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Preliminary Study on Fixture Layout Optimization Using Element Strain Energy

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***Abstract*—**The objective of positioning the fixture elements in the fixture is to make the workpiece stiff, so that geometric errors in the manufacturing process can be reduced. Most of the work for optimal fixture layout used the minimization of the sum of the nodal deflection normal to the surface as objective function. All deflections in other direction have been neglected. We propose a new method for fixture layout optimization in this paper, which uses the element strain energy. The deformations in all the directions have been considered in this way. The objective function in this method is to minimize the sum of square of element strain energy. Strain energy and stiffness are inversely proportional to each other. The optimization problem is solved by the sequential quadratic programming method. Three different kinds of case studies are presented, and results are compared with the method using nodal deflections as objective function to verify the propose method.

***Keywords*—**Fixture layout, optimization, strain energy, quadratic programming.

I. Introduction

FIXTURES are very important component in the manufacturing system. A fixture is used to hold and locate the workpiece in the desired orientation during the manufacturing process. The components that hold and locate the workpiece are called fixture elements. The arrangement of these fixture elements is very important to reduce the errors in manufacturing process. According to Prabhaharan et al. the position of the fixturing elements in the fixture is called fixture layout, and the layout, which minimizes the workpiece deformation is called optimal fixture layout [1].

The most usual optimization methods implemented are mathematical programming approaches, penalty function methods, simulated annealing, genetic algorithm, and ant colony algorithm. The mathematical programming methods can be classified as linear programming (LP), linear & quadratic integer programming (LQP), dynamic programming (DP), goal programming (GP) and sequential quadratic programming (SQP).

Menassa et al. proposed a method to determine the position of the fixture supports in a fixture. The objective function is the minimization of the workpiece deflection at specific points. The deflection is calculated by using the FEA. The method is verified with three numerical examples [2]. Meyet et al. presented a method to synthesize a fixture using dynamic conditions. Linear programming is used to solve the layout optimization. The objective function in this method is to minimize the deflection of the workpiece and this is achieved by using the minimum clamping force [3]. Roy et al. presented a technique based on the qualitative and quantitative reasoning to find the optimal supporting, locating, and clamping positions [4]. Tao et al. presented a computational geometry approach for arbitrarily shaped workpieces. All the possible clamping points are automatically found, and then optimal clamping points are chosen from a feasible clamping region. The method is verified by case studies [5]. Li et al. presented a fixture-workpiece elastic contact model to increase the workpiece location accuracy. Nonlinear programming method is used for solving the problem. The objective is to minimize the rigid body motion of the workpiece [6]. Liao et al. presented a technique for fixture layout optimization subjected to the dynamic conditions. The parameters affecting the fixturing stability are analyzed. These parameters are the clamping force magnitude, the application sequence, and the placement of the fixturing clamps. The deformation of a flexible workpiece under clamping and machining loads is estimated under dynamic conditions [7]. Li et al. presented an approach for fixture layout and clamping force optimization. This approach considers the workpiece dynamics during machining. The objective function of this approach is to minimize the maximum positional error at the machining point during machining. An iterative fixture layout and clamping force optimization algorithm yields the best results that are verified by simulations [8]. Tan et al. described an approach for the modeling, analysis and verification of optimal fixture design. The methods of force closure, optimization and finite element modeling (FEM) are used in this approach [9]. Amaral et al. developed a method to analyze the deformation of the contact area between modular fixture and tool to find the optimum support locations, using finite element analysis (FEA). ANSYS has been used for the analysis and optimization. The locators are placed in 3-2-1 principle. The objective function of this methodology is to minimize the maximum resultant deflection and assessing workpiece stability [10]. Most of the above studies are applied to the rigid bodies and use linear or nonlinear programming methods. The few studies applied to sheet metal parts are given below.

