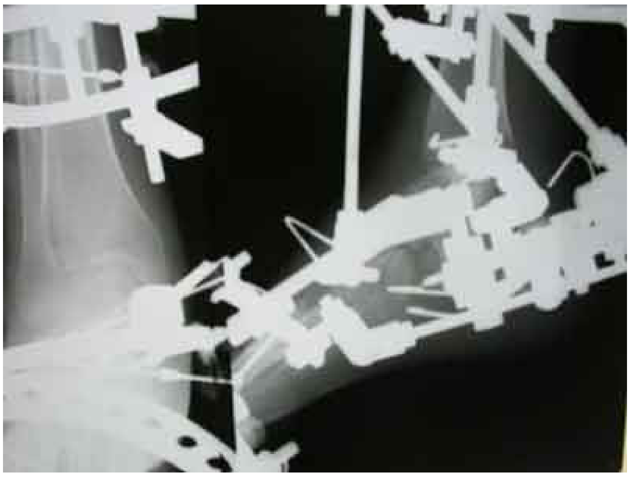


Fig. 4 Problems with high x-ray absorption (it is difficult to see broken limbs because there are too many metal parts)

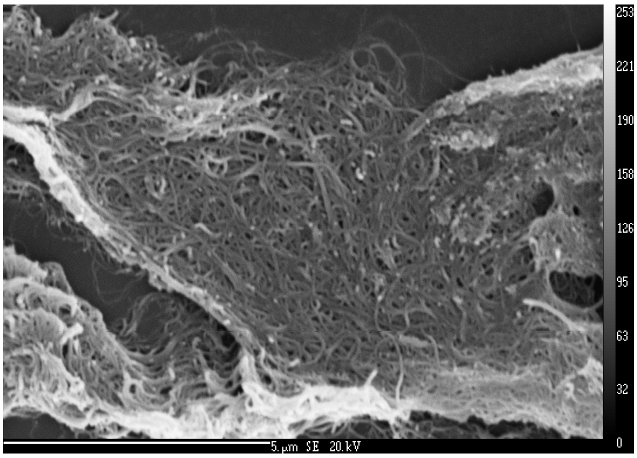


Fig. 5 Mechanical activated CNT for polymer composites preparation

B. New design

(According to shape, ecological perspective, patient's comfort, reducing the time of the operation, reducing the overall cost, "friendly-looking design", shape optimalization, see Fig. 2).

C. *Measuring of the real loadings*

Strain gauges etc., statistic evaluation, see Fig. 6

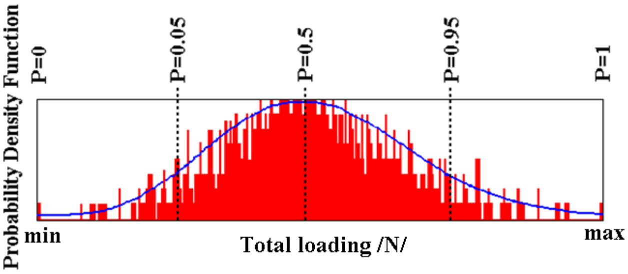


Fig. 6 Typical loading spectrum of an external fixator (SBRA Method – probabilistic input)

D. Numerical modelling and experiments

I.e. stochastic approach, SBRA Method application, FEM, etc., to avoid the overloading and undesirable situations, see Fig. 6 and 7.

