

# Planar Tracking Control of an Underactuated Autonomous Underwater Vehicle

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**Abstract**—This paper addresses the problem of trajectory tracking control of an underactuated autonomous underwater vehicle (AUV) in the horizontal plane. The underwater vehicle under consideration is not actuated in the sway direction, and the system matrices are not assumed to be diagonal and linear, as often found in the literature. In addition, the effect of constant bias of environmental disturbances is considered. Using backstepping techniques and the tracking error dynamics, the system states are stabilized by forcing the tracking errors to an arbitrarily small neighborhood of zero. The effectiveness of the proposed control method is demonstrated through numerical simulations. Simulations are carried out for an experimental vehicle for smooth, inertial, two dimensional (2D) reference trajectories such as constant velocity trajectory (a circle maneuver – constant yaw rate), and time varying velocity trajectory (a sinusoidal path – sinusoidal yaw rate).

**Keywords**—autonomous underwater vehicle, system matrices, tracking control, time – varying feed back, underactuated control.

## I. INTRODUCTION

MODERN developments in the field of control, sensing, and communication have made increasingly complex and dedicated underwater robot systems a reality. Used in a highly hazardous and unknown environment, the autonomy and control of the robot is the key to mission success. Though the dynamics of underwater vehicle system is highly coupled and non-linear in nature, decoupled linear control system strategy is widely used for practical applications. As autonomous underwater vehicle needs intelligent control system, it is necessary to develop control system that really takes into account the coupled and non-linear characteristics of the system. In addition, most of the AUVs are underactuated, i.e., they have fewer actuated inputs than the degrees of freedom (DOF), imposing non-integrable acceleration constraints. A summary of the recent development in this area can be found in [1], [2].

Control of underactuated system is a continuation of the research on nonholonomic system. In recent years, nonholonomic systems have been a topic of much interest in the control society. Control of nonholonomic systems has proved to be a challenging problem, inherently nonlinear and

not amenable to linear control theory. Since, Nonholonomic and underactuation systems do not satisfy the conditions of Brockett's theorem [3], several approaches have been proposed for the stabilization of these systems. A review of nonholonomic system control is given in [4]. Nonholonomic and underactuation systems rule out the use of trivial control schemes, e.g., full state-feedback linearization [5], and the complex hydrodynamics of non-actuated states exclude the kinematic control. Trajectory tracking control requires the design of control laws that guide the vehicle to track an inertial reference time varying geometric path (trajectory). In the past, the tracking controller designs for underactuated underwater vehicles used to follow classical approaches such as local linearization and decoupling of the multivariable model to steer as many DOF as the available control inputs, i.e., the six DOF vehicle is decoupled into two reduced dynamical systems: a depth—pitch model that considers the motion in the vertical plane and a plane—yaw model that studies the motion in the horizontal plane [6]. Note that when AUV is moving on a horizontal plane, it presents similar to the dynamic behavior of underactuated surface vessels [1], [7]. A practical result on stabilization and tracking was given in [8] using a dynamic feedback approach. Leonard [9] considered several control configurations, and a technique for synthesizing open-loop controls. A high-gain-based local tracking result was proposed in [10]. Based on Lyapunov's direct method and passivity approach, two global tracking solutions were proposed in [11]. Lefeber [12] proposed a simple global tracking controller using cascade approach and linear time-varying system theory. It is noted that in [10] – [12] the yaw reference velocities required being nonzero. This restrictive assumption was removed in [13]. In [14], a trajectory planning and a tracking control algorithm for an underactuated AUV moving on the horizontal plane was studied but the model of drag force used in this work was linear with respect to velocities; this confining assumption is rectified in [6]. However in [6] the system matrices are assumed to be diagonal. In [15], the above issue is conquered for the underactuated surface vessel by applying a simple back stepping control algorithm. However in this work the system matrices are assumed to be linear.

In most of the above works, the mass and damping matrices of the AUVs were assumed to be diagonal and / or linear. These restrictive assumptions imply that the vehicle must be a semi-submerged sphere. In reality, these assumptions do not hold for underwater vehicles. Relaxing these assumptions will affect the control design and stability analysis. In fact, the authors of the aforementioned papers recognized difficulties

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caused by the nonzero off-diagonal terms of system matrices. They neglected these terms and left the topic of dealing with these terms for the future work. In addition, the vehicles usually operate in real time subject to environmental disturbances. A good controller should compensate the constant or slow-varying biases of the disturbances. The controller should not react to high-frequency components of the disturbances, since this increases wear on the actuators. The above discussion poses a problem of designing a controller that forces the position and orientation of the underactuated vehicles with non-zero off-diagonal terms and nonlinearity in the system matrices and subject to disturbances to track a reference trajectory. In this paper, we propose a simple existing solution (developed for underactuated surface vessels [15]) to overcome these problems for the tracking control of underactuated AUVs moving in a horizontal plane (constant depth motion).