Cai et al. proposed N-2-1 locating principle algorithm for deformable sheet metal parts. It uses the finite element

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analysis and nonlinear programming methods to find the optimal location of the locators. The objective function for the optimization is the sum of the square of the nodal deflections. The total deformation of the sheet metal is minimized in this way. A simulation package OFixDesign is also introduced. The OFixDesign software utilizes three commercial softwares: IDEAS for preprocessing and post processing, MSC Nastran for finite element analysis, and VMCON for sequential quadratic programming [11]. Li et al. proposed the first method to determine the optimal fixture configuration design for sheet metal assembly with laser welding. This is the first method that considers both the number and the location of the locators. A powerful optimization technique using a genetic algorithm is used. A case study is presented to verify the effectiveness of the proposed method [12]. Li et al. developed a fixture configuration methodology based on a new proposed locating scheme for sheet metal laser welding. The case study of automotive assembly is investigated by applying the fixture configuration design method [13]. Cai developed a method for fixture optimization for sheet panel assembly considering welding gun variations. A fixture optimization model is formulated to minimize the assembly dimensional variations under welding gun variations. The method is verified by numerical examples [14].Ma et al. proposed a new method for compliant fixture layout design using a topology optimization method. The objective function is to minimize the overall deformation of the workpiece. Both 2-D and 3-D numerical examples are presented to verify the effectiveness of the proposed approach [15]. Cheng et al. developed a fixture layout method to minimize the assembly variation of Aeronautical Thin-Walled Structures (ATWS). This approach uses a genetic algorithm and ant's algorithm (GAAA) to optimize the fixture layout [16]. Xiong et al. proposed a new fixture layout optimization method N-2-1-1 for flexible aerospace workpiece. The objective function of the optimization algorithm is to minimize the maximum elastic deformation at the machined point [17].

We propose a new method for fixture layout optimization using the element strain energy. The propose method is verified by three different case studies. The results obtained by these case shows that workpiece is more stiff in optimal layout.

II. Fixture Locating Principles

A. 3-2-1Locating Principle for Rigid Bodies

A rigid body is fully constrained with minimum fixture elements by the 3-2-1 locating principle. This principle is the traditional principle for locating the prismatic shaped workpieces. According to this principle, three mutually perpendicular planes on the workpiece can be selected as datum planes. These planes are called primary (XZ), secondary (XY) and tertiary planes (YZ), and three, two, and one locators are required on each plane respectively.

P is the primary plane with fixture element (P1, P2, P3), S is the secondary plane with fixture elements (S1, S2), and T is the tertiary plane with fixture elements (T1) respectively. The six degree of freedom of the rigid body is fully constrained by six fixture elements.

B. N-2-1 Locating Principle for Metal Sheets

The locating principle 3-2-1 constrains the rigid body motion (six degree of freedom). This principle is not valid for metal sheet due to their flexible nature. When a force is applied to the metal sheet, like a drilling force or a resistance spot welding, the sheet deflects in direction normal to its surface.

Cai et al. (1996) proposed N-2-1 principle and showed that this principle is valid for sheet metal parts due to their flexible nature. According to this principle, 2-1 locators are enough to constrain the sheet metal in the secondary and tertiary plane, but 3 locators are not enough to constrain the metal sheet in the primary plane due to its flexible nature. So, the number of locators on the primary plane must be more than or equal to 3. This number of locators depends on the geometry and dimensional specification of the workpiece.

The arrangement of these locators is very important because the success of this principle depends on it. This arrangement can be achieved by fixture layout optimization method.

III. Fixture Layout Optimization

A. Strain Energy

When a force is applied on the body, this force is directly proportional to the deflection within the elastic limit. This is called Hook's law. The area under the force and deflection curve is the work done. This work is stored in the body as strain energy.

The total strain energy of the body can be written as

$$U = \frac{1}{2}\int_{V} \left\{\sigma\right\}^{T} \left\{\varepsilon\right\} dV \tag{1}$$

where U is the strain energy, σ and ε are the stress and strain vectors and V is the total volume of the body.

Consider the 2D element on which different normal stresses σ_{xx} and σ_{yy}, and shear stresses τ_{xy} and τ_{yx} are acting. The strain energy of this 2D element is given by

$$U = \frac{1}{2}\left\{\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right\} dV \tag{2}$$

Stiffness is the resistance of an elastic body to deformation by an applied force. This stiffness and strain energy are inversely proportional to each other. If the body has less strain energy values, then it will be stiffer than a body having high strain energy value.

The element strain energy values will be used as objective function in the fixture layout optimization proposed in this paper.

