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# Prediction of Soil Hydraulic Conductivity from Particle-Size Distribution

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***Abstract*****—**Hydraulic conductivity is one parameter important for predicting the movement of water and contaminants dissolved in the water through the soil. The hydraulic conductivity is measured on soil samples in the lab and sometimes tests carried out in the field. The hydraulic conductivity has been related to soil particle diameter by a number of investigators. In this study, 25 set of soil samples with sand texture. The results show approximately success in predicting hydraulic conductivity from particle diameters data. The following relationship obtained from multiple linear regressions on data ($R^2 = 0.52$):

$$K_S = 10.06 + 118.54(d_{10}) - 12.50(d_{50}) - 7.32(d_{60})$$

Where $d_{10}$, $d_{50}$ and $d_{60}$, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and $K_s$, saturated hydraulic conductivity is expressed in m/day. The results of regression analysis showed that $d_{10}$ play a more significant role with respect to $K_s$, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in $K_s$ calculation.

***Keywords*****—**Hydraulic conductivity, particle diameter, particle-size distribution and soil

## I. INTRODUCTION

SATURATED hydraulic conductivity represents the ability of a porous media to transmit water through its voids [2, 13, 15]. Since, direct measurement of hydraulic conductivity is time consuming and costly, indirect methods such as predicting from readily available soil properties e.g. particle-size distribution have been developed [2, 5, 16, 19 & 27]. Many different techniques have been proposed to determine estimate saturated hydraulic conductivity, including field methods, laboratory methods and calculations from empirical formulae [22]. Although in hydromechanics, it would be more useful to characterize the diameters of pores rather than those of the grains, the pore size distribution is very difficult to determine, so that approximation of hydraulic properties are mostly based on the easy-to-measure grain size distribution as a substitute [7]. There have been attempts to estimate saturated hydraulic conductivity based on particle-size distribution *(PSD)* [3, 16, 23, 25, 26, 27].Freeze and Cherry (1979) has long been recognized that hydraulic conductivity is related to the grain-size distribution of granular porous media [9]. Hazen (1982) proposed the following relationship between saturated hydraulic conductivity and soil particle diameter:

$$K_S = c(d_{10})^2 \tag{1}$$

Where $K_s$ is expressed in cm/sec, c is a constant that varies from 1.0 to 1.5, and $d_{10}$ is the soil particle diameter (mm) such that 10% of all soil particles are finer (smaller) by weight [8 & 11]. Shepherd (1989) extended Hazen's research by performing power regression analysis [20].

Also Uma et al. (1989) suggested an equation to estimate the Ks and transmissivity of sandy aquifers of the same form as Hazen Equation [24]. Puckett et al. (1985) sampled six soils at seven different locations in the Alabama lower coastal plain [17], and used regression analysis to determine that percentage of clay sized particles was the best predictor of Ks. Rawls and Brakensiek (1989) used field data across the U.S. to develop a regression equation that relates porosity, and the percentages of sand and clay-sized particles in the sample to $K_s$ [18]. Jabro (1992) estimated $K_s$ from grain-size and bulk density data [12].

Ahuja et al. (1989) estimated Ks using the generalized form of the Kozeny-Carmen equation [1]. Alyamani and Sen (1993) proposed the relationship between saturated hydraulic conductivity and soil particle diameters for 32 sandy soil samples obtained in Saudi Arabia and Australia with the equation [2]:

$$K_s = 1.505[I_o + 0.025(d_{50} - d_{10})]^2 \tag{2}$$

Where $K_s$ is expressed in cm/sec, $I_o$ is the x-intercept of the straight line formed by joining $d_{50}$ and $d_{10}$ of the grain-size distribution curve (mm). $d_{50}$ is the mean grain-size for which 50% of the particles are finer by weight (mm). Sperry and Peirce (1995) developed a linear model to estimate $K_s$ based on grain size, shape, and porosity [21]. [14] sought to improve upon $K_s$ prediction methods by quantifying the characteristics of the pore spaces at a microscopic scale. [8] developed multiple linear regression for southeastern U.S. sandy soils based on regional soil data.

[10] developed a new model to estimate saturated hydraulic conductivity from soil structural properties derived from water retention curve.