## II. MODELLING OF AUV KINEMATICS AND DYNAMICS

Here, we consider an experimental autonomous underwater vehicle that is not having any side thruster to control the sway direction (this is not implemented because of economical and weight considerations). There are only two stern propellers which are offering control inputs as the force in surge and the control torque in yaw on the horizontal plane (by differential mode operation of propellers) [refer Fig.1].

**Assumptions:** Vehicle has an  $xz$ -plane of symmetry; surge is decoupled from sway and yaw; heave, pitch and roll modes, and these axes terms are neglected. We leave relaxation of these assumptions as a topic for future research.

Under these realistic assumptions, the motion of the vehicle in the yaw plane is described by the following ordinary differential equations [1], [15]

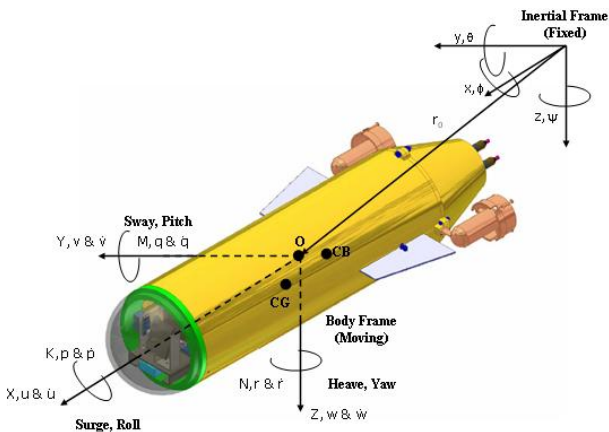


Fig. 1. Body-fixed frame and earth-fixed reference frame for AUV

The kinematics of the vehicle on the horizontal plane is given as:

$$\dot{\eta} = \mathbf{J}(\psi) \mathbf{v} \quad (1)$$

where,

$\eta = [x \ y \ \psi]^T$  is the displacement vector with respect to inertial frame,  $\mathbf{v} = [u \ v \ r]^T$  is the velocity vector with respect to body fixed frame and  $\mathbf{J}(\psi)$  is the transformation

$$\text{matrix and is given as : } \mathbf{J}(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The dynamic model of the vehicle in the presence of constant disturbances on the horizontal plane is as given by

$$\mathbf{M} \dot{\mathbf{v}} = -\mathbf{C}(\mathbf{v})\mathbf{v} - \mathbf{D}(\mathbf{v})\mathbf{v} + \boldsymbol{\tau} + \boldsymbol{\tau}_{dis} \quad (2)$$

where,

$\mathbf{M}$  – inertia matrix,  $\mathbf{C}(\mathbf{v})$  – Coriolis and Centripetal matrix,  $\mathbf{D}(\mathbf{v})$  – Damping matrix,  $\boldsymbol{\tau}$  – input vector and  $\boldsymbol{\tau}_{dis}$  – external disturbance vector, the components of which are as follows:

$$\mathbf{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}, \quad \begin{aligned} m_{11} &= m - X_{\dot{u}} \\ m_{22} &= m - Y_{\dot{v}} & m_{23} &= mx_g - Y_{\dot{r}} \\ m_{33} &= I_z - N_{\dot{r}} & m_{32} &= mx_g - N_{\dot{v}} \end{aligned}$$

$$\mathbf{C}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & c_{13} \\ 0 & 0 & c_{23} \\ -c_{13} & -c_{23} & 0 \end{bmatrix}, \quad \begin{aligned} c_{13} &= -m_{22}v - \frac{(m_{23} + m_{32})r}{2} \\ c_{23} &= m_{11}u \end{aligned}$$

$$\mathbf{D}(\mathbf{v}) = \mathbf{L}(\mathbf{v}) + \mathbf{NL}(\mathbf{v})|\mathbf{v}|$$

$$\mathbf{L}(\mathbf{v}) = \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & l_{23} \\ 0 & l_{32} & l_{33} \end{bmatrix}, \quad \begin{aligned} l_{11} &= X_u \\ l_{22} &= Y_v, l_{23} = Y_r \\ l_{32} &= N_v, l_{33} = N_r \end{aligned}$$

$$\mathbf{NL}(\mathbf{v}) = \begin{bmatrix} nl_{11} & 0 & 0 \\ 0 & nl_{22} & nl_{23} \\ 0 & nl_{32} & nl_{33} \end{bmatrix}, \quad \begin{aligned} nl_{11} &= X_{u|u|} \\ nl_{22} &= Y_{v|v|}, nl_{23} = Y_{r|r|} \\ nl_{32} &= N_{v|v|}, nl_{33} = N_{r|r|} \end{aligned}$$

$$\boldsymbol{\tau} = [\tau_u \ 0 \ \tau_r]^T \text{ and } \boldsymbol{\tau}_{dis} = [\tau_{u_{dis}} \ \tau_{v_{dis}} \ \tau_{r_{dis}}]^T$$

