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Predicting Bankruptcy using Tabu Search in the Mauritian Context

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***Abstract*—**Throughout this paper, a relatively new technique, the Tabu search variable selection model, is elaborated showing how it can be efficiently applied within the financial world whenever researchers come across the selection of a subset of variables from a whole set of descriptive variables under analysis. In the field of financial prediction, researchers often have to select a subset of variables from a larger set to solve different type of problems such as corporate bankruptcy prediction, personal bankruptcy prediction, mortgage, credit scoring and the Arbitrage Pricing Model (APM). Consequently, to demonstrate how the method operates and to illustrate its usefulness as well as its superiority compared to other commonly used methods, the Tabu search algorithm for variable selection is compared to two main alternative search procedures namely, the stepwise regression and the maximum R^2 improvement method. The Tabu search is then implemented in finance; where it attempts to predict corporate bankruptcy by selecting the most appropriate financial ratios and thus creating its own prediction score equation. In comparison to other methods, mostly the Altman Z-Score model, the Tabu search model produces a higher success rate in predicting correctly the failure of firms or the continuous running of existing entities.

***Keywords*—**Predicting Bankruptcy, Tabu Search

I. Introduction

BANKRUPTCY is the state of a firm or corporation being unable to repay its debts whereby it legally declares its inability to continue business. The primary purpose of bankruptcy is to either give a honest debtor a “fresh start” by relieving the debtor of most debts or repaying creditors in an orderly manner to the extent that the debtors has means available for payment. One type of bankruptcy is the liquidation bankruptcy whereby the trustee sells off all non-exempt assets held by debtors so that the debts can be repaid to the fullest extent possible. The occurrence of business failure all round the world is in constant rise. Statistics shows that in US itself, there are more than thirty companies which go out of business every week [1]. There are many factors accounting for these closures of firms. However, one of the main reasons is attributed to the combined effect of both rude competition in the market and heavier debt burdens carried by companies.

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The performance of companies is of great concern to the public and especially to stakeholders, both internal and external. Internal stakeholders such as managers and employees are mostly interested with those explicit skills invested in the firm but which may prove difficult to pass on to other enterprises. On the other hand, the external stakeholders namely, customers, suppliers, creditors and investors have quite different views of the firm at times of prosperity compared to situation of financial distress.

Information about the financial state of a firm allows auditors and security analysts to keep track and provide reports over the potential risk of bankruptcy or any other associated financial problems to external stakeholders. In this line, government regulations can be improved as more and more information are obtained in relation as to why individuals, firms and corporations declare bankrupt. Predicting bankruptcy of a company is vital for banks as they are in better position to predict which loans will default or which one will eventually become overdue.

In this paper we have used a Tabu Search selection model in choosing the best subset of variables among a whole set of explanatory data. Other common methods such as the Altman Z-Score model [3], Springate and Fulmer models have also been used to compare the efficiency of the Tabu search method.

This paper is organized as follows: section 2 gives an overview of the different methods used, section 3 we describe the Tabu search algorithm that we have proposed followed by section 4 in which we present our simulation results and discussion. Finally in the last section we conclude our findings on some Mauritian firms, a study which has never been done at the national level.

II. Bankruptcy Prediction Models

Various methods are used in trying to detect bankruptcy giving rise to different bankruptcy models. Three stages exist in the development of financial distress measures: univariate analysis, multivariate analysis, and logit analysis. The univariate model assumes that a single variable can be used for predictive purposes and achieves a moderate level of predictive accuracy meanwhile identifying factors related to financial distress. Yet, it does not provide a measure of relevant risk. In the next stage of financial distress measurement, multivariate analysis also known as multiple discriminant analysis or MDA attempts to overcome the potentially conflicting indications that may result from using single variables [2]. Normally failing firms exhibits

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significantly different ratio measurements than continuing entities. Three important questions arise concerning ratio in detecting failure: which ratios are most important in detecting bankruptcy, what weight should be attached to those selected ratio and how the weight should be established. We shall describe some classical models that have been frequently used in this field.

TABLE I
VARIABLES USED IN APPLICATION OF THE TABU SEARCH PROCEDURE TO PREDICTING CORPORATE BANKRUPTCY. PANEL A: DEFINITIONS OF THE INITIAL SET OF 20 VARIABLES AND SIZE

VARIABLE:	DEFINITION
SIZE:	Market Value of Equity = # Shares Outstanding × Price Per Share,
ALT 1:	Working Capital / Total Assets,
ALT 2:	Retained Earnings / Total Assets,
ALT 3:	Basic Earning Power = EBIT / Total Assets,
ALT 4:	Market Value of Equity / Book Value of Debt,
ALT 5:	Total Assets Turnover = Sales / Total Assets,
CR:	Current Ratio = Current Assets / Current Liabilities,
QR:	Quick Ratio = (Current Assets – Inventories) / Current Liabilities,
INV X:	Inventory Turnover = Sales / Inventories,
DSO:	Days Sales Outstanding = Receivables / (Annual Sales / 360),
FAT:	Fixed Assets Turnover = Sales / Net Fixed Assets,
CAP REG:	Capital Requirement = Operating Capital / Sales,
DEBT:	Debt Ratio = Total Debt / Total assets,
TIE:	Time-Interest Earned = EBIT / Interest Charges,
NOPAT:	Net Operating Profit (Margin) After Taxes = EBIT (1-T) / Sales,
PM:	Profit Margin on Sales = EBIT / Sales,
ROA:	Return on Total Assets = Net Income Available to Shareholders/ Total Asset,
ROE:	Return on Equity = Net Income Available to Shareholders / Common Equity,
PE:	Price Earnings Ratio = Price Per Share / Earrings per Share,
CD OBL:	Current Debt Obligation = Debt Due in One Year / SIZE,
MB:	Market-to-Book Ratio = Market Price per Share / Book Value per Share.
WC	Working Capital
TA	Total Assets
NPBIT	Net Profit before Interest
NPBT	Net Profit before Tax
S	Sales
RE	Retained Earnings
EBIT	Earnings before Interest and Tax
EQ	Equity
CL	Cash flow
ME	Market value of Equity
BVD	Book value of Debt
INT	Interest

