Portfolio Management: A Fuzzy Set Based Approach to Monitoring Size to Maximize Return and Minimize Risk

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Abstract— Fuzzy logic can be used when knowledge is incomplete or when ambiguity of data exists. The purpose of this paper is to propose a proactive fuzzy set- based model for reacting to the risk inherent in investment activities relative to a complete view of portfolio management. Fuzzy rules are given where, depending on the antecedents, the portfolio size may be slightly or significantly decreased or increased. The decision maker considers acceptable bounds on the proportion of acceptable risk and return. The Fuzzy Controller model allows learning to be achieved as 1) the firing strength of each rule is measured, 2) fuzzy output allows rules to be updated, and 3) new actions are recommended as the system continues to loop. An extension is given to the fuzzy controller that evaluates potential financial loss before adjusting the portfolio. An application is presented that illustrates the algorithm and extension developed in the paper.

Keywords— Portfolio Management, Financial Market Monitoring, Fuzzy Controller, Fuzzy Logic,

I. INTRODUCTION

RISK assessment models and strategies to control risks suggested in the literature do not take into account ambiguity. In recent models authors have relied on probability theory to present their solution to the problem of risk assessment. However, it has been long established that probability models do not address the problem of ambiguity [1].

Ellsberg   [2] was one of the earliest authors to demonstrate that the fundamental rules governing probability are rendered invalid if the likelihood of events cannot be expressed in binary terms. A number of subsequent empirical studies (for example, [3], [4], [5], [6])   have confirmed Ellsberg's paradox.

Since the assessment of risks in the course of portfolio selection cannot be expressed in quantifiable terms, experts have to often rely on verbal expressions. Such verbal expressions entail both uncertainty and ambiguity. Uncertainty is caused by the fact that the threats are

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prospective in nature and the possibility of their occurrence cannot be foreseen in precise terms. Ambiguity in responses to questions asked is attributable to the lack of precision in verbal descriptions, such as "high" 'moderate" or "low". In view of the fact that traditional two-valued logic of probability judgments is inadequate to handle the combined presence of uncertainty and ambiguity, the calculus of fuzzy sets is utilized in the proposed model to allow effective and systematic handling of imprecise and imperfect information.

A fuzzy set controller is discussed in Section III after presentation of the background on financial risk assessment in portfolio management with its primary reliance on non-fuzzy set based techniques and then a description of fuzzy logic fundamentals including fuzzy controllers in Section   II. Application is made to an investment situation within the typical limits of 99% in Section IV. Section V presents conclusions and benefits of this fuzzy controller approach.

II. BACKGROUND

A selection process for a stock portfolio was developed by Markowitz ([7], [8]). The seminal work by Markowitz [7] proposes a vector x of asset weights which sum to 1 be chosen such that linear combinations $\mu_p = x'\mu$ of the expected asset returns μ (expected excess returns) which represent the expected return on a portfolio, maximized for a specified level of "risk", σ_p, (the standard deviation of the portfolio). The efficient frontier is the curve of (μ_p, σ_p) traced by portfolios whose return/risk tradeoff is optimal in this sense [9].

Continuous time, mean-variance portfolio selection has been studied using a stochastic linear-quadratic (LQ) optimal control and backward stochastic differential equations (BSDEs). The LQ problem was reduced to solving a stochastic Riccati equation which was a fully nonlinear and singular BSDE with random coefficients. It was demonstrated that the efficient frontier in the mean-standard deviation diagram was a straight line and as such risk-free investment is a distinct possibility even when the interest rate is random [10].

A. Basics of Financial Risk:

As early as 1971, the estimation of risk in portfolio selection models was recognized to be problematic since the true values of the parameters of the distribution of returns are

not known with certainty but must be estimated from both objective and subjective information. A model was developed that considered the decision maker's lack of perfect information about the model's parameters, but Bayesian methods were assumed for risk measurement and, a normal linear regression structure was employed [11].

Almost three decades later, a decision method was utilized to determine an asset liability management model for casualty insurers based on a stochastic, method requiring scenario aggregation and ex ante decision rules that allow for multiple scenarios. Again, these findings suggest that the insurance companies would prefer accurate representations of uncertainties [12] which are lacking in stochastic applications. Yet, stochastic methods appear prevalent in the literature to address uncertainty and ambiguous information inherent in portfolio management having to do with risk and return.