( $x, y$ ) are the surge and sway displacements,  $\psi$  is the yaw angle in the earth fixed frame,  $u, v, r$  denote surge, sway and yaw velocities;  $m_{11} > 0, m_{22} > 0, m_{23}, m_{32}, m_{33} > 0, l_{11}, nl_{11} > 0, l_{22}, nl_{22} > 0, l_{23}, nl_{23}, l_{32}, nl_{32}, l_{33}, nl_{33} > 0$  denote the hydrodynamic damping and vehicle inertia including added mass, the controls  $\tau_u$  and  $\tau_r$  are the surge force and yaw moment.

## III. CONTROLLER DESIGN

The main objective of this controller design is that the vehicle tracks a reference trajectory  $\boldsymbol{\Omega}$  parameterized by ( $x_d(s), y_d(s)$ ) with  $s$  being the path parameter through the help of  $\tau_u$  and  $\tau_r$ . The vehicle's total linear velocity is tangential to the reference trajectory  $\boldsymbol{\Omega}$ , and the desired surge velocity can be adjusted on-line [15].

Since vehicle system given in the (2) is underactuated, it is not expected to force the vehicle to track an arbitrary path

$\Omega$ . We here impose the following sufficient conditions on the path  $\Omega$  [15]:

**Condition 1:** There exist strictly positive constants  $\varepsilon_i$ , where  $i = 1, 2, 3$  and 4 such that

$$\dot{x}_d^2(s) + \dot{y}_d^2(s) \geq \varepsilon_1, \forall s \quad (3)$$

$$0.5\sqrt{2}\alpha u_d(t) - |\gamma_1 + \gamma_2| \geq \varepsilon_2, \forall t \geq 0 \quad (4)$$

$$(\beta_1 + \beta_2)u_d(t) + \dot{u}_d(t) \geq \varepsilon_3, \forall t \geq 0 \quad (5)$$

and the solution of the following differential equation:

$$\{u_d \cos(\delta)(1 - \alpha) + (\gamma_1 + \gamma_2)\} \dot{\delta} = -\{(\beta_1 + \beta_2)u_d + \dot{u}_d\} \sin(\delta) - \{\alpha u_d \cos(\delta) - (\gamma_1 + \gamma_2)\} r_d, \forall s, \delta(t_0) = 0 \quad (6)$$

Satisfies

$$|\delta(t)| \leq \pi/4 - \varepsilon_4, \forall t \geq t_0 \geq 0$$

where,  $u_d$  and  $r_d$  are defined as

$$\bar{u}_d = \sqrt{\dot{x}_d^2(s) + \dot{y}_d^2(s)} \dot{s}$$

$$\bar{r}_d = (\dot{x}_d(s)\dot{y}_d(s) - \ddot{x}_d(s)\dot{y}_d(s)) / [\dot{x}_d^2(s) + \dot{y}_d^2(s)] \dot{s}$$

and the constants  $\alpha, \beta_1, \beta_2, \gamma_1$  and  $\gamma_2$  are

$$\alpha = m_{11} / m_{22}$$

$$\beta_1 = l_{22} / m_{22}$$

$$\beta_2 = nl_{22}|v| / m_{22}$$

$$\gamma_1 = l_{22}m_{23} / m_{22}^2 - l_{23} / m_{22}$$

$$\gamma_2 = nl_{22}m_{23}|v| / m_{22}^2 - nl_{23}|r| / m_{22}$$

#### A. Coordinate Transformation

We introduce the following coordinate transformation to the vehicle (changing the vehicle position, see Fig.2) for getting the vehicle system matrix in to a diagonal form:

$$\begin{aligned} \bar{x} &= x + \varepsilon \cos(\psi) \\ \bar{y} &= y + \varepsilon \sin(\psi) \\ \bar{v} &= v + \varepsilon r \end{aligned} \quad (10)$$

where, offset distance ( $\varepsilon$ ) =  $m_{23} / m_{22}$ .

