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Permanence and exponential stability of a predator-prey model with HV-Holling functional response

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Abstract—In this paper, a delayed predator-prey system with Hassell-Varley-Holling type functional response is studied. A sufficient criterion for the permanence of the system is presented, and further some sufficient conditions for the global attractivity and exponential stability of the system are established. And an example is to show the feasibility of the results by simulation.

Keywords—Predator-Prey system, Hassell-Varley-Holling, Delay, Permanence, Exponential stability

I. INTRODUCTION

AFTER the first model of predator-prey(short for PP) introduced by A.J. Lotka (1925) [1] and V. Volterra (1926) [2], there are extensively investigations on the dynamic behaviors of PP models, such as [3]- [10]. The dynamic relationship between predators and their preys will continue to be a dominant theme in ecology due to its universal existence and important applications. It was well known that the predator's functional response(i.e., the rate of prey consumption by an average predator) plays an very important role in the dynamic behaviors of predators and preys. Generally, the functional response can be classified into two types: prey-dependent and predator-dependent. Prey-dependent indicates that the functional response is only a function of the preys density, while predator-dependent means that the functional response is a function of both the preys and the predators densities. As noted by Cosner et al.(1999) in [11], the derived functional response maybe prey-dependent under the assumption of spatially homogeneous distributions of both predators and preys. However, when the spatial structure of one or both of the interacting populations are involved, it would be more plausible to take the predator-dependent functional form. The classical Holling types I-III functional responses presented by Holling(1959) [12], [13] are strictly prey-dependent; Beddington-DeAngelis type by Beddington(1975) [14] and DeAngelis et al.(1975) [15] as well as Crowley-Martin type(1989) [16] are predator-dependent functional responses.

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In 1969, Hassell and Varley [17] proposed a trophic function

$$g(x,y)=\frac{\alpha x}{y^{\gamma}},\ \gamma\in(0,1],$$

which is predator-dependent and called Hassell-Varley(HV for short) functional response, γ is called HV constant. In a typical predator-prey interaction where predators do not form groups, one can assume that $\gamma=1$, producing the so-called ratio-dependent predator-prey dynamics. For terrestrial predators that form a fixed number of tight groups, it is often reasonable to assume that $\gamma=1/2$. For aquatic predator that form a fixed number of tights groups, $\gamma=1/3$ maybe more appropriate. A unified mechanistic approach was provided by Cosner et al.(1999) [11] where the functional response was derived in the form,

$$g(x,y)=\frac{\alpha x}{my^{\gamma}+x}.$$

Hsu et al.(2008) [18] studied a PP system including this functional response and presented a systematic global qualitative analysis to the system. Wang et al.(2011) [19] investigated the permanence and global asymptotic stability of a non-autonomous PP model with delay.

Schenk et al.(2005) [20] derived the following functional response,

$$g(x,y)=\frac{\alpha\left(\dfrac{x}{y^{\gamma}}\right)^2}{1+\alpha h\left(\dfrac{x}{y^{\gamma}}\right)^2},$$

which is the HV functional response, adapted for the Holling type III response. Moreover, it is showing in [20] that their study provids the first experimental evidence discriminating between ratioand prey-dependence in a natural setting with unconfined predators and preys. Liu et al.(2010) [21] investigated the dynamics of a Gause type PP model with HV-Holling functional response,

$$\begin{cases} x'=rx\left(1-\dfrac{x}{K}\right)-\dfrac{\alpha x^2y}{\alpha hx^2+y^{2\sigma}},\\ y'=-dy+\dfrac{e\alpha x^2y}{\alpha hx^2+y^{2\sigma}}, \end{cases}$$

where $\sigma\in(0,1)$. But in the real world, the change of the environment is often dependent on time, such as the change of season. So it is naturel to consider time-dependent coefficients in the above model, namely non-autonomous system. Furthermore, Kuang(1993) [3] pointed out the fact that any model of species dynamics without delays is an approximation at best.