A. *Cash Flow Model*

Cash flow model based on financial principles states that the value of a firm is equal to the net present value of its expected future cash flows [1]. If actual cash flow predicts future financial status with great accuracy, then the past and present cash flows should be regarded as a good indicator of both the firm's value and the risk of bankruptcy. Previous research shows that the value of a firm can be written as the sum of the streams of discounted cash flows to and from operations, government, lenders and shareholders. After comparing bankrupt and non-bankrupt firms, it was found that the mean for both operating cash flows and cash taxes paid were significantly different for at least five years prior to bankruptcy. This difference in the operating cash flows between the two types of firms is obvious. Firstly because the quality of investment is better in non-bankrupt firms and secondly, there is a superior operational efficiency in continuous firms. So far, cash flow model are in better position to give early signals of bankruptcy.

B. *Springate [4] and Fulmer [5] Models*

Also known as the Canadian model, the Springate model was developed based on those procedures built up before by Altman [3]. The stepwise multiple discriminate analysis is used to select 4 out of 19 well known financial ratios that mostly differentiates between sound business and those that are actually failing. The model takes up the following form:

$$Z = 1.03A + 3.07B + 0.66C + 0.4D \tag{1}$$

where A=WC/TA, B=NPBIT/CL, C=NPBT/CL, D=S/TA. The abbreviations are defined in Table I. This model achieves an accuracy rate of 92.5% using the 40 companies tested by [4]. Botheras [6] and Sands [7] both tested Springate model on 50 companies and 24 companies respectively. They concluded that for the set of companies under consideration the following results were obtained respectively; with an average asset size of $2.5 million and $63.4 million, the accuracy rate was 88.0% and 83.3% respectively. Firms under the Springate model are classified as failing firms with scores strictly less than 0.862. In relation to the above model, we have the Fulmer [5] model which uses the same type of analysis procedure to evaluate 40 financial ratios applied to a total of 60 companies out of which 30 are failing. The model obtained is:

$$Z = 5.528A + 0.212B + 0.073C + 1.270D - 0.120E + 2.335F + 0.575G + 1.083H + 0.899I - 6.075, \tag{2}$$

where A=RE/TA, B=S/TA, C=EBIT/EQ, D=CF/DT, E=DEBT/TA, F=CL/TA, G=LOG(TA), H=WC/DT, I=LOG(EBIT/INT).

Firms under this model are considered as failing with a score strictly less than zero. However it accounts for an accuracy rate of 98% in classifying companies as bankrupt one year prior to bankruptcy and a rate of 81% for more than one year prior to bankruptcy with an average asset size of $ 455,000 for the 60 companies.

C. *The Z Score model.*

Detecting a company's operating and financial difficulties is a subject which has been particularly subject to ratio analysis. Altman Z-Score, also known as the zeta model, is one of the best known multiple discriminant analysis method for predicting corporate bankruptcy. The method can classify the bankrupt from non bankrupt firms during the first two year prior to bankruptcy better than other. The Z-Score formula for forecasting bankruptcy of Edward Altman is a multivariate method for measurement of the financial health of a company and a powerful diagnostic tool that forecast the probability of a company entering bankruptcy within a period of two years. Altman's Z-Score is the tried and tested formula for bankruptcy prediction. It has been shown to be quite reliable in numerous context and countries. However, it is not designed to be used in every situation; since before using the Z-Score for prediction, it must make sure that the firm being set under examination corresponds to the database. The Z-Score bankruptcy prediction combines five common business ratios using a weighing system. These ratios are Working Capital/Total Assets, Retained Earnings/Total Assets, Earnings before Interest and Tax/Total Assets, Market Value of Equity/Book Value of Debt and Sales/Total Assets.

Altman [3] initially compiled a list of 22 variables mostly financial ratios to be considered and added in the final set of variables. He explicitly uses the following procedures to select his final list of five variables: *Observation of the statistical significance of numerous alternative functions including determination of the relative contributions of each independent variable, evaluation of inter-correlations between the relevant variables, observation of the predictive exactness of the various profiles and judgement of the analyst.* Regardless of the positive results of that model, Altman's method has a key weakness since it assumes that variables from the sample data are normally distributed.

The Z-Score model is broken up into two categories of firm namely, the public manufacturing and private manufacturing firms. Under the public manufacturing group, mostly known as the original Z-Score model, the Z-Score formula is given as:

$$Z = 1.2A + 1.4B + 3.3C + 0.6D + 1.0E, \qquad (3)$$

where A=WC/TA, B=RE/TA, C=EBIT/TA, D=ME/BVD, E=S/TA.

Higher Z-Score value is more desirable. It is noted that for the original model with $Z \geq 3.0$, the company is considered safe with bankruptcy risk being very low while at $Z \leq 1.8$, this implies a very shattering situation. A Z-Score within 1.8 and 3.0 describes a zone of ignorance. The probability of bankruptcy within the following range is as such: ***95% for One Year***, meaning that the firm should be on alert and review its financial situation and ***70% within Two Years*** reflects a high probability of going bankrupt within two years of operation. Concerning the private manufacturing company, the Z-Score formula changes to

$$Z = 0.717A + 0.847B + 3.107C + 0.42D + 0.998E, \qquad (4)$$

with the definition of A,B,C,D and E being similar as for (3), except that now the market value of total liabilities is considered for the fourth term D. One may query why the prediction equation varies for the public and private firms. If a firm's stock is not publicly traded on the market value of equity over book value of debt, then the Z-Score can be calculated using the book values of equity as given in (4). This is considered as model A. Bankruptcy is inexistent for $Z \geq 2.90$ but for $Z \leq 1.23$ it is a strong indicator of a coming business failure. Within $1.23 \leq Z \leq 2.90$ the score is considered as a zone of ignorance. Model B for the Z-Score is mostly appropriate for non-manufacturing firms with the following criteria of consideration. If $Z \leq 1.10$ then there exist a high risk of bankruptcy while with $Z \geq 2.60$ bankruptcy is not likely to occur. Under model B a grey area is spotted for $1.10 \leq Z \leq 2.60$ whereby there exists neither the risk of bankruptcy nor the assurance of a long run continuing business.