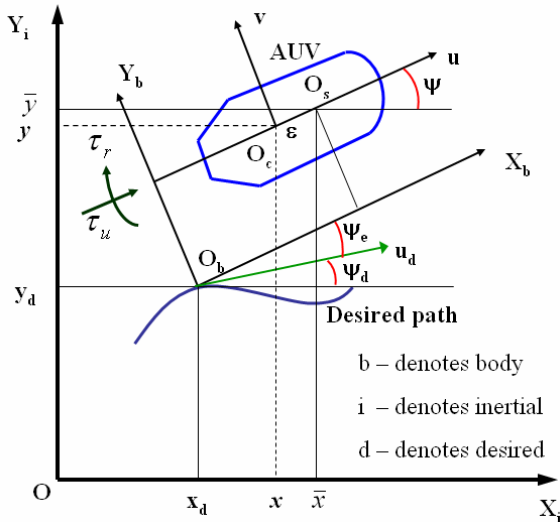


Fig. 2. AUV moving on a planar path and interpretation of path following errors [15]

Using the above change of coordinates, the vehicle dynamics in (2) (matrix form) can be rewritten as in the form of equations

$$\begin{aligned} \dot{\bar{x}} &= u \cos(\psi) - \bar{v} \sin(\psi) \\ \dot{\bar{y}} &= u \sin(\psi) + \bar{v} \cos(\psi) \\ \dot{\psi} &= r \\ \dot{u} &= \bar{\tau}_u \\ \dot{\bar{v}} &= -\alpha ur - (\beta_1 + \beta_2)\bar{v} + (\gamma_1 + \gamma_2)r \\ \dot{r} &= \bar{\tau}_r \end{aligned} \quad (11)$$

here, we have chosen primary controls  $\tau_u$  and  $\tau_r$  as

$$\begin{aligned} \tau_u &= m_{11}\bar{\tau}_u - m_{22}vr + l_{11}u + nl_{11}|u| - \{(m_{23} + m_{32})/2\}r^2 \\ \tau_r &= (m_{22}m_{33} - m_{23}m_{32})\bar{\tau}_r / m_{22} - \{(m_{11}m_{22} - m_{22}^2)uv \\ &\quad + \{m_{11}m_{32} - m_{22}(m_{23} + m_{32})/2\}ur \\ &\quad + (m_{32}l_{22} - m_{22}l_{32})v - (m_{22}l_{33} - m_{32}l_{23})r \\ &\quad + (m_{32}nl_{22} - m_{22}nl_{32})|v| - (m_{22}nl_{33} - m_{32}nl_{23})|r\} / m_{22} \end{aligned} \quad (12)$$

with  $\bar{\tau}_u$  and  $\bar{\tau}_r$  are considered as new controls which design is as follows in the next subsection. Now onwards  $(\bar{x}, \bar{y})$  are considered as the vehicle position instead of  $(x, y)$ . We now interpret the tracking errors in a frame attached to the path  $\Omega$  [15] as follows (refer Fig. 2)

$$\begin{bmatrix} x_e & y_e & \psi_e \end{bmatrix}^T = \mathbf{J}^T(\psi) \begin{bmatrix} \bar{x} - x_d & \bar{y} - y_d & \psi - \psi_d \end{bmatrix}^T \quad (13)$$

where  $\psi_d$  is the angle between the path and the  $X$ -axis defined by

$$\psi_d = \arctan(\dot{y}_d(s) / \dot{x}_d(s)) \quad (14)$$

In Fig.2,  $O$  denotes centre of earth-fixed inertial frame and  $O_x Y_i$  is the earth-fixed inertial frame;  $O_c$  denotes the center of gravity of the vehicle and  $O_s$  is referred to as the center of oscillation of the vehicle.  $O_b X_b Y_b$  is a moving frame attached to the path  $\Omega$  such that  $O_b X_b$  and  $O_b Y_b$  are parallel to the surge and sway axes of the vehicle;  $\bar{u}_d$  is tangential to the path; Therefore  $x_e, y_e$  and  $\psi_e$  can be referred to as tangential tracking error, cross-tracking error and heading error, respectively. Differentiating (13) along the solutions of the first three equations of (11) results in the kinematic error dynamics

$$\begin{aligned} \dot{x}_e &= u - \bar{u}_d \cos(\bar{\psi}_e) + r y_e \\ \dot{y}_e &= \bar{v} + \bar{u}_d \sin(\bar{\psi}_e) - r x_e \\ \dot{\bar{\psi}}_e &= r - \bar{r}_d \end{aligned} \quad (15)$$