Robust optimization is a field that does seek to address uncertainty in parameter estimation [13]. Utilizing this theory, a model was proposed that considered the total deviation of the realized expected returns from their nominal estimates to be less than or equal to some robustness budget. The magnitude of which stems from one's tolerance for total estimation error, protecting the investor against movements in expected returns up to the designated amount of the original. A norm can be set to measure distance of the deviation by either linear or non-linear methods [14].

Still, while emphasis on the mitigation of estimation and model risk in portfolio management has grown in importance and quantitative techniques have become commonplace in the investment industry, robust portfolio optimization remains typically rooted in statistical estimation methods [15] . In the robust portfolio optimization problem, the classical formulations of Value-at-Risk (VaR) can be extremely sensitive to errors in the mean and covariance matrix of the returns. Even with perfect knowledge of the distribution, the computation of VaR amounts is computationally cumbersome and not easily resolved by numerical techniques [16].

VaR is defined as the maximum loss on the portfolio where confidence levels are set between 95% to 99% and the time horizon is between one and ten days. Again incomplete knowledge is problematic since the actual profit and loss distribution can only be inferred not known with certainty. The assumption of a normal distribution has been utilized frequently but is often historically inaccurate. A bounded approach was proposed by Luciano and Marena [17] for its computational ease in situations where risk evaluation has to be performed quickly. The lower bound is thus interpreted as the worse-case scenario at a given confidence level when only marginal quantiles are known and uncertainty is evident.

Risk has frequently been modeled by utility theory. Utility implications from Markowitz's theory that the investor chooses a portfolio solely on expected value and variance (assumed to be the risk) have, therefore, been studied extensively. The optimal portfolio selection problem under Knightian uncertainty considers the decision maker's portfolio consisting of one risky and one risk-free asset. Expected utility is used to derive bounds on the no-transaction region for both optimistic and pessimistic decision makers resulting

in a closed interval for the standard expected utility but not necessary closed for the Choquet expected utility [18]. Thus, types of standard utility functions can impact the transaction area in both risk and risk-free scenarios.

Sharpe [19] was one of the first to propose a simplified method for the efficient set of portfolios under the assumptions of regression theory. The maximum Sharpe ratio determines the portfolio with the highest return/risk tradeoff achievable from the assets. Recently, therefore, an approach estimating mean excess returns and their covariance matrix has been developed to determine the maximum Sharpe matrix [9].

Since market and credit risks are generally modeled simultaneously, separate risk factors associated with pure market portfolio or pure credit portfolio models, have been tested by both linear and non-linear models [20]. Linear programming has been used to simultaneously minimize general risk or dispersion measures by first transforming the general risk minimization problem to a minimax problem , setting up necessary and sufficient conditions and solving the dual problem for both optimal portfolios and their sensitivities [21]. A minimax type risk function was also studied by Teo and Yang [22] in which the investor attempts to restrict the standard deviation for each of the available stocks. Again, the portfolio optimization problem is formulated as a linear programming problem based on capital asset pricing between the market portfolio and each individual return using nonsmooth methods. A conic programming has been used in a worst case Value-at-Risk (VaR) and robust portfolio optimization problem [16].

The above examples relate exclusively to the problems inherent in uncertainty in portfolio management yet each uses stochastic approaches, simulation or linear programming/regression to address the problems. Indeed. uncertainty utilizing fuzzy sets is underrepresented in the literature on portfolio management. Duval and Featherstone [23] developed a fuzzy logic alternative to the traditional mean variance model and compromise programming approach. The model's application was focused on the agribusiness industry. Thus, while it had results that suggested investments in publically traded food and agribusiness stocks allow farmers to capture additional benefits beyond market diversification options, its focus was limited. Importantly, given the previous reliance on stochastic approaches, minimax models, linear programming, and/or simulation models, the use of fuzzy logic provided realistic results with an innovative approach to address uncertainty in decision making.