III. Tabu Search Procedure

A: Tabu Search Procedure.

The word tabu (or taboo) comes from Tongan, a language of Polynesia, where it was used by the aborigines of Tonga Island to designate things that cannot be touched due to their sacredness. From Webster's dictionary however, the word also means "a prohibition imposed by social custom as a protective measure" or of something "banned as constituting a risk" [8]. The most important link to customary practice uproots from the fact that tabus as normally viewed upon are transmitted by means of a social memory which is modified over time, thus creating the fundamental link to the meaning of "tabu" in tabu search.

Tabu search, in itself, is meta heuristic that guides a local search procedure to explore the solution space beyond local optimality. Next to this meta heuristic attribution, Tabu search is also regarded as being a mathematical optimisation method belonging to the class of local search techniques whereby it enhances the performance of a local search method from the usage of memory structures.

On top of this, Tabu search is based on the grounds that problem solving, in order to be qualified as intelligent, must incorporate adaptive memory as well as responsive exploration. The adaptive memory feature of Tabu search suggests the importance of analysing current substitutes in relation to previous ascents of similar situation, on the other hand, the emphasis on responsive exploration in Tabu search is derived from the supposition that a bad calculated choice can yield more information than a good random selection. The following example of mountain climbing is a good representation of both the adaptive memory and the responsive exploration, whereby the climber must selectively remember (adaptive memory) key elements of the path travelled and must be able to make strategic choices (responsive exploration) along the way [8].

It is noted that the basic form of Tabu search was founded on ideas proposed by Fred Glover [9]. It was primarily developed to avoid replicated searching the same region or cycling between local minima by maintaining a simple map regions, already searched, up to date. Tabu search is different when compared to other improvement type methods in the sense that such search allows the exploitation of inferior results for the eventuality of an optimal answer.

B: The Purpose and Use of Memory in Tabu Search

The term memory is an integral part of any search system that is worth being referred to as "intelligent". Memory itself in Tabu search allows the search system to move forward to an optimal target rather than being stuck with what may be called the final solution. In Tabu search, the memory structure operates with reference to four basic dimensional elements being: *Recency, Frequency, Quality and Influence*. The recency based memory is mainly regarded as a short term memory. The first two elements, recency-based and frequency-based memory tend to complement each other. On the other hand, the frequency based memory being a long term memory provides information that adds to the information given by recency-based memory which consequently enlarges the possibility of selecting more desirable moves. Considering the quality dimension, it refers to the ability of differentiating between the merits of solutions visited during the search. The quality principle at this point tend to induce for reinforced actions that lead to good solutions and penalties to discourage actions that lead to poor solutions [10]. The last principle is that of influence whereby it takes into account the impact of those alternatives that are made during the exploration. Normally memory used in Tabu search is both explicit and attributive. The explicit part records the complete solutions inclusive of elite solutions obtained during the search, whereas the attributive memory is used for guiding purposes.

C: Selection of Variables using Tabu Search Method

Drezner [11] recommended seven general approaches for selecting variables in a regression analysis, yet the all-possible regressions turned out to be the more effective. However, if p independent variables are considered then the number of all-possible regressions equates to 2^p. Representing p by 40 variables, the number of different regressions required to be run would be roughly over one trillion with the computational prerequisite being unaffordable. Even with today's modern computers, it would necessitate lots of time to carry out this task. This is where the Tabu search system comes in to select the best required variables.

Frequently used search techniques such as stepwise or maximum R^2 methods proceed by studying the neighbourhood of a set of variables and additions or removals of variable are done in accordance with enhancements in the significance level of the neighbouring sets. The difference in Tabu search is that the exploration procedure does not limit the search to improving moves only but also allows for moves leading to inferior solutions. By adding a certain variable this may show way to a set whose significance level is inferior when compared to the previous one but from continuous additional changes of variables the significance level may improve. The purpose behind this Tabu status is to prevent rotation.

The main used criterion which assists in the selection of different regressions as always remains the R^2, Under the Tabu search algorithm, the significance level is computed repeatedly, hence the importance of having a competent procedure for the calculation of the significance level value. After getting the value of R^2, the significance level for a general case of p independent variables with n observations, can be calculated by the F probability distribution using the formulas as given

$$F_{p,n-p-1} = \frac{R_p^2}{p} \div \frac{1-R_p^2}{n-p-1} \text{ with } 1-\frac{|R|}{|R_{xp}|} = R_p^2, \quad (5)$$

Neighbourhood denoted by *N(s)* where s being the subset of reference, contains those subsets which include one additional variable, one variable less and subsets with one variable taken out and replaced by another, with a general size equal to *k+p(k-p)* subsets. The *move* concept specifies the variable has been transferred from the present subset to a subset in the neighbourhood either through the addition or removal or swapping process and the *status* concept stating the actual position of the variable. For example the status of a variable $\{x_3\}$ can either be "in" the selected subset or "out" of the selected subset [11].

D: Starting Solution.

The initial solution is acquired by applying an algorithm which is almost similar to the maximum R^2 improvement approach. We refer to such procedure as being a "greedy procedure" since the myopic best improvement is selected at each innovative iteration and such selection may not necessarily be the best move in the long run [11]. We start with an empty set of independent variables making selection of a variable between 1 and k . Each chosen variable is given a specific definition whereby the first chosen variable is variable #1, followed by variable #2 and so on. Then all the subsets in the neighbourhood are tested in a random order. The above procedure is repeated for the number of randomly selected variables whereby the total member of the neighbourhood being *k+p(k-p)* are without any doubt checked and none is done more than once. After creating the whole neighbourhood for the initial subset, the subset which yield the best significance level among others and the present subset are considered and used to replace the current subset. This procedure ends when no subset from the neighbourhood can provide a better significance level. Otherwise the process of creating the neighbourhood is repeated again.