From the above equation, we can see that the equilibrium point is origin i.e.  $(x_e, y_e, \bar{\psi}_e) = (0, 0, 0)$  if only the transformed sway velocity ( $\bar{v}$ ) is zero, which means that the vehicle must move on a straight line at the steady state. However our desired path  $\Omega$  to be different from straight line and the transformed sway velocity is generally different from zero. To overcome this problem, we introduce an angle  $\delta$  to the orientation error  $\bar{\psi}_e$  by defining

$$\psi_e = \bar{\psi}_e + \delta \quad (16)$$

For getting the sway velocity error ( $v_e$ ) as zero at the steady state, the desired sway velocity is chosen as follows

$$\dot{v}_d = -\alpha u_d r_d - (\beta_1 + \beta_2)v_d + (\gamma_1 + \gamma_2)r_d \quad (17)$$

Substituting (15), (16) and (17) in (11) and rewriting the path tracking error dynamics as:

$$\begin{aligned} \dot{x}_e &= u - u_d \cos(\psi_e) - v_d \sin(\psi_e) + r y_e \\ \dot{y}_e &= u_d \sin(\psi_e) + v_e - v_d (\cos(\psi_e) - 1) - r x_e \\ \dot{\psi}_e &= r - r_d \\ \dot{v}_e &= -\alpha(u r - u_d r_d) - (\beta_1 + \beta_2)v_e + (\gamma_1 + \gamma_2)(r - r_d) \\ \dot{u} &= \bar{\tau}_u \\ \dot{r} &= \bar{\tau}_r \end{aligned} \quad (18)$$

### B. Kinematic and Dynamic Control design

The control design is divided into two parts; kinematic and dynamic control. For kinematic control, first four equations from (18) are taken and in this case 'u' and 'r' are considered as controlled variables (controls). Similarly, the next two equations are considered for the dynamic control and for this case  $\bar{\tau}_u$  and  $\bar{\tau}_r$  are considered as controls which are derived from backstepping techniques [15], [16], and as given in (19).

$$\begin{aligned} \bar{\tau}_u &= -C_2 u_e + \frac{\partial \alpha_u}{\partial x_e} \dot{x}_e + \frac{\partial \alpha_u}{\partial u_d} \dot{u}_d + \frac{K_2 \alpha}{(\beta_1 + \beta_2) \Delta^2} z_2 r \\ \bar{\tau}_r &= \frac{\partial \alpha_r}{\partial z_1} \dot{z}_1 + \frac{\partial \alpha_r}{\partial z_2} \dot{z}_2 + \frac{\partial \alpha_r}{\partial \psi_e} \dot{\psi}_e + \frac{\partial \alpha_r}{\partial u_d} \dot{u}_d + \frac{\partial \alpha_r}{\partial v_d} \dot{v}_d \\ &\quad - [1 - K_2 \{\alpha u_d - (\gamma_1 + \gamma_2)\} (\beta_1 + \beta_2)^{-1} \Delta^{-2}] z_2 - C_3 r_e + \dot{r}_d \end{aligned} \quad (19)$$

where,  $C_1, C_2, C_3, K_1$  and  $K_2$  are positive constants.

$$\begin{aligned} u_e &= u - \alpha_u, \alpha_u = -K_1 x_e + u_d, K_1 = (\beta_1 + \beta_2) / (\gamma_1 + \gamma_2) \\ z_1 &= y_e + v_e / (\beta_1 + \beta_2), z_2 = \psi_e - \alpha_{\psi_e} \\ \alpha_{\psi_e} &= -\arcsin(K_2 z_1 / \Delta), \Delta = \sqrt{1 + (K_2 z_1)^2} \\ r_e &= r - r_d - \alpha_r \\ \alpha_r &= \frac{\left( -C_1 z_2 / \sqrt{1 + z_2^2} - K_2 / \Delta^2 \right)}{1 - K_2 \{\alpha u_d - (\gamma_1 + \gamma_2)\} (\beta_1 + \beta_2)^{-1} \Delta^{-2} \\ &\quad \times (u_d \sin(\psi_e) - v_d (\cos(\psi_e) - 1))} \end{aligned} \quad (20)$$