More in line with the focus of portfolio management, a fuzzy logic controller was posited by Khoshnevisan et al. [24] to minimize cumulative hedging error based on the Black-Scholes [25] options pricing functional form. This model formulated a risk-free scenario such that the minimum return was equal to the risk-free rate i and expected present value of the terminal option payoff seeks the maximum of the wealth invested or the expected return on the risk-free asset over the investment horizon. The selected tracking error function utilized utility functions where the replicating portfolio value at time t is linear. The model then considers the best performer to be least risky. Based on a simple hGA model, a fuzzy logic

controller (FLC) process was proposed as the ideal interface between man and machine allowing prioritization between exploitation and exploration for long term options. The FLC was utilized such that at each periodic rebalancing point, k percent of funds are allocated to the best-performing risk asset with (90-k) percent to the other risky asset to keep the portfolio self-balancing after the initial investment. A rule system determined that k would be raised or lowered slightly if the square of the risk differential is small. The decision process is thus considered over time and an interpretation of risk.

B. Basics of Fuzzy Logic for Risk

Fuzzy logic ([26], [27]) can be used to make decisions, where a fuzzy subset A of a set X is a function of X into [0,1]. We can write $A = \Sigma\ \alpha_i/x_i$ to mean that the value of the function A on x_i is α_i. The number α_i $(0\leq\alpha_i\leq1)$ denotes the degree of membership of x_i in A. Ordinary sets can be viewed in this way where $\alpha_i = 0$ or $\alpha_i = 1$. [For elementary operations on fuzzy sets, see Dubois and Prade [27].]

Of primary importance to this work is Zebda's [28] definition of fuzzy probabilities as:

$$Q_{ijk} = \Sigma\ \alpha_{ijk}\ /\ a_k \qquad (1)$$

where, if at time t the system is at state i and receiving input j, it goes to state k at time t+1 with fuzzy probability Q_{ijk}. Corresponding fuzzy benefits are defined by fuzzy sets B_{ijk} where

$$B_{ijk} = \Sigma\ \text{\ss}_{ijk}\ /\ b_k' \qquad (2)$$

Then using the extension principle ([27]), the averaged benefit is defined by:

$$E\ (B_{ij}) = \Sigma\ c_{ijl}\ /\ b_l \text{ where} \qquad (3)$$

$$c_{ijl} = \underset{(a_1,...a_p,b_1',...b_p')\varepsilon\ f^{-1}(b_l)}{\text{Max}} \quad \underset{k}{\text{Min}}\ \{\alpha_{ijk}, \text{\ss}_{ijk}\} \qquad (4)$$

$$\text{where } b_l = \begin{cases} \Sigma_k\ a_x\ b_x\ \text{ if } \Sigma_k\ a_x = 1 \\ 0 \text{ otherwise} \end{cases}$$

Here, $f(a_1, ...,a_p,b_1',...,b_p') = \Sigma\ a_x b_x'$ (5)

The preceding concepts are needed for the expected loss functions to be incorporated into the fuzzy controller as the cost is considered important to the quality program.

Fuzzy Controllers

A fuzzy controller system can be thought of as a variation of the typical expert system in which rules and available facts are used to draw a conclusion. The diagram of a general fuzzy controller is shown below:
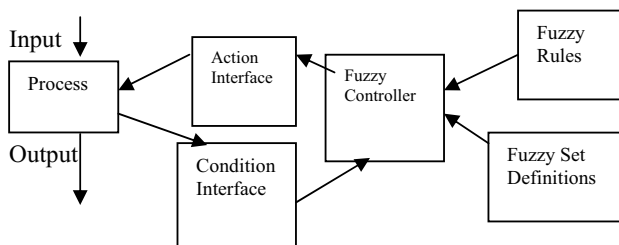

Figure 1. Fuzzy Controller

As diagrammed, the process module receives input in the form of a crisp or fuzzy data set. Additional input is in the form of a fuzzy rule or rules based on fuzzy set theory and relevant fuzzy set definitions that also act as input to the fuzzy controller. The Condition Interface determines the degree to which the input satisfies the "if" condition of the fuzzy-defined rule. Then based on the strength of each rule, and the definition of each fuzzy set in the rules, the Fuzzy Controller module fires each rule according to its strength to provide fuzzy output. The Action Interface defuzzifies this fuzzy output into a course of action. The result of the course of action taken provides input into the system. Thus, the Fuzzy Controller model allows learning to be achieved as 1) the firing strength of each rule is measured, 2) fuzzy output allows rules to be updated, and 3) new actions are recommended as the system continues to loop. This provides the Knowledge Base relevant for any expert system.