Motivated by these, in this paper we consider the following non-autonomous PP system with HV-Holling type functional response and delay,

$$\begin{cases} x' = x[a(t) - b(t)x(t-\tau)] - \dfrac{c(t)x^2y}{kx^2+y^{2\sigma}}, \\ y' = -d(t)y + \dfrac{e(t)x^2y}{kx^2+y^{2\sigma}}, \end{cases} \tag{1}$$

where $\sigma \in (0,1]$, $k > 0, \tau \geq 0$ are constants, a, b, c, d and e are continuously positive functions with positive upper and lower bounds. And consider the following initial conditions,

$$\begin{cases} x(t) = \varphi(t) \geq 0, \ t \in [-\tau, 0] \ \text{ with } \varphi(0) > 0, \\ y(t) = \phi(t) \geq 0, \ t \in [-\tau, 0] \ \text{ with } \phi(0) > 0. \end{cases} \tag{2}$$

The main problems we consider in this paper are as follows,

• The permanence of the prey and predator of the system.

• The global attractivity and exponential stability of the prey and predator of the system.

The organization of the rest part is that: In next section, we first give some lemmas and definitions, then present the main results of the paper. In section 3, we will give the proofs of all results. Lastly, an example is given to verify the feasibility of our results by simulation.

II. Lemmas and main results

In this section, we will give some lemmas, definitions and establish some criteria for guaranteeing the permanence and exponential stability of model (1).

Before presenting the main results we first give some useful notations: for any bounded function f defined on interval $[0, +\infty)$,

$$\overline{f} = \sup_{t\in[0,+\infty)} f(t), \ \underline{f} = \inf_{t\in[0,+\infty)} f(t)$$

and

$$\begin{aligned} M_1 =& \ \frac{\overline{a}e^{\overline{a}\,\tau}}{\underline{b}}, \ M_2 = \left[\frac{\overline{e}M_1^2}{\underline{d}}\right]^{\frac{1}{2\sigma}}, \\ H(t) =& \ a(t) - c(t)M_1M_2^{1-2\sigma}, \\ m_1 =& \ \min\left\{\frac{\underline{H}}{\overline{b}}, \ \frac{\underline{H}}{\overline{b}}e^{(\underline{H}-\overline{b}M_1)\tau}\right\}, \\ F(t) =& \ K^{-1}e(t) - d(t), \ m_2 = \left[\frac{(k\,m_1)^2\underline{F}}{\overline{e}}\right]^{\frac{1}{2\sigma}}, \end{aligned}$$

$$\begin{aligned} G_1(t) =& \ b(t) + (kM_1^2 + M_2^{2\sigma})^{-2}\Big[c(t)(Km_1^2m_2 + m_2^{2\sigma+1}) \\ & - 2\alpha^{-1}e(t)M_1M_2^{2\sigma} - (e^{\lambda\tau} - 1)\lambda^{-1}\Big] - \lambda m_1^{-1} \\ & - e^{-\lambda t}\Big[[a(t) + M_1b(t)]\int_t^{t+\tau} e^{\lambda l}b(l)dl + \\ & M_1b(t+\tau)\int_{t+\tau}^{t+2\tau} e^{\lambda l}b(l)dl\Big], \\ G_2(t) =& \ -\lambda m_2^{-1} + (kM_1^2 + M_2^{2\sigma})^{-2}\Big[2\sigma e(t)m_1^2m_2^{2\sigma-1} \\ & - \alpha c(t)M_1(2\sigma M_2^{2\sigma} - kM_1^2) \\ & - \alpha(e^{\lambda\tau} - 1)\lambda^{-1}[kM_1^4 + (2\sigma - 1)M_1M_2^{2\sigma}]\Big], \\ \widetilde{G}_1(t) =& \ b(t) + \frac{c(t)(k\,m_1^2m_2 + m_2^{2\sigma+1}) - 2\alpha^{-1}e(t)M_1M_2^{2\sigma}}{(kM_1^2 + M_2^{2\sigma})^2} \\ & - \Big\{[a(t) + M_1b(t)]\int_t^{t+\tau} b(l)dl \\ & + M_1b(t+\tau)\int_{t+\tau}^{t+2\tau} b(l)dl\Big\}, \\ \widetilde{G}_2(t) =& \ 2\sigma e(t)m_1^2m_2^{2\sigma-1} - \alpha c(t)M_1\left(2\sigma M_2^{2\sigma} - kM_1^2\right), \end{aligned}$$

where α and λ are positive constants.