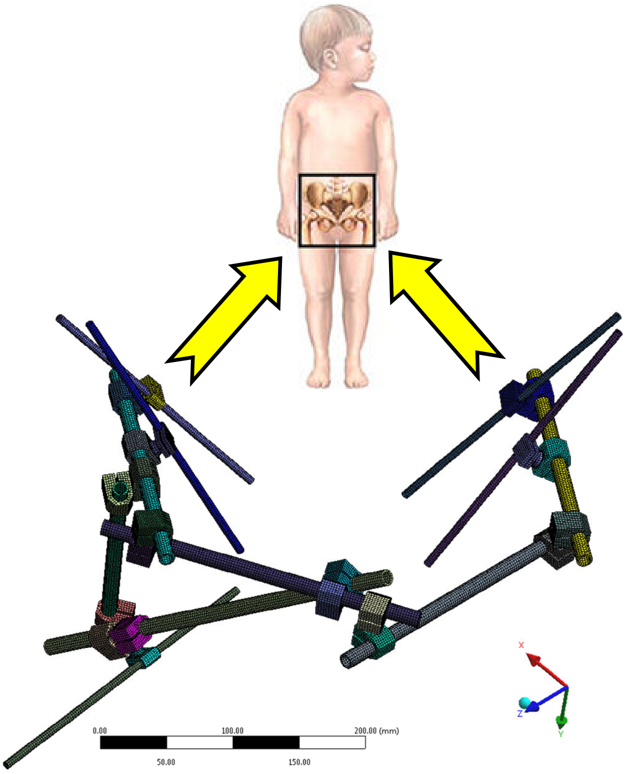


Fig. 7 Numerical modelling for treatment of pelvis and acetabulum

The new proposed designs of the external fixators cannot be more specifically described here, for confidentiality reasons. However, the methodology of designing is based on the probabilistic approach (SBRA Method), see also the end of the chapter III in this text.

III. SBRA METHOD IN MINING – REPORT ABOUT THE SOLUTION OF A HARD ROCK DISINTEGRATION PROCESS

The provision of sufficient quantities of raw materials is one of the main limiting factors of further industry development.

It is therefore very important to understand the ore disintegration process, including an analysis of the bit (i.e. excavation tool) used in mining operations. The main focus is on modelling of the mechanical contact between the bit and the platinum ore and its evaluation (i.e. practical application in the mining technology), see Fig. 8. However, material properties of the ore have a large stochastic variability. Hence, the stochastic approach (i.e. SBRA Method in combination with FEM is applied). MSC.Marc/Mentat software was used in modelling this problem, see Fig. 9 and reference [1], [2] and [4].

Fig. 8 Typical example of mechanical interaction between bits and hard rock (ore disintegration process)

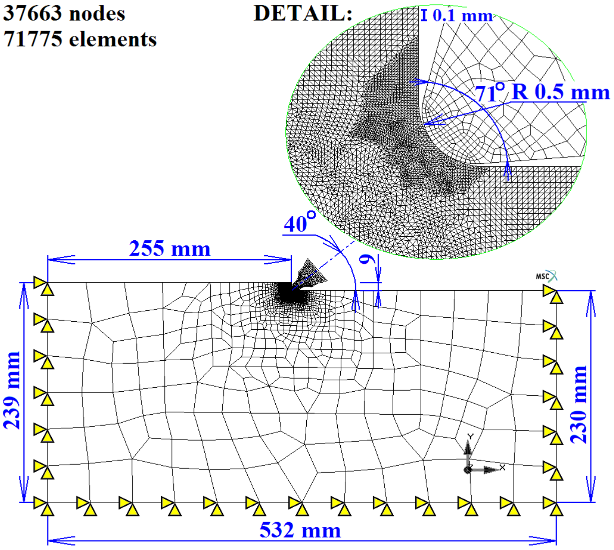


Fig. 9 Ore disintegration process (FEM model)

The bit moves into the ore with the prescribed time dependent function and subsequently disintegrates it. When the bit moves into the ore (i.e. a mechanical contact occurs between the bit and the ore) the stresses increase. When the equivalent stress is greater than the tensile strength in some elements of the ore, then these elements break off. Hence, a part of the ore disintegrates. This is done by deactivating the elements, see Fig. 10 and 11.