III. MODEL

Several versions of fuzzy controllers have been developed for use in a wide variety of fields [see, [29]. [30], [31], [32], [33], [34], [35])] The fuzzy controller proposed in this paper is based upon the work of Yager [36] which has also been applied to measuring productivity by controlling cost variance [1].

First, we assume that our Knowledge Base is made up of rules that have the following form: When V is A_j and U is B_j, then W is C_j. The index j refers to the jth rule in the Knowledge Base and the sets A_j, B_j, and C_j are allowed to be fuzzy sets. We further assume that all fuzzy sets are normal convex. A fuzzy set A is a trapezoidal subset of R if there exists four points of R, a, b, c, and d such that :

$$A(x) = 0 \text{ if } x \leq a \qquad (6)$$
$$A(x) \geq A(y) \text{ if } a \leq y \leq x \leq b \qquad (7)$$
$$A(x) = A(y) \text{ if } b \leq y \leq x \leq c \qquad (8)$$
$$A(x) \leq A(y) \text{ if } c \leq y \leq x \leq d \qquad (9)$$
$$A(x) = 0 \text{ for } x \geq d \qquad (10)$$

The most common trapezoidal set is the triangular shape where b = c. Then, if A is a fuzzy set by the α-level of A we mean the set $A\alpha = \{x \mid A\ \alpha\ (x) \geq 0\}$. Of course, $A\alpha$ is identified with its characteristic function $A\alpha\ (x) = 1$ if $A(x) \geq \alpha$ and $A\alpha\ (x) = 0$ otherwise.

A. Algorithm

1. Consider the R_j rule:
   R_j: when V is A_j and U is B_j then W is C_j

2. Associated to the rule R_j, define a fuzzy set F_j by:
   $F_j = A_j \cap B_j \cap C_j$.

3. Define the measure of how well x,y,z fits rule R_j as:
   $F_j(x,y,z) = \text{Minimum } \{A_j(x), B_j(y), C_j(z)\}$
   where x,y,z are in the domains of A_j, B_j, and C_j.

4. Assume the following: V is A and U is B where A and B are fuzzy sets.

5. Define $D_j = F_j \cap A \cap B$ where D_j measures how well the input "V is A" and "U is B" fits the rule R_j.

6. Let maximum$_x$ denote the maximum as x ranges over all of its allowable value, such that Maximum$_x$ Minimum

$\{A_j(x),A(x)\}$ denotes the possibility of having A_j given A. We denote this possibility by Poss$[A_j| A]$.

7. Define fuzzy set S_j for rule R_j by:

$S_j(z) = $ Minimum $\{$Poss $(A_j|A)$, Poss $(B_j|B)$, $C_j(z)\}$ where $S_j(z)$ measure for R_j how much z fits the then part of the rule, given that the inputs are U is A and V is B.

8. Let $T_j = $ Minimum$\{A_j(x_0)$, $B_j(y_0)\}$ where x_0, Y_0 is crisp input (given by sensors) Minimum $\{$Poss $(A_j|A)$, Poss $(B_j|B)] = T_j$ if A and B are crisp x_0 and Y_0

9. Then T_j measure how strongly R_j should apply given that we have inputs $U= X_0$ and $V = Y_0$ where X_0 and Y_0 are crisp values.

10. Then, $S_j = $ Minimum $\{T_j$, $C_j(z)\}$ denotes how rule R_j is fired when the then part of the rule $C_j(z)$is dampened by the firing strength T_j of the rule R_j.

11. Define $S(z) = $ Maximum$_j$ $S_j(z)$ as how much action z should be taken given the input and given the fact that we wish to fire all rules R_j with strength T_j.

12. Define $S^*(z) = [\Sigma_j T_j \times S_j(z)]/\Sigma_j T_j$ as the weighted average of all information coming from all rules.

13. Consider the z_j level of each S_j as: $C_{Tj} = \{z/ C_j(z) \quad T_j\}$ as an interval on the convex set; $C_{Tj} = [a_j, b_j]$.