Tabu list, as the word itself specifies, is a list containing all those moves, more precisely, all those variables which are not allowed to be taken out. If the move concerns the swapping of two variables, then both variables are added to the Tabu list. The following represents the algorithm for the addition of variables in the Tabu list. The latter does not record whether a particular variable was added to a particular selected group of independent variable or whether a particular variable was last added or removed in the list. It is emptied whenever a new best solution is obtained.

Algorithm 1: Addition of Variable in the Tabu List.

$tabulist = addlist(tabulist, l)$,

$tabulist = [tabulist, l\ ]$;

where l represents the new variable to be added.

Tabu size, being interrelated with the tabu list, is a list with a a predefined capacity which is not infinite. The Tabu size reflects the length of the Tabu list which whenever exceeded will always eliminate existing variables in the list in a first in first out manner. The FIFO property works as follows, that variable which was added at the very first move is deleted from the Tabu list whenever the Tabu size is exceeded.

Algorithm 2: Deleting Variable from the Tabu List.

$tabulist = delet(tabulist)$,

$[m, n] = size(tabulist)$;

$tabulist = tabulist(:, 2:n)$;

$|R|$ - Determinant of the correlation matrix inclusive of the dependent variable, $|R_{xp}|$ - Determinant of the correlation matrix with the p independent variables only.

Any subset in the neighbourhood is judged ***admissible*** only if none of the variable that is shifted in or out of the subset appears in the Tabu list

The Search Parameter, in general, the length of the Tabu list is a critical parameter in most Tabu search algorithm. A wrong choice of the size may lead to a very inefficient algorithm and consequently affect the result produced. In this paper, the Tabu size was set as lying in between 10% of the neighbourhood size and the number 10, $10\% * (k + p(k - p)) < Tabu\ Size < 10$, leading to a length of 7. Glover [9] recommended that a good Tabu length would be 7, while Anderson [12] proposed that a length within 7 and 15 is most suitable for path assignment problem, meanwhile Lokketangen [13] cited that the most efficient algorithm is that which is inclusive of more than 200 items in the Tabu list [14].

However, Taillard [15] reported that a successful list is one that arbitrarily modifies its length at a certain point of time. Morton and Pentico [16] suggested a list that keeps all solutions and such lists work best for scheduling problems [14]. On the overall, it seems that the best length to choose for the Tabu list depends mainly on the type of problem being considered and the algorithm being used. The ***stopping criterion*** is set to be terminated at 30 successive Tabu search iterations if no new best solution is found. The complete Tabu search algorithm is given below as per the discuss parameters.

Algorithm 3: Tabu Search Algorithm

k= number of independent variable,
g = number of observation,
s = rand(g,k);s(:,p)=randperm(k);q=randperm(b);
Y = rand(g,1)
Accessibleset = setdiff(p,q);
FAset = []; Fremset =[]; Fswapset=[];
For iter = 1:30

(a) Initial = [set consisting of a randomly selected variable between 1 and k],

(b) Calculate the significance level of the set Initial,

$RXP = corrcoef\ (initial)$,
$R = corrcoef\ (initial\ set\ \&\ Y)$
$F1 = ((1 = (\det R / \det RXP) / k) / ((\det R / \det RXP) / (g - k - 1)))$,

(c) Set bestsol = [F1];,

(d) Create the neighbourhood $N(s)$ of set Initial:

Addition of Variable:

$for\ \ i = (size(initial, 2) + 1) : k$, $add = [initial\ \ s(:, p(i))]$,

Removal of Variable:

$for\ \ j = 1 : size(initial, 2)$,

$rem = initial(:, j)$,

Swapping of Variable:

$for\ \ m = (size(initial, 2)) + 1 : size(s, 2)$, $for\ \ h = 1 : size(initial, 2)$,

$swap = [initial2(:, j)\ \ s(:, p(m))]$,

$initial2 = initial1$,

$initial1(:, h) = []$

Repeat step (b) for each new add set, rem set & swap set.

$FA\min$, $Frem\min$, $Fswap\min$: min sig. level,

$FAset$, $Fremset$, $Fswapset$: set with min sig. level

(e) Use algorithm 1 for addition of variable in tabulist

(f) $if\ \ size(tabulist, 2) > 7$, use algorithm 2.2

(g) $F\min = \min[FA\min\ \ Frem\min\ \ Fswap\min\}$

(h) $if\ \ F\min < bestsol$: initial set = *FAset* or *Fremset* or *Fswapset*

$else\ \ initial = best\ accessible\ set$

IV. Simulation Results and Discussions

All relevant data were obtained from the annual report of each bankrupt and non-bankrupt firm at the Registrar of Companies of Mauritius over a period ranging between 1989 and 2006. The number of firms for which data is collected over these 17 years amounts to a total of 30 firms both bankrupt and non-bankrupt. However for reason of confidentiality none of the firms' individual data or their name or the specific years for which they were considered had been disclosed. A general T variable is used to specify the event years prior to bankruptcy instead of the year under consideration itself. That is, $T-1$ represents one year prior to bankruptcy while $T-2$ considers data for two years prior to bankruptcy with respect to the year of reference *T*.

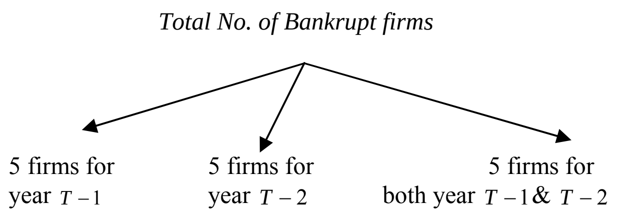

Fig. 1 Illustration of Data Collection for Bankrupt and Non-Bankrupt Firms

Furthermore, those firms considered for appraising the Tabu search method in predicting business failure have been chosen from different sectors of the economy. However the data for financial firms have been excluded because they are not stable. The panel A from Table I below shows the 21 variables for which data were collected for all the 30 firms. The SIZE variable, appearing in the panel, is used to match a non-bankrupt firm with a bankrupt firm that is, a continuing "industrial firm" is strictly compared to a failing "industrial company". Since Mauritius is a small developing island, the number of firms going bankrupt within a year is limited and hence the SIZE variable is not really required for matching firms. Nevertheless it is included only for completeness purpose for those ratios requiring the SIZE value.