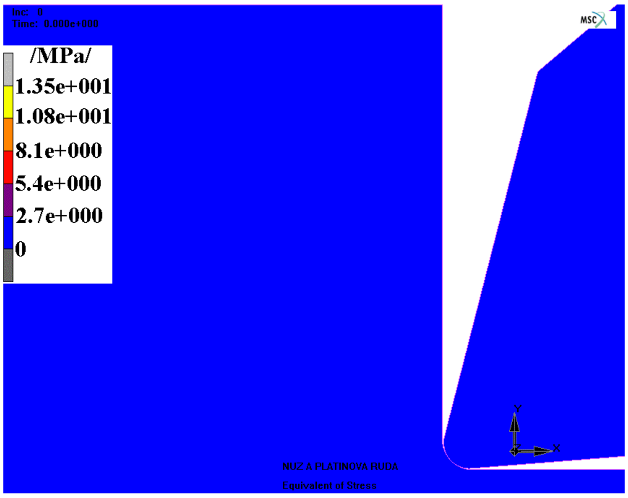


Fig. 10 Disintegration of the ore and movement of the bit – beginning of the solutions (equivalent von Misses stresses distributions, results of one Monte Carlo simulation)

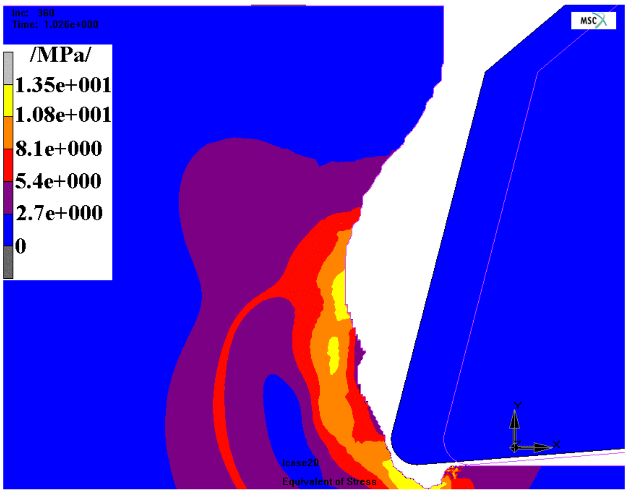
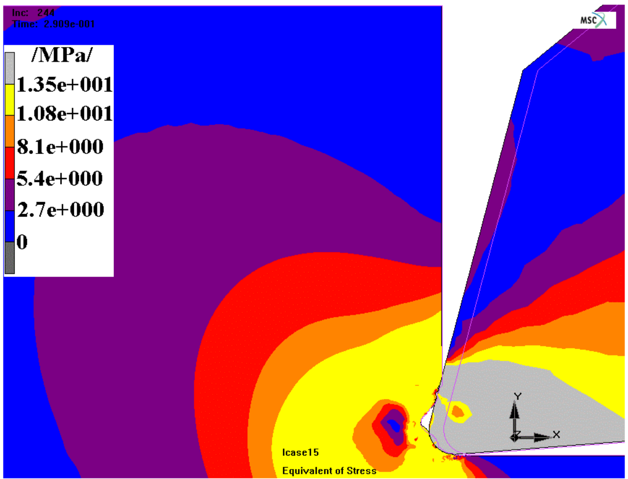
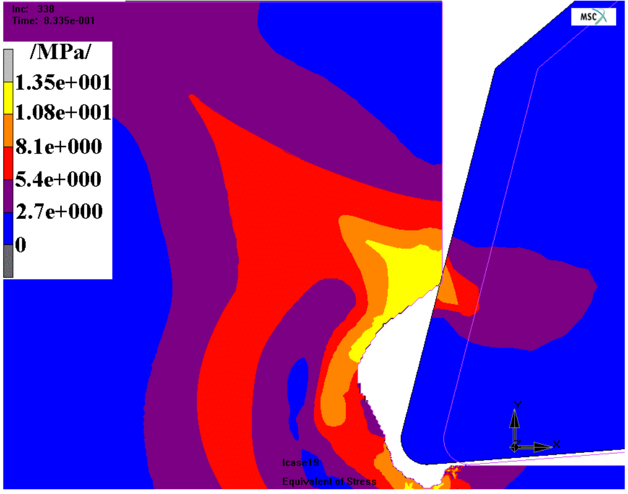