Definition 2.1. System (1) is said to be permanent, if there exist positive numbers m_i, $M_i, i = 1, 2$ such that

$$m_1 \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq M_1$$

and

$$m_2 \leq \liminf_{t\to+\infty} y(t) \leq \limsup_{t\to+\infty} y(t) \leq M_2$$

for any positive solution $(x(t), y(t))$ of system (1) with initial condition (2).

Definition 2.2. System (1) is called exponential stability, if

$$\lim_{t\to+\infty} e^{\lambda t}|x_1(t) - x_2(t)| = 0 \ \text{ and } \ \lim_{t\to+\infty} e^{\lambda t}|y_1(t) - y_2(t)| = 0$$

for any two positive solutions $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ of system (1) with initial condition (2). More accurately, that is there exist positive constants T, M_0, $\widetilde{M}_0$ and λ such that

$$|x_1(t) - x_2(t)| \leq M_0e^{-\lambda t} \ \text{ and } \ |y_1(t) - y_2(t)| \leq \widetilde{M}_0e^{-\lambda t}$$

for $t \geq T$.

Definition 2.3. System (1.2) is called globally attractivity, if

$$\lim_{t\to+\infty}(|x_1(t) - x_2(t)| + |y_1(t) - y_2(t)|) = 0,$$

for any two positive solutions $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ of system (1).

Lemma 2.1.(*See [4]*) If $a > 0, b > 0$ and $\dot{z}(t) \geq (\leq)\, b - a\, z(t)$ for $t \geq 0$, $z(0) > 0$, then the following inequality holds:

$$z(t) \geq (\leq) \frac{b}{a} + \left[z(0) - \frac{b}{a}\right] \exp\{-a\,t\}$$

or

$$\liminf_{t\to+\infty} y(t) \geq \frac{b}{a} \left(\limsup_{t\to+\infty} y(t) \leq \frac{b}{a} \right).$$

Lemma 2.2.(*See [5] [6]*) If $a > 0,\ b > 0,\ \tau \geq 0$ and $\dot{y}(t) \leq y(t)\,[b - a\,y(t-\tau)]$ for $t \geq 0$, then there exists a constant $T > 0$ such that

$$y(t) \leq \frac{b}{a} \exp\{b\,\tau\} \ \text{ for } t \geq T.$$

Lemma 2.3.(*See [5] [6]*) If $a > 0,\ b > 0,\ \tau \geq 0$ and $\dot{y}(t) \geq y(t)\,[b - a\,y(t-\tau)]$ for $t \geq 0$, and there exists positive constant M such that $\limsup_{t\to+\infty} y(t) \leq M$, then

$$\liminf_{t\to+\infty} y(t) \geq \min \left\{ \frac{b}{a} \exp\{(b - aM)\tau\}, \frac{b}{a} \right\}.$$

Assumption 1. $\underline{H} > 0$ and $\underline{F} > 0$.

Assumption 2. $\liminf_{t\to+\infty}\{G_i(t)\} > 0,\ i = 1, 2.$

Assumption 3. $\liminf_{t\to+\infty}\{\widetilde{G}_i(t)\} > 0,\ i = 1, 2.$

Theorem 2.1. If Assumption 1 holds, then system (1) is permanent.

Theorem 2.2. If Assumptions 1 and 2 hold, then system (1) is global attractive and exponential stable with the exponent $\lambda \in I$, where $I = \{\lambda > 0 | \liminf_{t\to+\infty} G_i(t) > 0\}$.

Theorem 2.3. If Assumptions 1 and 3 hold, then system (1) is global attractive.

Remark If replace the delay τ by function $\tau(t)$, namely the delay is time-dependent, some similar results can be obtained by the methods used below.