[5] reported that considerable success in predicting hydraulic conductivity from PSD data of soils. [13] reported that the lower content of both silt and organic matter and lower values of bulk density had increased $K_s$.

The results showed that the hydraulic conductivities calculated by the USBR and Slitcher methods are in all cases lower than for the other methods [6, 28 & 29]. Hazen formula which is based only on the $d_{10}$ particle size is less accurate than the Kozeny-Carman formula which is based on the entire particle size distribution and particle shape [4 & 29].

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The aim of the study was to determine relationship between saturated hydraulic conductivity and particle-size distribution.

## II. MATERIALS AND METHODS

The 25 sets of soil samples were collected to estimate hydraulic conductivity based on particle-size distribution (PSD). Standard methods were applied to investigate particle size distribution (grain size curve), and finally determine of parameters of $d_{10}$, $d_{50}$ and $d_{60,}$ Where $d_{10}$, $d_{50}$ and $d_{60}$, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight.

Soil texture was classified according to the International Society of Soil Science (ISSS) classification system. The soil texture was sand.

The values of parameters of $d_{10}$, $d_{50}$, $d_{60}$ and saturated hydraulic conductivity are summarized in Table 1. The mean values of parameters of $d_{10}$, $d_{50}$ and $d_{60}$ were 0.253, 0.707 and 0.936 [mm], respectively, also the mean values of saturated hydraulic conductivity was 24.38 (m/day).

In this study saturated hydraulic conductivity was measured by the constant head method. The samples were first wetted by capillarity for 24 hours. This was done from the bottom so that air could escape from the upper surface. The water is then allowed to flow through the soil with maintaining a constant pressure head and saturated hydraulic conductivity was measured when outflow rate becomes constant.

The results were analyzed with SPSS 16.0 and EXCEL software with statistics such as Correlation Coefficient (R), Root Mean Square Error (RMSE), Mean Bias Error (MBE), Mean Absolute Error (MAE), and Relative Error (RE) that calculated using equation (3), (4), (5) and (6) respectively, where $n$ represents the number of instances presented to the model and $O_i$ and $P_i$ represents measured and predicted, and $O_{ave}$ and $P_{ave}$ represents mean values of measured and predicted respectively.

TABLE I
SUMMARIZE OF STATISTICS OF $D_{10}$, $D_{50}$, $D_{60}$ AND SATURATED HYDRAULIC CONDUCTIVITY

| Statistics | PARAMETERS | | | |
|---|---|---|---|---|
| | $d_{10}$ | $d_{50}$ | $d_{60}$ | $K_s$ |
| Mean | 0.253 | 0.707 | 0.936 | 24.38 |
| Minimum | 0.16 | 0.42 | 0.61 | 15.1 |
| Maximum | 0.36 | 1.10 | 1.38 | 36.1 |
| Std. Deviation | 0.061 | 0.185 | 0.248 | 5.96 |
| Skewness | -0.171 | 0.179 | 0.287 | 0.204 |

$d_{10}$, $d_{50}$ and $d_{60}$, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and $K_s$, saturated hydraulic conductivity is expressed in m/day

$$R=\left[\frac{\sum_{i=1}^{n}(O_i-O_{ave})(P_i-P_{ave})}{\sqrt{\sum_{i=1}^{n}(O_i-O_{ave})^2(P_i-P_{ave})^2}}\right]^{1/2} \tag{3}$$

$$RMSE=\left[\frac{\sum_{i=1}^{n}(P_i-O_i)^2}{n}\right]^{1/2} \tag{4}$$

$$MAE\ =\sum_{i=1}^{n}\left[\left|P_i-O_i\right|/n\right] \tag{5}$$

$$RE=(MAE/O_{ave})100 \tag{6}$$

## III. RESULT AND DISCUSSION

The following equations for $Ks$, saturated hydraulic conductivity (m/day), were obtained from multiple regressions on data.