Fig. 11 Disintegration of the ore and movement of the bit (equivalent von Misses stresses distributions, results of one Monte Carlo simulation)





The ore material is elasto-plastic with isotropic hardening rule. The probabilistic inputs, i.e. elastic properties (Modulus of elasticity “E (Pa)” and Poisson's ratio “μ (1)”) and plastic properties (yield stress “R_p (Pa)” and fracture stress “R_m (Pa)”) are described by bounded histograms, see Fig. 12 and 13.

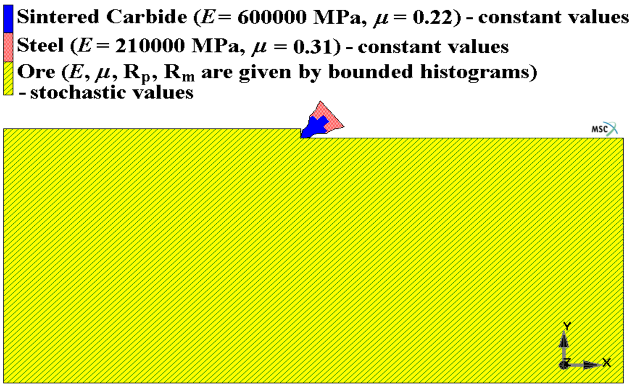


Fig. 12 Material properties (Finite element model)

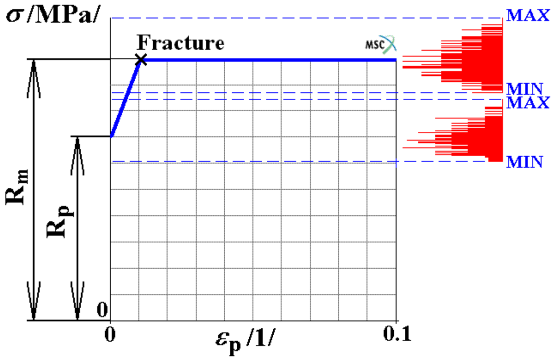
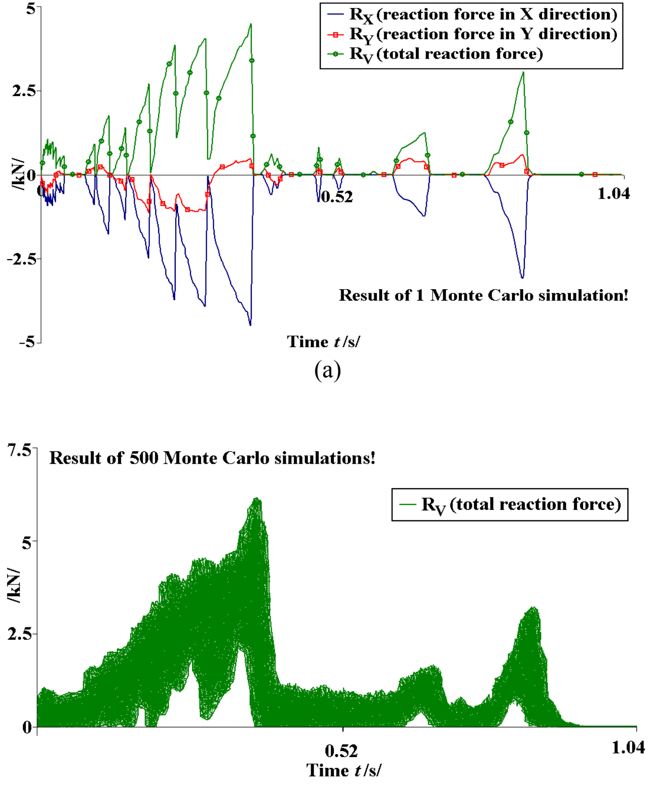