III. Proofs

Proof of Theorem 2.1.

Step 1. The estimating of the upper bounds of all positive solutions of system (1).

From the first equation of Eqs.(1) we obtain

$$x' \leq x[a(t) - b(t)x(t-\tau)],$$

which together with Lemma 2.2 yields

$$\limsup_{t\to+\infty} x(t) \leq M_1. \tag{3}$$

It follows from the second equation of Eqs.(1) that, for sufficient large t,

$$(y^{2\sigma})' \leq 2\sigma[\overline{e}M_1^2 - \underline{d}\, y^{2\sigma}],$$

which implies

$$\limsup_{t\to\infty} y(t) \leq M_2. \tag{4}$$

Step 2. The estimating of the lower bounds of all positive solutions of system (1).

It follows from the first equation of Eqs.(1) that, for sufficient large t,

$$x' \geq x[\underline{H} - \overline{b}\, x(t-\tau)],$$

where $H(t) = a(t) - c(t)M_1M_2^{1-2\sigma}$. By using Lemma 2.3 we have

$$\liminf_{t\to\infty} x(t) \geq m_1. \tag{5}$$

Meanwhile, from the second equation of Eqs.(1) we get, for sufficient large t,

$$y' \geq y\left[\underline{F} - \frac{ey^{2\sigma}}{(k\,m_1)^2}\right],$$

where $F(t) = K^{-1}e(t) - d(t)$, which gives

$$(y^{-2\sigma})' \leq 2\sigma[\overline{e}(k\,m_1)^{-2} - \underline{F}\, y^{-2\sigma}].$$

Thus, according to Lemma 2.1, we have

$$\liminf_{t\to\infty} y(t) \geq m_2,$$

which together with (3)-(5) verifies Theorem 2.1. This completes the proof.

Proof of Theorem 2.2.

Choose the Lyapunov functional in the form:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{6}$$

where

$$V_1(t) = \alpha e^{\lambda t}\left|\log\frac{x(t)}{\hat{x}(t)}\right| + e^{\lambda t}\left|\log\frac{y(t)}{\hat{y}(t)}\right|,$$

$$\begin{aligned} &V_2(t)\\ &= \alpha \int_t^{t+\tau} \int_{l-\tau}^{t} e^{\lambda l}\Big\{ b(l)[(a(s) + b(s)x(s-\tau))|x(s) - \hat{x}(s)| \\ &\quad + b(s)\hat{x}(s)|x(s-\tau) - \hat{x}(s-\tau)|] \\ &\quad + c(s)(\Delta\hat{\Delta})^{-1}(s)[kx^2(s)\hat{x}^2(s)|y(s) - \hat{y}(s)| \\ &\quad + x(s)y(s)\hat{y}(s)|y^{2\sigma-1}(s) - \hat{y}^{2\sigma-1}(s)| \\ &\quad + \hat{y}(s)y^{2\sigma-1}(s)|x^2(s) - \hat{x}^2(s)|]\Big\} ds dl \end{aligned}$$

and

$$V_3(t) = \alpha \int_t^{t+\tau} \int_s^{s+\tau} e^{\lambda l} b(l)b(s)\hat{x}(s)|x(s-\tau) - \hat{x}(s-\tau)| dl ds,$$

where $\Delta(t) = kx^2(t) + y^{2\sigma}(t)$, $\hat{\Delta}(t) = K\hat{x}^2(t) + \hat{y}^{2\sigma}(t)$.