B. Fixture Layout Optimization Process

The formulation of the fixture layout optimization problem is

$$\text{Minimize} \quad F = \sum_{i=1}^{n} u_i^2 \qquad i = 1, 2, 3 \ldots n \qquad (3)$$

$$\text{Subject to} \quad \begin{aligned} a_j &\le x_j \le b_j \\ c_j &\le y_j \le d_j \\ g_j &\le z_j \le k_j \end{aligned} \qquad j = 1, 2, 3 \ldots m \qquad (4)$$

where,

F: objective function

u: strain energy of finite elements

n: number of finite elements

m: number of locator

a_i, b_i, c_i, d_i, g_j, k_j : limitation of locator in the x, y, z direction

The objective function of the optimization is defined as the sum of the square of the element strain energy. The fixture layout optimization problem is solved by sequential quadratic programming, which is one of the most efficient numerical optimization algorithms. The design variables are the positions of the locators. Since there is no analytical relation between the objective function and the design variables, so the sensitivity information is calculated by the finite difference method. We have selected the very small element for meshing so that locators move only on the mesh node in order to avoid the complexities in the problem.

The fixture layout optimization is shown in the flow chart in Fig.1. First the locating area for each locator is defined depending on the geometry of the workpiece and the manufacturing process requirements. Then the initial fixture layout is defined. A finite element analysis is performed to calculate the deflection and the strain energy of each finite element for this initial fixture layout. Then the optimization problem is solved by sequential quadratic programming method. If $k = 0$, then the process go to the next cycle $k = 1$. The Karush-Kuhn-Tucker (KKT) necessary conditions are checked at the end of each cycle. If the problem satisfies the Karush-Kuhn-Tucker (KKT) necessary condition, the process is terminated, because optimum has been achieved at this stage. If the KKT necessary conditions are not satisfied, then it will go for next cycle by updating the layout. This process continues, until the optimum layout is achieved.

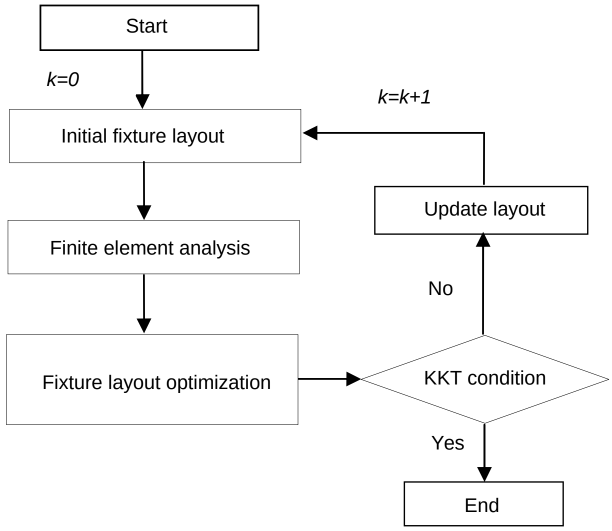


Fig. 1 Flow chart for fixture layout optimization

IV. Fixture Layout Optimization Case Studies

Three different kinds of case studies are presented to verify the propose method. The preprocessing is done in Hypermesh, analysis in MSC NASTRAN, and optimization in Matlab. A sequential quadratic programming method is used as optimization method.

The fixture layout optimization problem may be defined as: finding the position of the locators, so that the stiffness of the workpiece is maximized. This stiffness is achieved in terms of strain energy, because minimization of strain energy is equal to maximization of the stiffness.

The objective function of the propose method is the sum of the square of the strain energy of the finite elements. The previous studies have the objective function as sum of the square of the nodal deflections normal to the surface of the workpiece. The optimal layout of both the methods will be determined, and strain energy values for both optimal layouts will be calculated. The method with the less strain energy value will be preferred because the workpiece in this layout has more stiffness.

It has been verified by FEA that four clamps are enough for each case study. So, the locating principle will be 4-2-1. Three different sub cases are solved for each case study. In case 1, two locators will be used as design locator, and each locator will be moved independently. In case 2, four locators will be used as design variable, and these four locators L4, L6, L5, and L7 will be moved in two pairs. In case 3, four locators are design variable, and these four locators will be moved together.

The material used for optimization is steel having young's modulus of elasticity 207 GPa and Poisson ratio 0.3. These material properties are used for all three case studies. We named the propose method and the previous method as A and B to simply describe the method

A: Sum of square of nodal displacement normal to the surface as objective function

B: Sum of square of the element strain energy values as objective function

A. Case Study 1 – Plate Example

The workpiece used here is a sheet metal plate. The dimensions of the sheet metal are 800 mm x 600 mm x 1 mm. The finite element model of the plate is shown in Fig.2. The finite elements are QUAD4 element with size 10 mm x 10 mm. O (0, 0, 0) is the origin point. A force of magnitude 50 N is applied on the sheet at (300, 200, 0) in the Z-direction.