Fig. 13 Material properties of the ore (probabilistic inputs, stress vs. plastic strain)

From the results, the total reaction forces “Rv (N)” can be calculated. These forces act in the bit, see Fig. 14 (distribution of the total reaction forces acquired from 500 Monte Carlo simulations - stochastic result, i.e. print of 500 curves). The calculated maximum forces (i.e. SBRA-FEM solutions) can be compared with the experimental measurements. However, the experimental results also have large variability due to the anisotropic and stochastic properties of the material and due to the large variability of the reaction forces.



(a)

(b)

Fig. 14 Reaction forces in the bit (probabilistic outputs)

Reliability function “RF” (in this case, measured in newtons), see equation (1) and references [1], [10] and [11], can be defined by:

$$\mathrm{RF} = \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}} - \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,FEM}}}\,, \tag{3}$$

where “$\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}}$ (N)” is the allowable reaction force in the cutting bit, which can be acquired from the real capacity of the whole cutter-loader system in the mine and $\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,\ FEM}}} = 5068^{+1098}_{-984}$ N is the maximum total reaction force (acquired from 500 Monte Carlo simulations and solved by FEM). If situations when: $\mathrm{RF} \le 0$ occur, then the cutter-loader system is overloaded. Else if $\mathrm{RF} > 0$, then safe situations of loading occurs.

Hence, fully probabilistic assessment can be calculated by comparing of probabilities, see equation (2):

$$P(\mathrm{RF} \le 0) \le P_{\mathrm{ALLOWABLE}}\,, \tag{4}$$

where, “$P_{\mathrm{ALLOWABLE}}$” is the acceptable probability of overloading of the cutting-loader system. This overloading sometimes really occurs in the mine. Value of “$P_{\mathrm{ALLOWABLE}}$” can be given by chosen performance requirements of the client (i.e. investor), see Fig. 15.

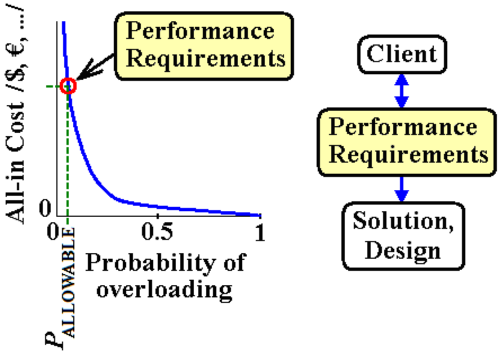


Fig. 15 Definition of the acceptable probability of overloading

All the results presented here were applied for optimizing and redesigning of the cutting bit (excavation tool for platinum ore), see Fig. 16 and 17.