14. Consider the midpoint m_j of C_{Tj} as $m_j = (a_j + b_j)/2$

15. Defuzzifying for action z^* is:

$z^* = [\Sigma_j m_j T_j]/\Sigma_j T_j$

(This algorithm is due to Yager [36] and Shipley, et al. [37].)

IV. APPLICATION

An example of how a fuzzy controller can be used to monitor and react to variations in a typical portfolio management problem is presented. The application concerns re-evaluation of a portfolio over the previous 20 days (approximately one month of trading opportunity) in which the risk/return ratio has been as high as 14% but in which the average is 4 %. The following application is based on Table 1 below.

TABLE 1.
Risk/Return over 20-Day Personal Investments of 50-Stock Portfolio

Day Number	Risk/Return Ratio	Day Number	Risk/Return Ratio
1	.04	11	.00
2	.06	12	.02
3	.08	13	.04
4	.02	14	.02
5	.00	15	.00
6	.04	16	.06
7	.08	17	.14
8	.02	18	.04
9	.02	19	.02
10	.06	20	.04

Using 99% limits the following chart can be constructed with the risk/return of the portfolio plotted as shown in Figure 2 below.

```
                              *
 _____  .124

    *     *                                  .080
  *              *        *
-*---------*--------------*----------*---*--------  .040
     *     * *    *   *         *
                                             
      *          *      *                    .000
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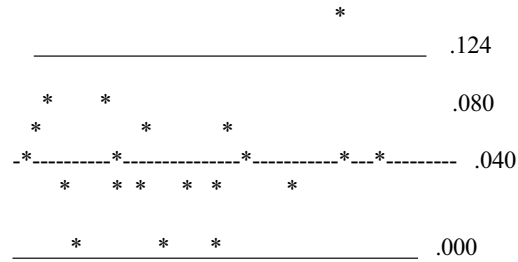
Figure 2. Interval Plotting of Risk/Return for 20-Day Portfolio

Generally, the risk/return for one day is out of the acceptable limits of the investor. Since this could significantly impact the financial status of the investor depending on the amount of the trade, the controller could be used to make adjustments to reduce the portfolio size. On the other hand, if all risk/return proportions are clustered close to the mean, the controller would suggest appropriate reduction in monitoring, freeing the investor for other risk versus risk-free asset investigations and necessitating minor adjustments to trading frequency and thus changes in portfolio size. For simplicity, while numerous rules may be proposed based on changes to market risk and return realized, this application considers only the following three rules from the perspective of a risk-averse trader:

R_1 : If within the upper and lower bounds and a positive change occurred in the percentage of risk with a negative change in return observed since the last trade, then decrease significantly the portfolio size.

R_2: If within the upper and lower bounds and a positive change occurred in the percentage of risk with no change in return since the last trade, then decrease slightly the portfolio size.

R_3: If within the upper and lower bounds and a negative change occurred in the percentage of risk with positive change in return observed since the last trade, then increase significantly the portfolio size.

For the risk averse investor, the assumption is made that 12% risk will be preferable with a return on investment of 20%. However, further assuming that it is desired that the portfolio risk/return be clustered within 1-sigma (.028) of the mean (.04) function F_1 will be deemed "Acceptable" as indicated in Figure 3.