Apart from the five variables ratio used for the Altman model, the remaining 15 variables excluding the current debt obligation was considered by Brigham, Gapenski and Ehrhardt [17]. It is noticed that for bankrupt firms a large amount of data are mostly either missing or not available as from the second year prior to bankruptcy. This is considered as a normal situation for failing firms.

The sample data is elaborated as follows. A total number of 15 bankrupt firms and 15 non-bankrupt firms are considered. Any firm with complete data for the first year (event year $T-1$), the second year (event year $T-2$) or the first two years just prior to bankruptcy remains in the sample. If each firm is counted as an observation (or "separate" firm), the total number of observation for the bankrupt firms is 20 with 10 observation for event year $T-1$ and 10 for event year $T-2$ and similarly for non-bankrupt firms. Total number of bankrupt firms in observation is equal to 20 and similarly for non-bankrupt firms.

TABLE II
DESCRIPTIVE STATISTICS: DATA FOR 10 NON-BANKRUPT FIRMS T-1 YEAR PRIOR TO BANKRUPTCY

VAR	Mean	Median	Std Dev	Min	Max
Size	281594 6	391218	745593 7	1482 2	2400944 7
ALT1	3.03	0.05	9.38	0.00	29.73
ALT2	0.41	0.03	1.05	0.00	3.38
ALT3	4.36	0.08	9.76	0.00	29.26
ALT4	8.50	0.32	18.04	0.00	53.31
ALT5	0.95	0.61	1.13	0.08	3.88
CR	1.55	0.89	2.13	0.06	7.41
QR	1.34	0.63	2.06	0.06	6.98
INV X	42.75	18.37	46.35	1.69	129.82
DSO	0.32	0.14	0.57	0.01	1.91
FAT	2.86	0.65	4.80	0.00	13.31
CAP REQ	4.23	0.09	12.04	0.00	38.41
DEBT	2.18	0.09	6.49	0.00	20.65
TIE	912.05	4.06	2758.07	0.00	8759.07
NOPAT	3.99	0.07	11.89	-0.01	37.82
PM	5.65	0.12	12.37	0.02	37.81
ROA	52.72	0.39	140.78	0.06	447.47
ROE	0.54	0.16	1.20	0.00	3.94
PE	1.30	0.59	2.24	-2.31	5.95
CD OBL	40.42	0.29	105.78	0.00	337.43
MB	2.95	0.83	6.10	0.00	20.00

Thus by matching the 15 non-bankrupt firms with the 15 bankrupt firms yields a total of 30 firms in the final sample with 40 observations. The table II shows the data collected for both type of firms – continuing and failing entities, at one time with data from one year prior to bankruptcy and at another from two years to bankruptcy. From a descriptive statistical representation, we notice that the non-bankrupt firms produce more favourable values with the maximum value coming from ratio TIE with 8,759.07 meanwhile the most minimum value from the price earning ratio with -2.31. Yet the bankrupt firms generate results demonstrating the real convergence of the firms towards failure with the mean being zero or negative, the least maximum value not exceeding 400 and the greatest minimum value being -243.49 from the price earnings ratio again.

A: *Prediction Scores and Results*

The Tabu search procedure is used to select a subset of variables among the original set of 20 variables from panel A which best foresee bankruptcy. This procedure is applied twice, once for the whole sample ($T-1$ and $T-2$), then for the $T-2$ event year observation and finally for the $T-1$ event year observations. However only the first two applications will be considered in depth. The second application, that is, for $T-2$ event year observations, is most appropriate since firstly there is an adequate amount of observations $n = 20$ for analysis and secondly the consistency of using the data for one year only does increase predictability as a result of a higher degree of homogeneity among the sample. Thus the reason for not considering the $T-1$ event year observations since it will not contribute much in altering the existing results.

TABLE III
DESCRIPTIVE STATISTICS: DATA FOR 10 BANKRUPT FIRMS T-1 YEAR PRIOR TO BANKRUPTCY

VAR	Mean	Median	Std Dev	Min	Max
SIZE	0.00	0.00	0.00	0.00	0.00
ALT1	0.25	0.00	0.78	0.00	2.47
ALT2	-1.95	-0.01	6.05	-19.16	0.50
ALT3	-0.05	0.00	0.09	-0.22	0.06
ALT4	2.16	1.01	3.03	0.00	9.75
ALT5	-0.05	-0.41	0.78	-1.93	0.74
CR	1.10	0.69	1.71	0.00	5.85
QR	0.18	0.12	0.20	0.00	0.44
INV X	-6.54	0.00	16.19	-52.23	0.00
DSO	-40.96	-8.05	72.18	-171.32	59.84
FAT	-0.78	0.00	1.55	-4.32	1.05
CAP REQ	0.01	0.00	0.03	0.00	0.08
DEBT	0.88	0.38	1.05	0.00	3.14
TIE	-0.85	0.00	2.13	-6.47	0.96
NOPAT	28.52	2.26	105.11	-59.40	322.34
PM	0.05	0.00	0.09	-0.02	0.22
ROA	0.15	0.24	0.91	-2.22	1.00
ROE	0.27	0.35	0.97	-2.22	1.26
PE	-24.43	0.00	76.97	-243.39	0.00
CD OBL	0.00	0.00	0.00	0.00	0.00
MB	0.60	1.00	0.52	0.00	1.00

The 20 original variables considered in the previous panel is set as the independent variables with the data collected for the two category of firms being the number of observations. However, the dependent variable, denoted as y, is a dummy variable which is set to zero for data analogous to a non-bankrupt firm and equal to 1 for a bankrupt firm. For our first application, the Tabu search procedure illustrated earlier selects the following two variables: *ALT5 (Total Assets Turnover) and INV X (Inventory Turnover)*

An ordinary least square (OLS) regression is applied to the two selected variables, as the independent variables, and to the dummy (bankrupt/non-bankrupt) dependent variable. The resulting regression equation is in fact the Tabu-generated prediction score:

$$\text{Tabu Prediction Score} = 0.627 - 0.162\ \text{ALT5} - 0.00652\ \text{INV X} \tag{5}$$

The results following the use of Tabu prediction score along with the Altman Z-Score for public firms, the Springate [4] and Fulmer [5] score equations are given in Table IV and Table V.