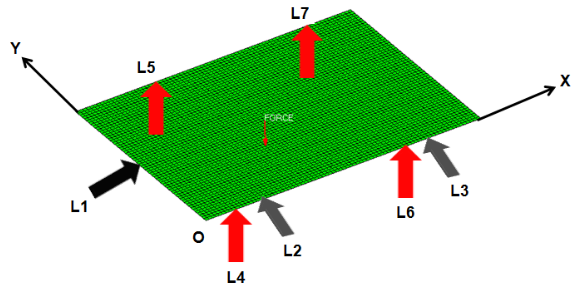


Fig. 2 Finite element model of plate example

TABLE I
INITIAL AND OPTIMAL CLAMPING POSITION OF THE LOCATORS

Cases	Locator type	Parameters		Optimal position (mm)	
		Loc ator	Initial position (mm)	A	B
All Cases	Fixed Locator	L1	(0,300,0)		
		L2	(100,0,0)		
		L3	(700,0,0)		
Case-1	Fixed Locator	L5	(200,600,0)		
		L6	(600,0,0)		
	Design Locator	L4	(0,0,0)	(190,0,0)	(250,0,0)
		L7	(800,600,0)	(530,600,0)	(500,600,0)
Case-2	Design Locator	L4	(0,0,0)	(150,0,0)	(220,0,0)
		L5	(0,600,0)	(150,600,0)	(220,600,0)
		L6	(800,0,0)	(500,0,0)	(500,0,0)
		L7	(800,600,0)	(500,600,0)	(500,600,0)
Case-3	Design Locator	L4	(0,0,0)	(200,0,0)	(240,0,0)
		L5	(0,600,0)	(200,600,0)	(240,600,0)
		L6	(800,0,0)	(600,0,0)	(560,0,0)
		L7	(800,600,0)	(600,600,0)	(560,600,0)

The fixed locators L1 constraint the workpiece in X and the locators L2 and L3 constrain the workpiece in the Y direction, while the other locators L4, L5, L6 and L7 constrain the workpiece in the Z direction. The clamping length of each design locator is 300 mm along the long edge of the workpiece from the corner point of the workpiece. The two locators L4 and L6 move on the edge along the X axis and the other two locators L5 and L7 move on the other edge parallel to the X axis. The initial position of the locators with their optimal position obtained by fixture layout optimization is given in Table I for all cases.

B. Case Study 2 – Spacer Grid

The second case study selected to verify the propose method is the spacer grid. The Z and the X dimensions of the workpiece are 800 mm x 640 mm. The finite element model of the spacer grid is shown in Fig.3. The finite elements are QUAD4 elements. The point O (0,0,0) is the origin. The force of magnitude 200 N is applied to the sheet metal in the Y-direction.

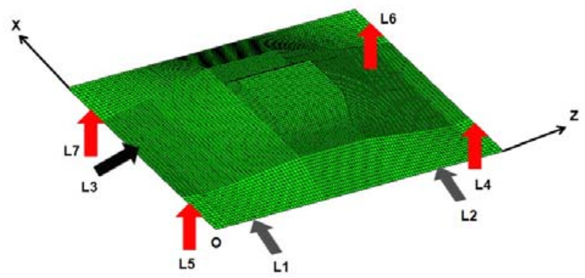


Fig. 3 Finite element model of spacer grid

The fixed locators L1 and L2 constraint the workpiece in X and the locator L3 constrain the workpiece in Z direction, while the other locators L4, L5, L6 and L7 constrain the workpiece in the Y direction. The clamping length of each design locator is 200 mm along the short edge of the workpiece from the corner point of the workpiece. The two locators L5 and L7 move along the edge on the X axis, while the other two locators L4 and L6 move along the edge parallel to the X axis. The initial position of the locators with their optimal position is given in Table II for all cases.