The Dini derivative of $V_1(t)$ along the solution of model (1) is (in the following, we will drop the parenthesis and t of $x(t), \hat{x}(t), y(t)$ and $\hat{y}(t)$ for simplify in the case of no confusion),

$$\begin{aligned} D^+V_1(t) = \lambda V_1(t) + \alpha e^{\lambda t} sgn(x - \hat{x})\left[\frac{x'}{x} - \frac{\hat{x}'}{\hat{x}}\right] \\ + e^{\lambda t} sgn(y - \hat{y})\left[\frac{y'}{y} - \frac{\hat{y}'}{\hat{y}}\right]. \end{aligned} \tag{7}$$

In view of

$$\begin{aligned}
&sgn(x-\hat{x})\left[\frac{x'}{x}-\frac{\hat{x}'}{\hat{x}}\right]\\
&= sgn(x-\hat{x})\left[-b(t)(x-\hat{x})+b(t)\int_{t-\tau}^{t}(x'(s)-\hat{x}'(s))ds\right.\\
&\quad\left.-c(t)\left(\frac{xy}{\Delta}-\frac{\hat{x}\hat{y}}{\hat{\Delta}}\right)\right]\\
&\le -b(t)|x-\hat{x}|+b(t)\int_{t-\tau}^{t}\Big\{a(s)|x(s)-\hat{x}(s)|\\
&\quad+b(s)x(s-\tau)|x(s)-\hat{x}(s)|\\
&\quad+b(s)\hat{x}(s)|x(s-\tau)-\hat{x}(s-\tau)|\\
&\quad+c(s)\left|\frac{x^2(s)y(s)}{\Delta(s)}-\frac{\hat{x}^2(s)\hat{y}(s)}{\hat{\Delta}(s)}\right|\Big\}ds\\
&\quad-c(t)sgn(x-\hat{x})\left(\frac{xy}{\Delta}-\frac{\hat{x}\hat{y}}{\hat{\Delta}}\right),
\end{aligned}$$

$$\begin{aligned}
x^2y\hat{\Delta}-\hat{x}^2\hat{y}\Delta = \; & kx^2\hat{x}^2(y-\hat{y})+xy\hat{y}(\hat{y}^{2\sigma-1}-y^{2\sigma-1})\\
&+\hat{y}y^{2\sigma-1}(x^2-\hat{x}^2)
\end{aligned}$$

and

$$\begin{aligned}
xy\hat{\Delta}-\hat{x}\hat{y}\Delta = \; & (kxy\hat{x}+y\hat{y}^{2\sigma})(x-\hat{x})+k\hat{x}x^2(y-\hat{y})\\
&+\hat{x}\hat{y}(\hat{y}^{2\sigma}-y^{2\sigma}),
\end{aligned}$$

we have

$$\begin{aligned}
&sgn(x-\hat{x})\left[\frac{x'}{x}-\frac{\hat{x}'}{\hat{x}}\right]\\
&\le -b(t)|x-\hat{x}|+b(t)\int_{t-\tau}^{t}\Big\{[a(s)\\
&\quad+b(s)x(s-\tau)]|x(s)-\hat{x}(s)|\\
&\quad+b(s)\hat{x}(s)|x(s-\tau)-\hat{x}(s-\tau)|\Big\}ds\\
&\quad+\int_{t-\tau}^{t}c(s)(\Delta\hat{\Delta})^{-1}\Big\{kx^2(s)\hat{x}^2(s)\left[y(s)-\hat{y}(s)\right]\\
&\quad+x(s)y(s)\hat{y}(s)\left[\hat{y}^{2\sigma-1}(s)-y^{2\sigma-1}(s)\right]\\
&\quad+\hat{y}(s)y^{2\sigma-1}(s)\left[x^2(s)-\hat{x}^2(s)\right]\Big\}ds\\
&\quad-c(t)(\Delta\hat{\Delta})^{-1}sgn(x-\hat{x})\Big\{\left[kxy\hat{x}+y\hat{y}^{2\sigma}\right](x-\hat{x})\\
&\quad+k\hat{x}x^2(y-\hat{y})+\hat{x}\hat{y}(\hat{y}^{2\sigma}-y^{2\sigma})\Big\}\\
&\le -\left\{b(t)+c(t)(\Delta\hat{\Delta})^{-1}\left[kxy\hat{x}+y\hat{y}^{2\sigma}\right]\right\}|x-\hat{x}|\\
&\quad+kc(t)(\Delta\hat{\Delta})^{-1}\hat{x}x^2|y-\hat{y}|+c(t)(\Delta\hat{\Delta})^{-1}\hat{x}\hat{y}|\hat{y}^{2\sigma}-y^{2\sigma}|\\
&\quad+b(t)\int_{t-\tau}^{t}\Big\{[a(s)+b(s)x(s-\tau)]|x-\hat{x}|\\
&\quad+b(s)\hat{x}|x(s-\tau)-\hat{x}(s-\tau)|\Big\}ds\\
&\quad+\int_{t-\tau}^{t}c(s)(\Delta\hat{\Delta})^{-1}\Big\{kx^2\hat{x}^2|y-\hat{y}|\\
&\quad+xy\hat{y}|\hat{y}^{2\sigma-1}-y^{2\sigma-1}|+\hat{y}y^{2\sigma-1}|x^2-\hat{x}^2|\Big\}ds
\end{aligned} \tag{8}$$