TABLE IV
RESULTS OF PREDICTION SCORES FOR T-1 AND T-2 EVENT YEAR FOR NON - BANKRUPT FIRMS

	MEAN	MEDIAN	STD DEV	MIN	MAX
ALTMAN Z SCORE	22.75	8.29	29.97	1.37	72.58
SPRINGATE	27.85	1.21	59.74	0.59	134.72
FULMER	16.98	7.58	16.50	5.91	44.46
TABU	0.18	0.19	0.25	-0.09	0.45

TABLE V
RESULTS OF PREDICTION SCORES FOR T-1 AND T-2 EVENT YEAR FOR BANKRUPT FIRMS

	MEAN	MEDIAN	STD DEV	MIN	MAX
ALTMAN Z SCORE	-5.45	-0.50	15.34	-32.34	4.73
SPRINGATE	-0.71	-0.37	1.06	-1.88	0.26
FULMER	-18.89	4.82	53.43	114.38	7.89
TABU	0.71	0.65	0.18	0.49	0.93

TABLE VI
COMPARISON OF MEANS FOR 20 BANKRUPT AND 20 NON-BANKRUPT FIRMS USING TABU PREDICTION SCORE

	NonBankrupt Firms		Bankrupt Firms
Mean	0.300		0.735
Variance	0.057		0.058
Std Dev	0.238		0.240
N	20		20
E(XY)		63.960	
Sx1x2		0.239	
Pearson Correlation		0.254	
t-test		-5.755	
Mean Difference		-0.435	
P(T≤t) Two Tail tα/2		2.536	

We note that the Springate model produces the greatest maximum value with 134.71 for continuing firms. When compared to the other model, the Tabu score for the operational firms for the five different situations was the lowest. Table VI and Table VII show whether statistical difference exist between the two categories of matched firms and thus report the results of a paired two sample t-test of the difference in means for both the Z- Score and the Tabu prediction score. Both the Z –Scores and the Tabu Scores are significantly different across the bankrupt and non-bankrupt samples. The Pearson value detects the linear dependencies within the two categories of firms producing a no relationship between the bankrupt and non-bankrupt firms. On the other hand, the two tailed t-test is conducted under the hypothesis that $H_0 : \textit{Mean Difference} = 0$ and $H_1 : \textit{Mean Difference} \neq 0$. Under the Tabu score situation, we do not reject H_0 since we have a negative mean difference while under the Altman case H_0 was rejected stating a non-zero mean difference. The test was done for $\alpha/2 = 0.10$ for the Tabu prediction score and $\alpha/2 = 0.05$ for the Z-score.

To determine whether the Tabu selection procedure can outperform the Z-Score model even more, we consider a more homogenous sample of the $T-2$ observations. The following variables were selected for $T-1$ event year - *ALT 3, QR, DSO, Debt* and *PE* whereas for $T-2$, get *ALT 1, ALT 2, INVX, DSO* and *DEBT* ,followed by their respective regression equation

$T-1$:

$$\text{Tabu Prediction Score} = 0.631 + 0.0014\text{Alt3} - 0.55\text{QR} - 0.0018\text{DSC} + 0.157\text{DEBT} - 0.00115\text{PE} \tag{6}$$

T-2

$$\text{Tabu Prediction Score} = 0.641 - 2.02\text{Alt1} - 0.0141\text{Atl3} - 0.00944\text{INV} + 0.000001\text{DSO} + 0.654\text{DEBT} \tag{7}$$

From the Tabu prediction score for T-2 event year (7), the following results are obtained with the Tabu score giving better results.

TABLE VII
COMPARISON OF MEANS FOR 20 BANKRUPT AND 20 NON-BANKRUPT FIRMS USING ALTMAN Z-SCORE

	Non-Bankrupt Firms		Bankrupt Firms
Mean	25.168		-2.776
Variance	2112.46		130.82
Std Dev	45.96		11.44
N	20		20
E(XY)		10.671	
Sx1x2		1	
Pearson Correlation		0.153	
t-test		2.639	
Mean Difference		27.945	
P(T≤t) Two Tail tα/2		1.734	

TABLE VIII
RESULTS OF PREDICTION SCORES FOR T-2 EVENT YEAR FOR NON-BANKRUPT FIRMS

Score	MEAN	MEDIAN	STD DEV	MIN	MAX
ALTMAN Z SCORE	25.67	6.11	51.08	1.10	164.58
SPRINGATE	1.92	0.80	3.18	0.46	10.88
FULMER	38.27	6.75	73.65	5.44	235.94
TABU	0.28	0.37	0.29	-0.24	0.58

TABLE IX
RESULTS OF PREDICTION SCORES FOR T-2 EVENT YEAR FOR BANKRUPT FIRMS

Score	MEAN	MEDIAN	STD DEV	MIN	MAX
ALTMAN Z SCORE	-3.76	0.58	13.35	-40.63	4.02
SPRINGATE	-1.53	-0.42	2.29	-5.64	0.35
FULMER	-10.71	4.62	45.70	-140.38	7.59
TABU	0.72	0.66	0.23	0.31	1.03

We notice again the non-existent relationship between the bankrupt and non-bankrupt firms for both the scores. The t-test reported in Table X and Table XI show again the difference in means across the two categories of firms for both the prediction scores which are statistically considerable.