From (18), (19) and (20), the time derivatives of the tracking errors are rewritten as follows:

$$\begin{aligned} \dot{x}_e &= -K_1 x_e - u_d (\cos(\psi_e) - 1) - v_d \sin(\psi_e) \\ &\quad + (r_d + \alpha_r + r_e)(z_1 - (\beta_1 + \beta_2)^{-1} v_e) + u_e \\ \dot{z}_1 &= -K_2 u_d z_1 / \Delta - v_d (\cos(\psi_e) - 1) - \{(\beta_1 + \beta_2)^{-1} [\alpha u_d \\ &\quad - (\gamma_1 + \gamma_2)] (\alpha_r + r_e)\} + u_d \{\sin(z_2) \cos(\alpha_{\psi_e}) \\ &\quad + (\cos(z_2) - 1) \cos(\alpha_{\psi_e})\} - \alpha (\beta_1 + \beta_2)^{-1} u_e (r_d + \alpha_r + r_e) \\ \dot{z}_2 &= -C_1 z_2 / \sqrt{1 + z_2^2} + \{1 - K_2 [\alpha u_d - (\gamma_1 + \gamma_2)] (\beta_1 + \beta_2)^{-1} \Delta^{-2}\} r_e \\ &\quad - K_2 \alpha (\beta_1 + \beta_2)^{-1} \Delta^{-2} u_e r \\ \dot{v}_e &= -\alpha \{(u_e + \alpha_u)(r_d + \alpha_r + r_e) - u_d r_d\} - (\beta_1 + \beta_2) v_e \\ &\quad + (\gamma_1 + \gamma_2) \{\alpha_r + r_e\} \\ \dot{u}_e &= -C_2 u_e + K_2 \alpha (\beta_1 + \beta_2)^{-1} \Delta^{-2} z_2 r \\ \dot{r}_e &= -C_3 r_e - \{1 - K_2 [\alpha u_d - (\gamma_1 + \gamma_2)] (\beta_1 + \beta_2)^{-1} \Delta^{-2}\} z_2 \end{aligned} \quad (21)$$

For appropriate choices of  $C_1, C_2, C_3, K_1$  and  $K_2$ , the controls  $\tau_u$  and  $\tau_r$  given by (13) and (19) forces the

transformed tracking errors  $(x_e, y_e, \psi_e)$  to converge to zero asymptotically. This can be proved with the help of Lyapunov's method and Barbalat's lemma [15], [16].

## IV. SIMULATION RESULTS

To demonstrate the performance of the proposed scheme, typical simulation results are presented. A large number of simulation results have shown that the proposed control scheme performs well in terms of smooth transient response, quick convergence of tracking errors to zero, less control input, and robustness, even in the case of disturbed conditions.

For this study, JUBILEE, a test bed AUV being developed at IITM, is selected as an experimental set-up. Fig. 3 shows the first prototype of this AUV, along with its specifications in Table I. and the parameters used for simulation are shown in Table II.

TABLE I  
SPECIFICATIONS OF JUBILEE AUV

Items	Specifications and details
Hull dimension	0.25m (Diameter) x 1.065m (Length) Cylinder shape
Buoyancy	Neutrally buoyant in water
Working Depth	50m
Power	Lithium ion batteries (DC 12V,20Ahr & 24V,22Ahr)
Endurance	1hr
Speed	2m/s (4knots)
Thrusters	Two twin blade Thrusters with differential actuation (2.9kgf)
Control Planes	Two independent stern planes (actuated by brushless DC motors)
Sensors	Miniature Pressure sensor (for Depth) 2 axis Tilt sensor ( for Attitude) 3 axis Accelerometer Leak sensor (leakage measurement)
Controller	Atmega 128 micro controller
Camera	CCD camera with servo actuated Pan tilt mechanism
Allowable error limits	In linear displacements (in x, y and z axes) : $\pm 0.5m$ , In angular displacements (in $\phi, \theta$ , and $\psi$ angles) : $\pm 2^\circ$

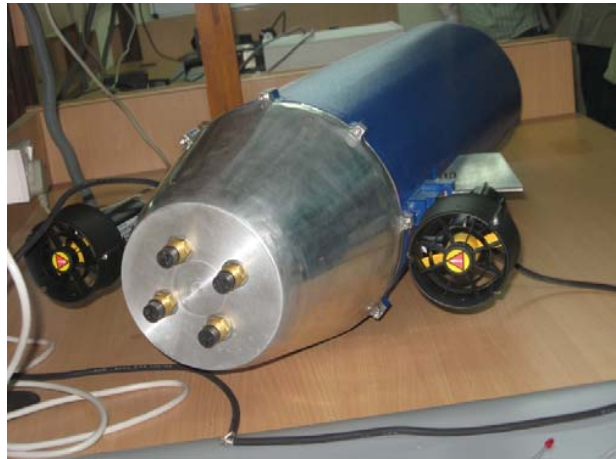


Fig. 3. Photo of JUBILEE AUV

There are two desired trajectories considered here: a constant velocity trajectory (circular path), and a time varying

velocity trajectory (sinusoidal path). The simulation results were obtained with controller constants chosen as:

$$C_1 = 0.1, C_2 = 0.1, C_3 = 1.5, K_1 = 2 \text{ and } K_2 = 2.$$

The vehicle starts from rest, accelerates for first 50 seconds to achieve the desired forward speed of 1m/s and to attain this speed for the vehicle accelerated for first 50 seconds. For the circular path tracking, the desired path is chosen as follows:

For first 50 seconds:  $x_d(s) = 0.01s^2$ ,  $y_d(s) = 0$ , and  $x_d(s) = 25 \sin(0.08(s-50)) + 25$ ,  $y_d(s) = 25 \cos(0.08(s-50)) - 2$  for the rest of testing time, similarly, for sinusoidal tracking, the chosen desired path is as follows:

For first 50 seconds:  $x_d(s) = 0.01s^2$ ,  $y_d(s) = 0$ , and  $x_d(s) = (s-50) + 25$ ,  $y_d(s) = 10 \sin(0.32(s-50))$  for the rest of testing time. The initial errors are assumed to be zero for all the states. The disturbances are assumed to be constant and which magnitudes are 5N, 5N and 2Nm acting in the surge, sway and yaw axis respectively.

TABLE II  
RIGID BODY AND HYDRODYNAMIC PARAMETERS OF THE JUBILEE AUV

Parameter	Symbol	Value	Unit
Mass	$m$	39	kg
Rotational Inertia	$I_z$	3.7	kg.m <sup>2</sup>
Added mass in surge	$X_{\dot{u}}$	-1.17	kg
Added mass in sway	$Y_{\dot{v}}$	-34.84	kg
Added (mass) inertia in yaw	$N_{\dot{\psi}}$	-1.04	kg.m
Surge linear drag	$N_{\dot{u}}$	-2.66	kg.m <sup>2</sup> /rad
Surge quadratic drag	$X_{ u }$	-7.41	kg/s
Sway linear drag	$Y_v$	-62.45	kg/s
Sway quadratic drag	$Y_{ v }$	-112.21	kg/m
Yaw linear drag	$Y_{r }$	0.25	kg.m/rad
Yaw quadratic drag	$N_v$	1.2	kg.m/s
	$N_r$	-31.25	kg.m <sup>2</sup> /s
	$N_{ v }$	2.24	kg
	$N_{r }$	-59.75	kg.m <sup>2</sup> /rad <sup>2</sup>

The circular tracking simulation results are presented in Figs. 4-6. From the Fig.4, it is found that the control performance is quite well even in the disturbed condition. The tracking position errors are converging into a small bounded value which is near to zero (refer Fig. 5).

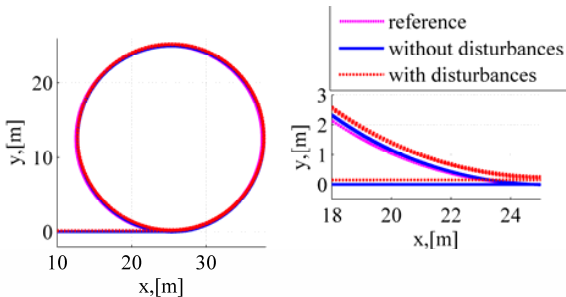


Fig. 4. AUV reference and actual path (with and without disturbances) for circular tracking, (right side plot: enlarged view)