$$K_S=8.91+61.08(d_{10}) \tag{7}$$

$$K_S=16.88+10.60(d_{50}) \tag{8}$$

$$K_S=16.55+8.32(d_{60}) \tag{9}$$

$$K_S=16.16+121.5(d_{10})^2 \tag{10}$$

$$K_S=20.90+6.52(d_{50})^2 \tag{11}$$

$$K_S=20.79+3.84(d_{60})^2 \tag{12}$$

$$K_S=10.14+114.67(d_{10})-20.93(d_{50}) \tag{13}$$

$$K_S=9.80+116.39(d_{10})-15.92(d_{60}) \tag{14}$$

$$K_S=16.68-2.85(d_{50})+10.38(d_{60}) \tag{15}$$

$$K_S=10.06+118.54(d_{10})-12.50(d_{50})-7.32(d_{60}) \tag{16}$$

Where $d_{10}$, $d_{50}$ and $d_{60}$, are the soil particle diameter (mm) that 10%, 50% and 60% of all soil particles are finer (smaller) by weight and $K_s$, saturated hydraulic conductivity is expressed in m/day.

Table II was indicated the various statistics of equations mentioned above.

The results showed as per the table the equation (16) was the best model for predicting $K_s$, saturated hydraulic conductivity (m/day), with 0.719 R, 4.06 RMSE, 3.32 MAE and 13.62 RE. Comparison of observed vs. predicted values of saturated hydraulic conductivity obtained from the equation (16) as a 1:1 scale has been depicted in figure (1) that indicates good match.

TABLE II
SUMMARIZE OF STATISTICS OF VARIOUS EQUATIONS OF HYDRAULIC CONDUCTIVITY

| Equation | STATISTICS PARAMETERS | | | |
|---|---|---|---|---|
| | R | RMSE | MAE | RE |
| 7 | 0.621 | 4.58 | 3.68 | 15.08 |
| 8 | 0.329 | 5.52 | 4.58 | 18.79 |
| 9 | 0.346 | 5.48 | 4.51 | 18.49 |
| 10 | 0.617 | 4.60 | 3.74 | 15.33 |
| 11 | 0.295 | 5.58 | 4.65 | 19.08 |
| 12 | 0.311 | 5.55 | 4.56 | 18.72 |
| 13 | 0.715 | 4.09 | 3.37 | 13.82 |
| 14 | 0.712 | 4.10 | 3.26 | 13.39 |
| 15 | 0.346 | 5.48 | 4.50 | 18.44 |
| 16 | 0.719 | 4.06 | 3.32 | 13.62 |

R is the Correlation Coefficient; RMSE is the Root Mean Square Error; MAE, Mean Absolute Error and RE, is the Relative Error

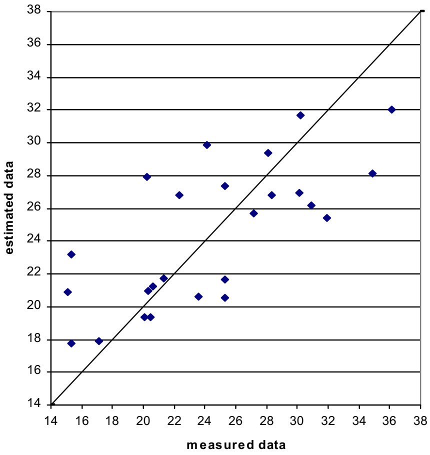

Fig. 1 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (16)

The results showed that among single parameter linear equations (equation 7, 8 and 9) in this study, the equation that predicted $K_s$, saturated hydraulic conductivity (m/day), from $d_{10}$ estimated better (less prediction error) than $d_{50}$ and $d_{60}$ with 0.621 R; 4.58 RMSE; 3.68 MAE; and 15.08 RE and the equation that predicted $K_s$, from $d_{50}$ and $d_{60}$ estimated with larger prediction error and the higher trend is evident between $Ks$ and $d_{10}$. The results of single parameter regression analysis showed that when $d_{10}$, $d_{50}$ and $d_{60}$ increase, $Ks$, saturated hydraulic conductivity (m/day), increases. The results showed that among single parameter quadratic equations (equation 10, 11 and 12) in this study, the equation that predicted $K_s$, saturated hydraulic conductivity (m/day), from $d_{10}$ estimated better (less prediction error) than $d_{50}$ and $d_{60}$ with 0.617 R; 4.60 RMSE; 3.74 MAE; and 15.33 RE. Comparison between linear and quadratic single parameter equations showed $K_s$, saturated hydraulic conductivity predicted from linear equations, estimated rarely better than quadratic single parameter equations.