TABLE X
COMPARISON OF MEANS FOR 10 BANKRUPT AND 10 NON-BANKRUPT FIRMS FOR T-2 EVENT YEAR USING TABU PREDICTION SCORE

	Non-Bankrupt Firms		Bankrupt Firms
Mean	0.276		0.719
Variance	0.084		0.051
Std Deviation	0.289		0.226
N	10		10
E(XY)		-5.630	
S_{x1x2}		0.259	
Pearson Correlation		0.178	
t- test		-3.818	
Mean Difference		-0.443	
$P(T \leq t) - TwoTail \quad t_{\alpha/2}$		2.306	

TABLE XI
COMPARISON OF MEANS FOR 10 BANKRUPT AND 10 NON-BANKRUPT FIRMS FOR T-2 EVENT YEAR USING ALTMAN Z-SCORE

	Non-Bankrupt Firms		Bankrupt Firms
Mean	25.671		-3.763
Variance	2609.068		178.136
Std Deviation	51.079		13.347
N	10		10
E(XY)		23.712	
S_{x1x2}		37.331	
Pearson Correlation		0.176	
t- test		0.353	
Mean Difference		29.434	
$P(T \leq t) - TwoTail \quad t_{\alpha/2}$		1.860	

The critical part remains the predictive power of the Tabu prediction score and the Altman Z-Score. Sensitivity analysis is conducted for the Z-Score using values between 1.80 and 3.00 as explained earlier. Meanwhile the value of 1.81 reflects the cut-off point for the Z-Score, for the Tabu prediction score this shows the optimal value. With a score less than 1.81 under the Tabu prediction score, the firm is set to being bankrupt else for greater values it is non-bankrupt. The Altman Z-Score was applied for both the public* and the private** manufacturing firms using (3) and (4) respectively. For the whole sample, the Tabu prediction score predicted correctly 100% and 80% of the cases versus only 30%, 45% and 60% for both of Altman's Z-Scores as shown in Table XII, Table XIII and Table XIV. The superiority of the Tabu score and method is highlighted even more under application of the T-2 event year sample with again the Tabu score predicting fully 100% and 90% of the cases with the Altman Z-Score predicting only 60% to 70% of the cases.

The Altman Z-Score was applied for both the public* and the private** manufacturing firms as explained in chapter 1 using (3) and (4) respectively. For the whole sample, the Tabu Prediction score predicted correctly 100% and 80% of the cases versus only 30%, 45% and 60% for both of Altman's Z-scores. The superiority of the Tabu score and method is highlighted even more under application of the T-2 event year sample with again the Tabu score predicting fully 100% and 90% of the cases with the Altman Z-score predicting only 60% to 70% of the cases.

TABLE XII
PREDICTIVE POWER OF THE ALTMAN Z-SCORE* FOR THE COMPLETE SAMPLE V/S ALTMAN'S Z-SCORES

	No. of Firms	Success Rate
Total	40	40
Right Prediction:		
Non-Bankrupt Firms	10	50.0%
Bankrupt Firms	12	60.0%
Total Correct Prediction	22	55.0%
Wrong Prediction:		
Non-Bankrupt Firms	10	50.0%
Bankrupt Firms	4	20.0%
Total Incorrect Prediction	14	35.0%

TABLE XIII
PREDICTIVE POWER OF THE ALTMAN Z-SCORE** FOR THE COMPLETE SAMPLE V/S ALTMAN'S Z-SCORES

	No. of Firms	Success Rate
Total	40	40
Right Prediction:		
Non-Bankrupt Firms	9	45.0%
Bankrupt Firms	13	65.0%
Total Correct Prediction	22	55.0%
Wrong Prediction:		
Non-Bankrupt Firms	7	35.0%
Bankrupt Firms	2	10.0%
Total Incorrect Prediction	9	22.5%

The wrong prediction cases are quite high for the Altman's scores under both analyses. The Tabu prediction score predicts less and less incorrect cases to such extent that in some cases, mostly bankrupt firms, the Tabu prediction score has no wrong prediction. Under both aspects, the Tabu prediction score has a higher success rate than the Altman score. A difference of success rate which is worth considering when attempting to predict corporate or personal bankruptcy and choosing between these two prediction scores.

TABLE XIV
PREDICTIVE POWER OF THE TABU PREDICTION SCORE FOR THE COMPLETE SAMPLE

	No. of Firms	Success Rate
Total	40	40
Right Prediction:		
Non-Bankrupt Firms	16	80.0%
Bankrupt Firms	20	100.0%
Total Correct Prediction	36	90.0%
Wrong Prediction:		
Non-Bankrupt Firms	4	20.0%
Bankrupt Firms	NONE	0.0%
Total Incorrect Prediction	4	10.0%

TABLE XV
PREDICTIVE POWER OF THE ALTMAN'S Z-SCORES* FOR T-2 EVENT YEAR

	No. of Firms	Success Rate
Total	20	20
Right Prediction:		
Non-Bankrupt Firms	6	60.0%
Bankrupt Firms	7	70.0%
Total Correct Prediction	13	65.0%
Wrong Prediction:		
Non-Bankrupt Firms	4	40.0%
Bankrupt Firms	2	20.0%
Total Incorrect Prediction	6	30.0%

TABLE XVI
PREDICTIVE POWER OF THE ALTMAN'S Z-SCORES** FOR T-2 EVENT YEAR

	No. of Firms	Success Rate
Total	20	20
Right Prediction:		
Non-Bankrupt Firms	5	50.0%
Bankrupt Firms	7	70.0%
Total Correct Prediction	12	60.0%
Wrong Prediction:		
Non-Bankrupt Firms	2	20.0%
Bankrupt Firms	1	10.0%
Total Incorrect Prediction	3	15.0%