TABLE II
INITIAL AND OPTIMAL CLAMPING POSITION OF THE LOCATORS

Cases	Locator type	Parameters		Optimal position (mm)	
		Loc ator	Initial position (mm)	A	B
All Cases	Fixed Locator	L1	(0, 0,100)		
		L2	(0,0,700)		
		L3	(320,0,0)		
Case-1	Fixed Locator	L5	(105,0,0)		
		L6	(440,0,800)		
	Design Locator	L4	(0,0,800)	(115,0,800)	(160,0,800)
		L7	(640,0,0)	(525,0,0)	(525,0,0)
Case-2	Design Locator	L4	(0,0,800)	(105,0,800)	(155,0,800)
		L5	(0,0,0)	(105,0,0)	(155,0,0)
		L6	(640,0,800)	(525,0,800)	(485,0,800)
		L7	(640,0,0)	(525,0,0)	(485,0,0)
Case-3	Design Locator	L4	(0,0,800)	(115,0,800)	(155,0,800)
		L5	(0,0,0)	(115,0,0)	(155,0,0)
		L6	(640,0,800)	(525,0,800)	(485,0,800)
		L7	(640,0,0)	(525,0,0)	(485,0,0)

C. Case Study 3 – Central Tunnel

The Central tunnel is the third case study to verify the propose method. The finite element model of the central tunnel is shown in Fig.4. The finite elements are QUAD4 and TRIA3 elements. O (0,0,0) is the origin point. The force of magnitude 1000N is applied to the central tunnel in the Z-direction.

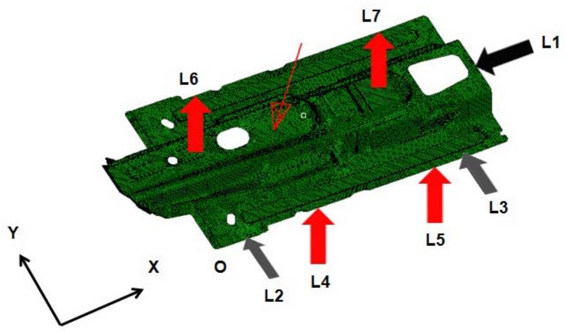


Fig. 4 Finite element model of central tunnel

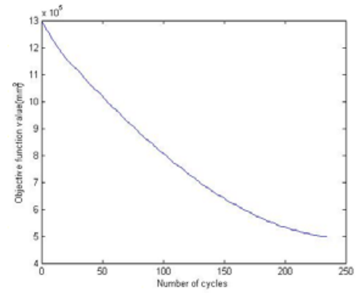
The fixed locator L1 constraint the workpiece in X and the locators L2 and L3 constrain the workpiece in the Y direction, while the other locators L4, L5, L6 and L7 constrain the workpiece in the Z direction. The clamping length of each design locator is 490 mm along the long edge of the workpiece from the corner point of the workpiece. The two locators L4 and L5 move along the edge on X axis, while the other two locators L6 and L7 move along the edge parallel to the X axis. The initial position of the locators with their optimal position is given in Table III for all cases.

TABLE III
INITIAL AND OPTIMAL CLAMPING POSITION OF THE LOCATORS

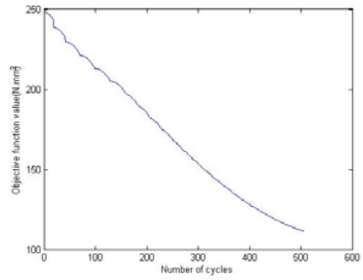
Cases	Locator type	Parameters		Optimal position (mm)	
		Locator	Initial position (mm)	A	B
All Cases	Fixed Locator	L1	(1145,289,0)		
		L2	(71,0,0)		
		L3	(1046,24,0)		
Case-1	Fixed Locator	L5	(897,24,0)		
		L6	(222,579,0)		
	Design Locator	L4	(0,0,0)	(383,24,0)	(122,0,0)
		L7	1115,579,0)	(847,579,0)	(857,579,0)
Case-2	Design Locator	L4	(0,0,0)	(393,24,0)	(122,0,0)
		L5	(1115,24,0)	(984,24,0)	(984,24,0)
		L6	(0,604,0)	(392,579,0)	(122,604,0)
		L7	(1115,579,0)	(984,579,0)	(984,579,0)
Case-3	Design Locator	L4	(0,0,0)	(272,24,0)	(272,24,0)
		L5	(1115,24,0)	(842,24,0)	(842,24,0)
		L6	(0,604,0)	(272,579,0)	(272,579,0)
		L7	(1115,579,0)	(842,579,0)	(842,579,0)

V. OPTIMIZATION RESULTS

The fixture layout optimization is performed as shown in Fig.1. The optimization process is started from the initial layout, and an optimum position is achieved in many cycles. The convergence of the objective function value from initial position to optimum position for both the methods A and B is shown in Fig.5 to Fig.7 for the case-1 of all case studies.