1.0

Proportion Risk/Return

.068
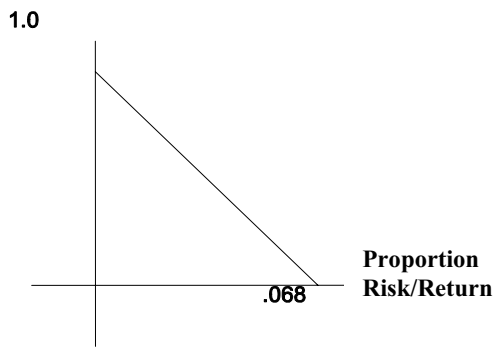
Figure 3. " Acceptable" According to Proportion Risk/Return

Moderately Acceptable" indicates some state of flux such that the risk/return proportion on most days are within 2-sigma of the mean. Although some may be close to the UCL or outside of the UCL this is generally only one or two stocks at high risk or low return impacting the portfolio. Thus F_2 would be the convex function shown in Figure 4.

1.0

.040   .096   .152   **Proportion Risk/Return**
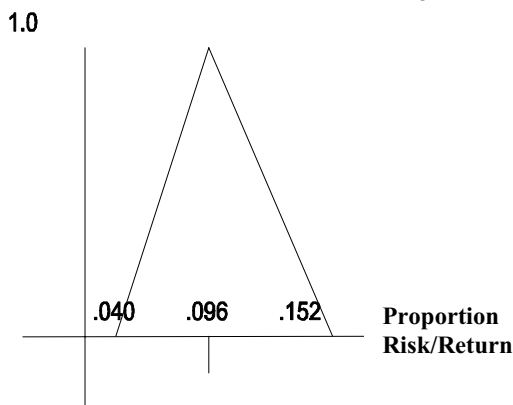
Figure 4. "Moderately Acceptable" According to Proportion Risk/Return

Finally, "Unacceptable" would be any portfolio with consistent risk/return proportions above the 3-sigma UCL such that F_3 would be described by Figure 5.

1.0

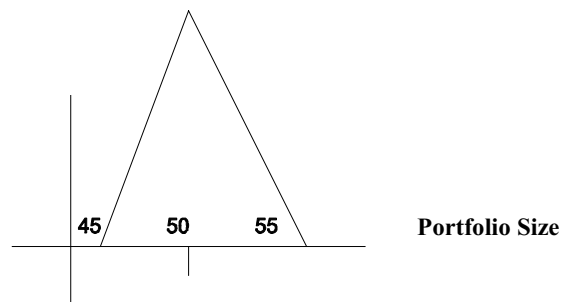**Proportion Risk/Return**

.124   .20
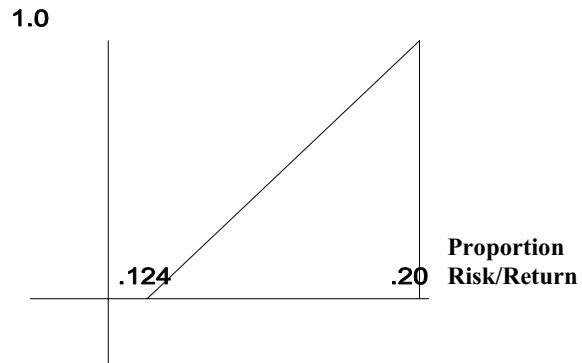
Figure 5. "Unacceptable" According to Proportion Risk/Return

With the "if" part of the rule defined as above, we define the "then" part to be the portfolio size necessary for assuring risk/return by the representation for $S_j(z)$ as give in Figures 6 through 9.

1.0

Portfolio Size

40
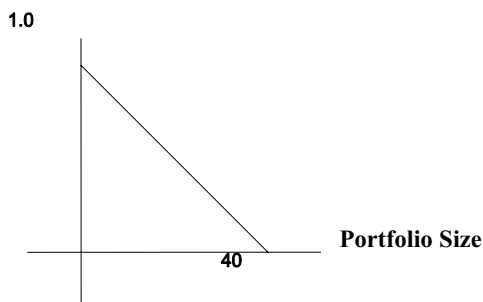
Figure 6. Portfolio Size for "Acceptable"

45   50   55   **Portfolio Size**

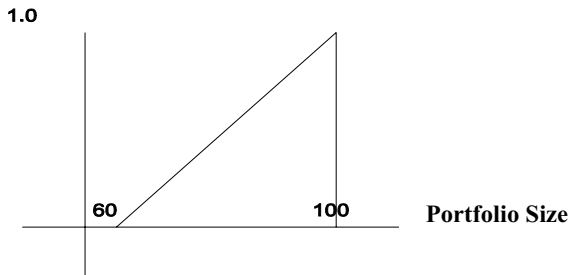**Figure 7.** Portfolio Size for "Moderately Acceptable"

Figure 8. Portfolio Size for "Unacceptable"

To determine the firing strength of each rule, T_j, we combine the convex functions and evaluate each rule with respect to some risk/return proportion, say 0.060 (3 poorly performing stocks in a portfolio of 50) when it was known that at the last trading day of the month the risk/return percentage was higher at 0.08 due to increased risk.
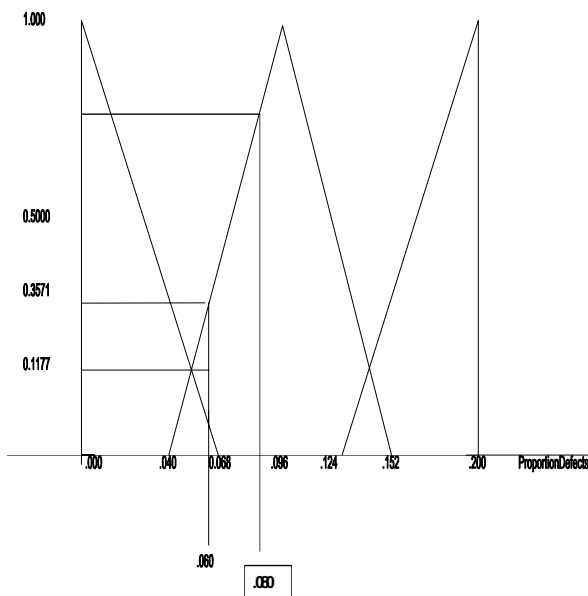

Figure 9. Fuzzy Functions Defined for Input Risk/Return Situation

Applying the Algorithm in Section 3.1, when R_1, T_1 = 0.1177 and $C_1(z)$ = 0.8823, and S_1 = minimum {0.1177, 0.8823} = 0.1177 denotes the extent to which rule R_1 is fired. When R_2, T_2 =0.3571 and $C_2(z)$ = 0.6429, so S_2 = 0.3571 denotes the extent to which rule R_2 is fired. When R_3, T_3 =0, $C_3(z)$ = 1.00, and S_3 = 0.00, which is that rule R_3 is not fired. Hence, with the proportion of risk/return, 0.060 as input, R_1 and R_2 apply and R_3 does not apply since 0.060 does not fit the "if" condition of R_3.

Next, for each R_j an interval on the convex set and the mean of the interval is defined such that for R_1, $a_1 = 0$ and $b_1 = 35.292$ with $m_1 = 17.646$; and for R_2, $a_2 = 46.7855$ and $b_2 = 53.2145$ with $m_2 = 50$. This yields, $Z^* = [(.1177)(17.646)+(.3571)(50)]/ (.1177+.3571) = 41.9796$ units which indicates that the portfolio size can be reduced to

42 investments, since the proportion of risk/return is within the limits, but barely so.