TABLE XVII
PREDICTIVE POWER OF THE TABU PREDICTION SCORE FOR T-2 EVENT YEAR

	No. of Firms	Success Rate
Total	20	20
Right Prediction:		
Non-Bankrupt Firms	9	90.0%
Bankrupt Firms	10	100.0%
Total Correct Prediction	19	95.0%
Wrong Prediction:		
Non-Bankrupt Firms	1	10.0%
Bankrupt Firms	-	0.0%
Total Incorrect Prediction	1	10.0%

V. Conclusions

We have shown how the Tabu search procedure can be applied with promising outcomes to financial problems. Tabu search when compared to the stepwise regression and the Maximum R^2 improvement procedure reaches the exact optimal set of variables under all conditions and at a faster rate. The method confirmed its superiority it always selected the optimal set of explanatory variables more frequently than other procedures in an efficient manner. Moreover, the proposed Tabu search method can be readily applied to problems in finance both for the selection of variables for the APT and also for the selection of variables for scoring and bankruptcy models in corporate, personal and real estate finance. Our results showed that based on the comparison made between bankrupt and non-bankrupt firms at two different periods prior to bankruptcy, between the Altman Z-score and the Tabu prediction score, the Tabu search method to be better. Results indicated that the Tabu prediction has a greater superiority over the Altman Z-score with up to 100% correct prediction for the bankrupt and non bankrupt cases in the Mauritian milieu.

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References

[1] C.E. Mossman, G.G. Bell, L.M. Swartz, and H.Turtle, "An Empirical Comparison of Bankruptcy Models," *The Financial Review*, Vol. 33, pp. 35-54.1998

[2] B.N. Gibson, "Bankruptcy Prediction: The Hidden Impact of Derivatives", 1998. Available http://www.trinity.edu/rjensen/acct/5341/1998sp/gidson/bankrupt.htm [May 2008].

[3] E. Altman, "Financial Ratios, Discriminant Analysis and The Prediction of Corporate Bankruptcy," *Journal of Finance*. Vol. 23 No. 4, pp. 589-609. 1968

[4] G.L.V. Springate, "Predicting the Possibility of Failure in a Canadian Firm," M.B.A. Research Project, Simon Fraser University, January 1978. Unpublished.

[5] J.G. Jr. Fulmer, J.E. Moon, T. A.Gavin, , M.J. Erwin, “A Bankruptcy Classification Model for Small Firms,” *Journal of commercial Bank Lending*, pp. 25-37.1984
[6] D.A. Botheras, (1979), “Use of A Business Failure Prediction Model for Evaluating Potential and Existing Credit Risk,” M.B.A. Research Project, Simon Fraser University, March 1979. Unpublished.
[7] E.G. Sands, “Business Failure Prediction and the Efficient Market Hypothesis,” M.B.A. Research Project, Simon Fraser University, November 1980. Unpublished
[8] F. Glover, and M. Laguna, “Tabu Search,” Kluwer Academic Publishers, Boston. 1997.
[9] F. Glover, “Future Paths for Integer Programming and Links to Artificial Intelligence,” *Computers and Operations Research*, Vol. 13, pp. 533-549. 1986
[10] F. Glover, and M. Laguna, “Tabu Search,” 1997 Available *http://www.dei.unipd.it/~fisch/ricop/tabu_search_glover_laguna.pdf* [May 2008].
[11] Z. Drezner, G.A. Marcoulides, G.A and S. Salhi, “Tabu Search Model Selection in Multiple Regression Analysis,” *Communications of Statistics*, Vol. 28 No. 2, pp.349-367. 1999
[12] C. A. Anderson, K. Fraughnaugh, M. Parker, and J. Ryan, , “Path Assignment for Call Routing: An Application of Tabu Search,” *Annals of Operations Research*, Vol. **41**, pp.301-312. 1993
[13] A. LØkketangen, “Tabu Search as a Metaheuristic Guide for Combinatorial Optimisation Problems,” *Dr. Scient. Thesis*, Institute for Informatics, University of Begen, Bergen, Norway. 1995
[14] A. Thesen, “Design and Evaluation of Tabu Search Algorithm for Multiprocessor Scheduling,” *Journal of Heuristics*, Vol. **4**, pp. 141-160. 1998
[15] E.D. Taillard, “Parallel Taboo Search Techniques for the Job Shop Scheduling Problem,” *ORSA Journal on Computing*, Vol. 6 No. 2, pp. 108-117. 1994
[16] T. E. Morton, and D. W. Pentico, “Heuristic Scheduling Systems,” New York, NY: John Wiley & Sons. 1993
[17] E. F. Brigham, L.C. Gapenski, and M.C. Ehrhardt, “Financial Management – Theory and Practice,” Nine Edition, The Dryden Press, New York. 1999
[18] Z. Drezner, G.A. Marcoulides, and M.H Stohs, “Financial Applications of a Tabu Search Variable Selection Method,” *Journal of applied Mathematics and Decisions Sciences*, Vol. 5 No. 4, pp. 215-234. 2001
[19] G. J. Eidleman, , “A Guide to Failure Prediction,” *The CPA Journal*, February. 1995 Available *http://www.nysscpa.org/cpajournal/old/16641866.htm* [May 2008].
[20] R.J. Freund, and R.C. Littell, R.C. “SAS System for Regression,” Third Edition, SAS Publishing. 2000. Available from:http://www.books.google.mu/books?isbn=1580257259 [May 2008].
[21] M. Gendreau, M. “An Introduction to Tabu Search,” (2002), Available *http://www.ifi.uio.no/infheur/bakgrunn/Intro_to_TS_Gendreau.htm* [May 2008].
[22] R. Roll, “ R^2 ,” *Journal of Finance*, Vol. 43, pp. 541-566. 1988
[23] L. Shilton, and J. Teall, (1994), “Option-Based Prediction of Commercial Mortgage Defaults,” *The Journal of Real Estate Research*, Vol. 9, pp. 219-236. 1994
[24] J. D. Thayer, “Stepwise Regression as an Exploratory Data Analysis Procedure,” 2002. Available: *http://www.eric.ed.gov/ERICWebPortal/recordDetail?accno=ED393890* [May 2008].