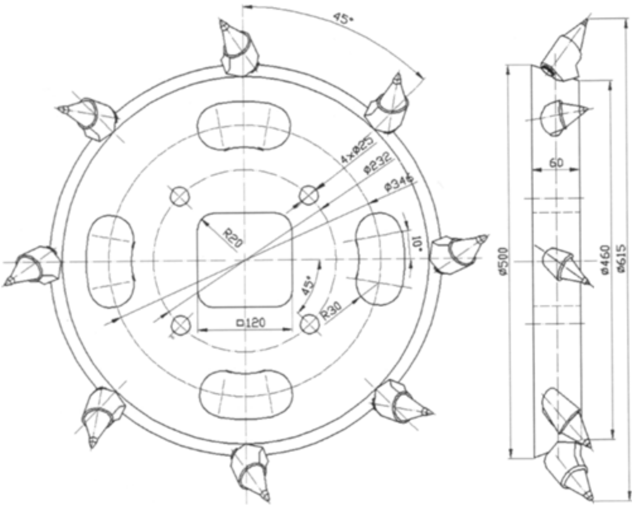


Fig. 16 Final shape of excavation tool for platinum ore disintegration process

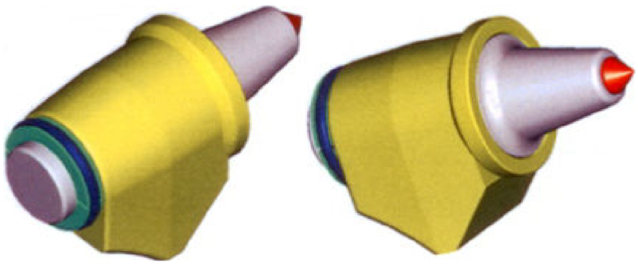


Fig. 17 Final shape of cutting bit for platinum ore disintegration process

For more information see references [1], [2] and [4].

IV. OTHER APPLICATION OF SBRA METHOD

Other examples of the applications of SBRA Method in designing are shown in references [1], [5], [10] and [11], see Fig. 18, 19, 20 and 21. Other probabilistic approaches (i.e. other interesting examples) are presended in reference [8].

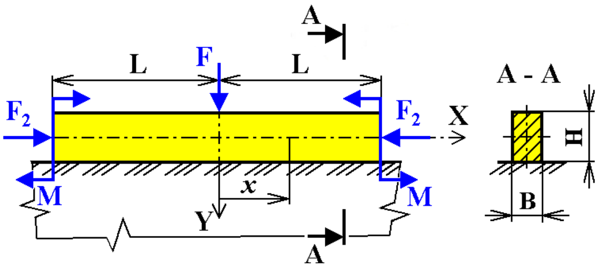


Fig. 18 Beam on Elastic Foundation is exposed to loads (see references [1] and [5])

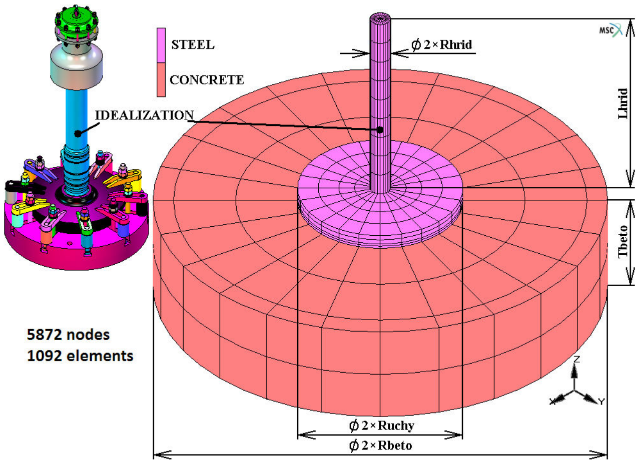


Fig. 19 Finite element model of a massive machine for the dynamic testing of railroad axles (see reference [1])

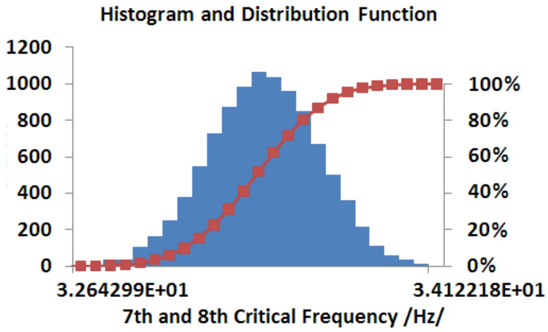


Fig. 19 Histogram of the critical frequency for a massive machine for the dynamic testing of railroad axles (i.e. application of SBRA Method, see reference [1])

Fig. 20 Experiments on a massive machine for the dynamic testing of railroad axles (see reference [1])

V. CONCLUSION

Application of the SBRA Method in the area of biomechanics (design of external fixators in traumatology) and rock mechanics (hard rock disintegration process - design of excavation tool) are reported and other application are mentioned. Application of the SBRA Method (especially with connection with FEM) is a modern and innovative trend in engineering, because this method includes the real variability of the inputs and outputs.

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