In the preceding model, the decision maker is reacting to the quantified rules determined by the fuzzy controller. Yet, the decision maker will be the one to pay the penalty if the controller selects an inappropriate rule. The penalty for most investors is obviously lost revenue and increased cost of trading to react to market risk.

If we investigate the situation where the portfolio size as suggested by the controller based on firing strength is reduced to 42 from 50 stocks, but should not have been so drastically reduced, we have the situation where R_1 was applied when R_2 or even R_3 should have been applied. Assuming that revenue generated from rapid trading is potentially much less than that of maintaining functioning stocks, the loss could be, for example, \$600.00 per day or \$27,000 over the 20-day period if the portfolio size is reduced (L_{12} = 27,000). Reducing the portfolio size when it should have been increased could be higher in lost revenue than simply the cost of massive trading, for example, \$1000.00 per day or \$45,000 for the period (L_{13} = 45,000). Increasing the portfolio size in order to protect from excessive risk would be the cost of increased trading. The added cost of increasing the trading could be \$160 per day or \$7200 for the period (L_{31} = 7200). Maintaining the present portfolio size when it could have been reduced could be s \$3600(L_{21} = 3600). (For simplicity sake, crisp values have been used to represent the losses. The model does, of course, allow losses to be fuzzy values.)

The decision maker may believe that the likelihood of reducing the portfolio size inappropriately instead of maintaining the 50 units or increasing the portfolio size as a precautionary measure is: p_{12} = .2/.7 + .3/.6 and p_{13} = .9/0 + .8/.1 , respectively. This indicates that the decision maker believes that there is a significant likelihood of a penalty being incurred if R_1 is applied inappropriately and the portfolio size is reduced, but has little belief that reducing the portfolio size is the wrong decision and, thus a low belief that the loss will be incurred. Similarly, the second fuzzy probability indicates that the decision maker believes strongly that no penalty will be incurred due to a wrong decision to reduce the portfolio size (R_1) rather than increase the portfolio size to at least 60 (R_3). All of this indicates that, in general, the decision maker does not think the portfolio size needs to remain at 50 units and very strongly that the portfolio size does not need to be increased.

It should be noted that p_{ij} is not necessarily the same as p_{ji}. The uncertainty of the decision maker with respect to selecting R_i when R_j should be applied is not the same as applying R_j when R_i should be applied. Similarly, the penalty of selecting R_i instead of R_j is not the same as the penalty of R_j instead of R_i. Again, differences in costs and uncertainty relate to the phase in the portfolio management process.

According to the scenario above:

$E(L_1) = L_{12}p_{12} + L_{13}p_{13}$
  $= (27,000)(.2/.7 + .3/.6) + (45,000)(.9/0 + .8/.1)$
  $= .2/18,900 + .3/16,200 + .9/0 + .8/4500$
$E(L_2) = L_{21}p_{21} = 36,000(.6/.3 + .5/.4) = .6/1080 + .5/1440$
$E(L_3) = L_{31}p_{31} = (7200)(1/0)$
$M = \{0; 1080; 1440; 16,200; 18,900\}$