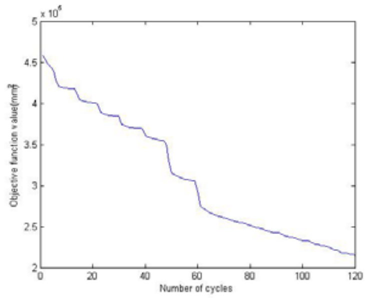


(a) Method A

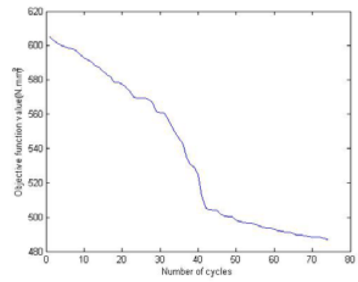


(b) Method B

Fig. 5 Plate example convergence results for Case-1

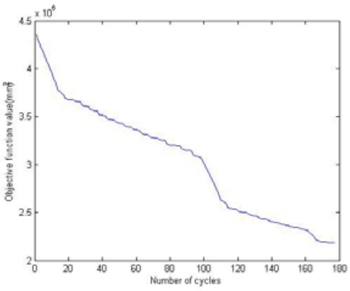


(a) Method A

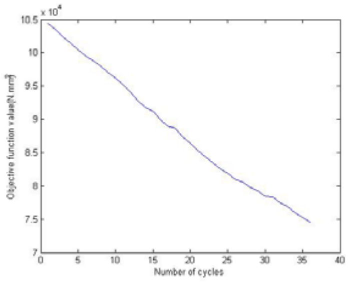


(b) Method B

Fig. 6 Spacer grid convergence results for Case-1



(a) Method A



(b) Method B

Fig. 7 Central tunnel convergence results for Case-1

The fixture layout optimization is performed and different kinds of parameters are calculated like Initial and optimal layout objective function values, number of cycles in which the problem is converged, and the strain energy of the workpiece in the optimal layout are given in Table IV for both the A and B method.

TABLE IV
OPTIMIZATION RESULTS OF THREE CASE STUDIES

Sr. No.	Parameters	Case-1		Case-2		Case-3	
		A	B	A	B	A	B
Case Study 1	F (Initial)	1.29 E +06	2.48 E+02	7.32 E+06	7.29 E+02	7.32 E+06	7.29E +02
	F (Optimum)	4.98 E+05	1.11 E+02	4.50 E+05	1.07 E+02	5.05 E+05	1.10E +02
	Number of cycles	234	506	59	62	21	25
	Strain energy	4.48 E+02	4.33 E+02	4.35 E+02	4.24 E+02	4.46 E+02	4.35E +02
Case Study 2	F (Initial)	4.58 E+05	6.05 E+02	1.59 E+06	2.18 E+03	1.59 E+06	2.18E +03
	F (Optimum)	2.15 E+05	4.87 E+02	1.73 E+05	4.67 E+02	1.73 E+05	4.67E +02
	Number of cycles	120	74	58	82	12	15
	Strain energy	6.77 E+02	6.77 E+02	6.62 E+02	6.24 E+02	6.62 E+02	6.24E +02
Case Study 3	F (Initial)	4.35 E+06	1.04 E+05	6.66 E+06	1.49 E+05	6.66 E+06	1.49E +05
	F (Optimum)	2.18 E+06	7.45 E+04	1.88 E+06	8.32 E+04	2.27 E+06	4.68E +04
	Number of cycles	177	36	53	34	23	20
	Strain energy	6.03 E+03	5.86 E+03	6.50 E+03	6.14 E+03	5.00 E+03	5.00E +03

The optimum value of the objective function F is less than the objective function value of the initial position. The strain energy value of the propose method A is less than the strain energy values of the deflection method B. The strain energy and stiffness are the inversely proportional to each other. The lower value of the strain energy with the method B shows that workpiece is stiffer for the method B as compared to the method A. This reduces the errors in the manufacturing process.

VI. CONCLUSION

The element strain energy concept is introduced for fixture layout optimization especially for metal sheet. Three kinds of case studies are solved to verify the propose fixture layout optimization method. The results obtained by this fixture layout optimization method are compared with the previous available optimization method. The fixture layout optimization using element strain energy results shows that the stiffness of the workpiece in this optimal layout is higher. So, the geometric errors in the workpiece during the manufacturing process are reduced.

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