Also the results showed that among tow parameter linear equations (equation 13, 14 and 15), the equation 13 that predicted $K_s$, from $d_{10}$ and $d_{50}$ (without $d_{60}$) estimated better than other tow parameter equations with 0.715 R; 4.09 RMSE; 3.37 MAE; and 13.82 RE and $K_s$ predicted based on $d_{50}$ and $d_{60}$ (without $d_{10}$) estimated with largest prediction error. Then it is concluded that $d_{10}$ play a more significant role with respect to $K_s$, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in $K_s$ calculation.

Variations between predicted and observed $K_s$ are reported in the literature [2, 5, 12, 16, 18, 23, 24, 25, 26 & 27], and the results showed that when three parameter was used as input of linear equations for predicting $K_s$, estimated $K_s$ better than single and tow parameter equations.

Also the Comparison between observed and predicted data obtained from the equation (7), (8), (9), (10), (11), (12), (13), (14) and (15) have been depicted (Fig. 2-10).

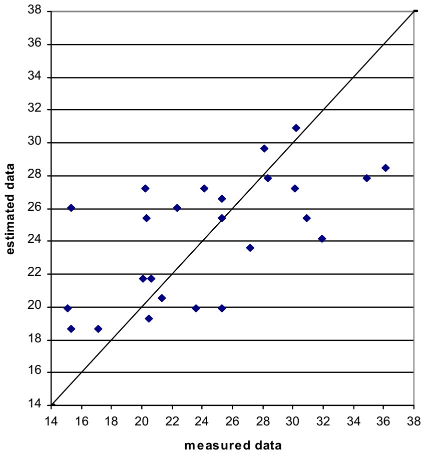

Fig. 2 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (7)

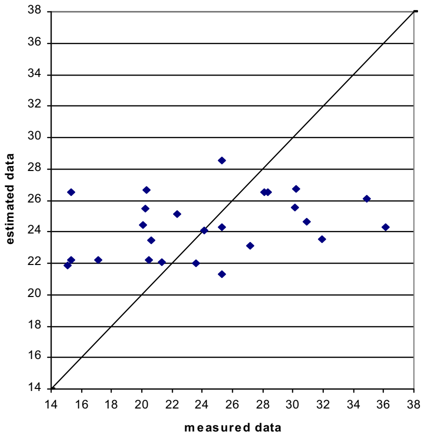

Fig. 3 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (8)

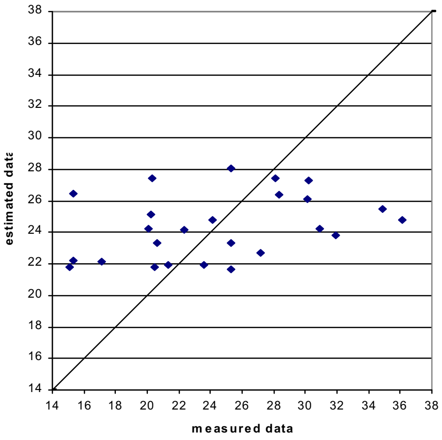

Fig. 4 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (9)

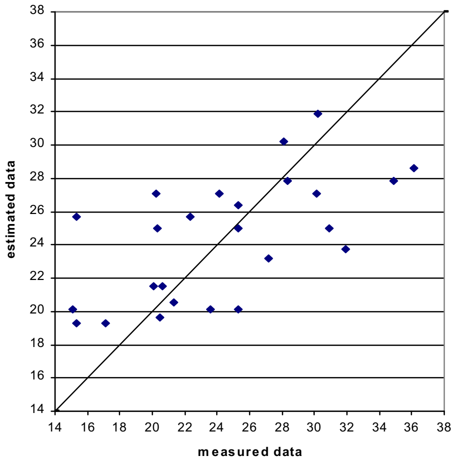

Fig. 5 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (10)