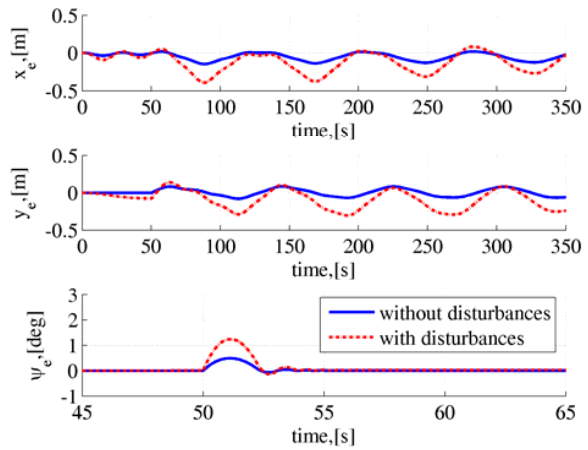


Fig. 5. Tracking errors (with and without disturbances) for circular tracking

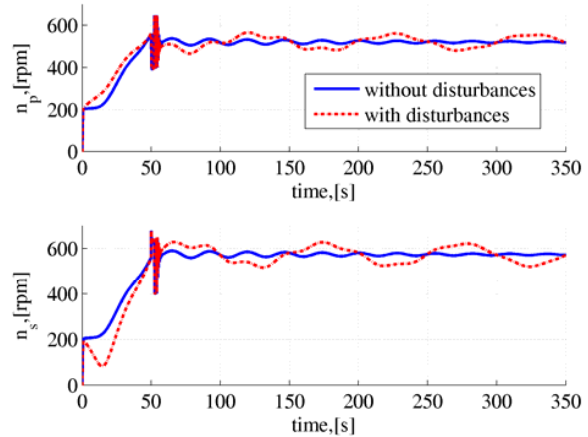


Fig. 6. Port side ( $n_p$ ) and starboard side ( $n_s$ ) thruster rotations (with and without disturbances) for circular tracking

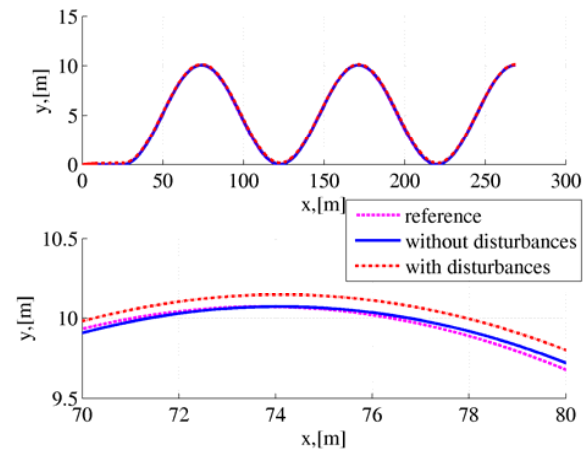


Fig.7. AUV reference and actual path (with and without disturbances) for sinusoidal tracking, (Plot in down: enlarged view)

Simulation results for sinusoidal path are presented in Figs. 7-9, where the required velocities are not to be constant.

The results show that the control strategy presented is applicable in the case where the velocities vary with time.

It was observed that the errors are converged into a small bound after a smooth transient time interval. However, in the disturbed condition the errors of the earlier case are high compared to the present case, but those errors are also converging to a bounded region after a short interval.

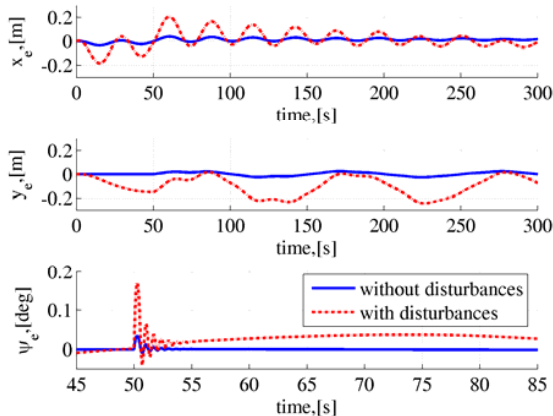


Fig. 8. Tracking errors (with and without disturbances) for sinusoidal tracking

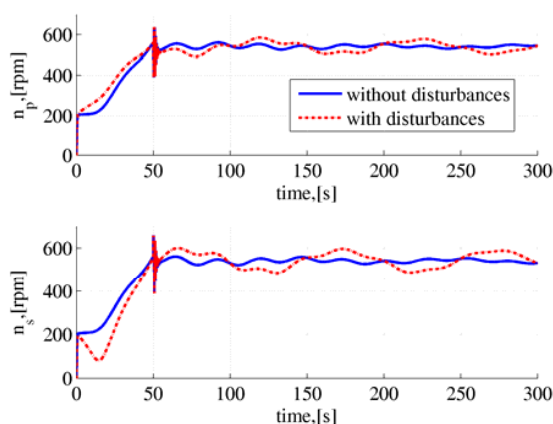


Fig. 9. Port side ( $n_p$ ) and starboard side ( $n_s$ ) thruster rotations (with and without disturbances) for sinusoidal tracking

## V. CONCLUSION

In this paper, a trajectory tracking control scheme for an underactuated underwater vehicle in the horizontal plane is presented, in which the system matrices of the vehicle are not assumed to be diagonal and linear. It is shown that the backstepping technique can be used for stabilizing the AUV and bringing the tracking error to a neighborhood of zero for given reference, smooth, inertial 2D trajectories. Computer simulations showed very good tracking performance and robustness of the proposed method even in the presence of constant disturbances. The proposed control design is simple in structure and it provides a chance to extend the same technique to the three dimensional tracking control as well.

Experimental verification of the planar tracking control will be taken up in the near future to validate the simulation results.

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