$M = \{0/0; \ .057/1080; \ .076/1440; \ .238/4550; \ .857/16,200; \ 1/18,900\}$

$\alpha_1 = .3 \ \alpha_2 = .076 \ \alpha_3 = 0$

$\beta_1 = .7 \ \beta_2 = .924 \ \beta_3 = 1$

Using the multiplicative model,

$T_1' = .0824 \ T_2' = .32996 \ T_3' = 0$

Then $Z^{*\prime} = [(17.646)(.0824) + (50)(.32996)]/ \ (.0824+.32996) = 43.5$

This indicates that the potential for loss has had an impact on the portfolio size such that the decision maker would reduce the portfolio size, but not as dramatically as originally indicated by the fuzzy controller. The loss-based fuzzy controller recommended in this paper reacts to the low belief that reducing the sample size is the wrong decision, while also considering that if the portfolio size is reduced to 42 stocks, the probability of incurring a loss is relatively high.

In contrast, if $p_{12} = .7/.2 + .8/.3$ such that the decision maker believes that R_1 may be the wrong decision but a relatively low penalty will be incurred, $Z^{*\prime} = 47.60$. Thus, the portfolio size would be only moderately reduced, and R_2 would take precedence over R_1. For this model, R_3 has no impact on the portfolio size since $T_3 = 0$.

Note that if the decision maker believes that the appropriate rule has been applied by the controller, $p_{ij} = 0$ for all $i \neq j$ then $E(L_i) = 0$ for i=1,2,3. Then $\alpha_i=0$ and $\beta_j = 1$ such that $T_j' = \beta_j T_j$ indicates that T_j' reduces to T_j such that $Z^{*\prime} = Z^*$. For the example, the rule would be to reduce the portfolio size according to both the fuzzy controller and the loss-based fuzzy controller.

V. CONCLUSIONS

The application as presented is obviously incomplete. Other rules may be used that relate to the fact that most of the portfolio stocks are within bounds found acceptable to the decision maker. If most of the stocks are yielding satisfactory risk/return ratios, the decision maker may decide to hold these, incurring more risk on some that he/she believes may return higher revenue at a later time. The cost of trading and frequency thereof also impacts the decision to allow the fuzzy controller to adjust automatically on a daily basis. The rules and frequency of firing are at the discrimination of the decision maker. The fuzzy controller may, therefore, be set to control and adjust or as simply a monitoring technique. The addition of the concept of the fuzzy loss-based controller puts more decision power into the hands of the investor. However, in order to be a strength of the model, the values determined (unlike simply setting them as in the example) must be realistic. This will require considerable investigation by the decision maker, perhaps utilizing simulations, linear programming or other stochastically based models or other fuzzy logic models to address the uncertainly of the loss figures.

The model proposed is capable of handling any number of rules, and any method for determination of loss. It can be tailored easily to the investor and reacts to the risk-taking or risk-averseness of the person managing his/her personal portfolio.

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