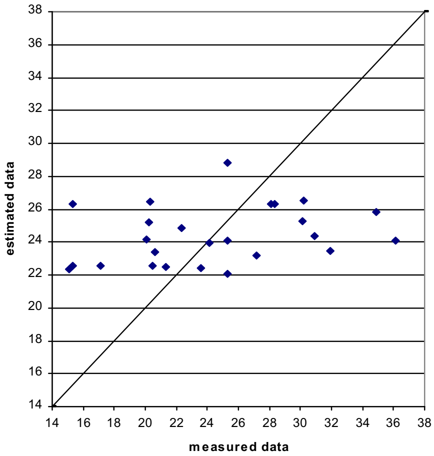

Fig. 6 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (11)

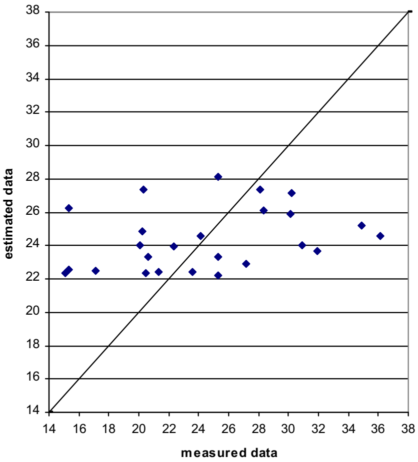

Fig. 7 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (12)

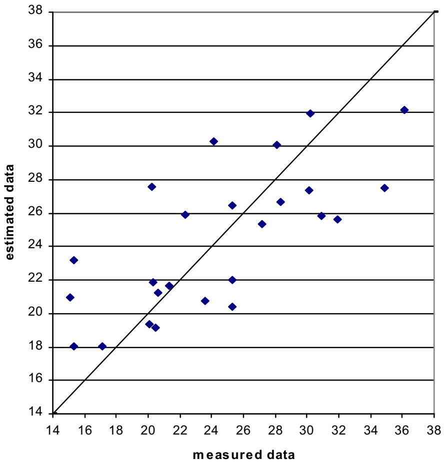

Fig. 8 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (13)

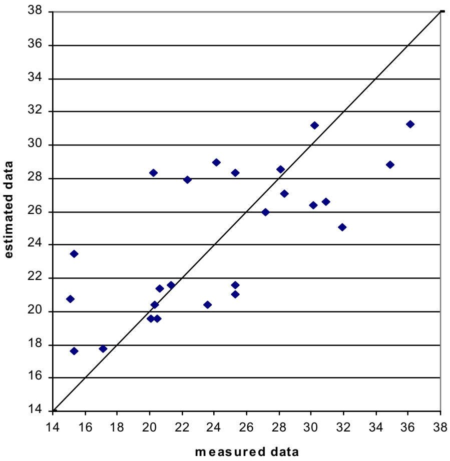

Fig. 9 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (14)

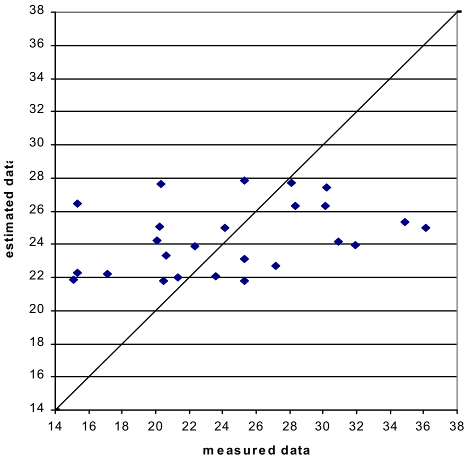

Fig. 10 Comparison of measured saturated hydraulic conductivity, $K_s$ (m/day) and $K_s$ estimated by equation (15)

## IV. CONCLUSION

In this study described equations to estimate Ks, saturated hydraulic conductivity, from $d_{10}$, $d_{50}$ and $d_{60}$ data. The results showed approximately success in predicting hydraulic conductivity from particle diameters data. The results of regression analysis showed that $d_{10}$ play a more significant role with respect to $K_s$, saturated hydraulic conductivity (m/day), and has been named as the effective parameter in $K_s$ calculation. Comparison between linear and quadratic single parameter equations showed $K_s$, saturated hydraulic conductivity predicted from linear equations, estimated rarely better than quadratic single parameter equations.

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