and

$$sgn(y-\hat{y})\left[\frac{y'}{y}-\frac{\hat{y}'}{\hat{y}}\right]\le\frac{e(t)\left[\hat{y}^{2\sigma}|x^2-\hat{x}^2|-\hat{x}^2|\hat{y}^{2\sigma}-y^{2\sigma}|\right]}{\Delta\hat{\Delta}}. \tag{9}$$

Then, the Dini derivation of (6) and the substitution of (7)-(9) imply

$$\begin{aligned}
&D^+V(t)\\
&\le \alpha\lambda e^{\lambda t}\left|\log\frac{x}{\hat{x}}\right|+\lambda e^{\lambda t}\left|\log\frac{y}{\hat{y}}\right|\\
&\quad-\alpha e^{\lambda t}\left\{b(t)+c(t)(\Delta\hat{\Delta})^{-1}\left[kxy\hat{x}+y\hat{y}^{2\sigma}\right]\right\}|x-\hat{x}|\\
&\quad+k\alpha e^{\lambda t}c(t)(\Delta\hat{\Delta})^{-1}\hat{x}x^2|y-\hat{y}|\\
&\quad+\alpha e^{\lambda t}c(t)(\Delta\hat{\Delta})^{-1}\hat{x}\hat{y}|\hat{y}^{2\sigma}-y^{2\sigma}|\\
&\quad+\alpha\int_{t}^{t+\tau}e^{\lambda l}b(l)dl\left[a(t)+b(t)x(t-\tau)\right]|x-\hat{x}|\\
&\quad+\alpha e^{\lambda t}\frac{e^{\lambda\tau-1}}{\lambda\Delta\hat{\Delta}}\left[kx^2\hat{x}^2|y-\hat{y}|+xy\hat{y}|y^{2\sigma-1}-\hat{y}^{2\sigma-1}|\right.\\
&\quad\left.+\hat{y}y^{2\sigma-1}|x^2-\hat{x}^2|\right]\\
&\quad+\alpha\int_{t+\tau}^{t+2\tau}e^{\lambda l}b(l)dl\,b(t+\tau)\hat{x}(t+\tau)|x-\hat{x}|\\
&\quad+e^{\lambda t}e(t)(\Delta\hat{\Delta})^{-1}\left[\hat{y}^{2\sigma}|x^2-\hat{x}^2|-\hat{x}^2|\hat{y}^{2\sigma}-y^{2\sigma}|\right]\\
&\le -e^{\lambda t}[\alpha G_1(t)|x-\hat{x}|+G_2(t)|y-\hat{y}|].
\end{aligned}$$

It follows from the assumptions in Theorem 2.2 that there exist positive numbers β_1,β_2 and sufficient large $T>0$ such that

$$D^+V(t)\le -\beta_1e^{\lambda t}|x-\hat{x}|-\beta_2e^{\lambda t}|y-\hat{y}|<0 \;\text{ for } t\ge T.$$

So $V(t)$ is decreasing in $[T,+\infty)$, thus $V(t)\le V(T)$ for $t\ge T$. Consequently, from the mean value theorem we get

$$\alpha M_1^{-1}|x(t)-\hat{x}(t)|+M_2^{-1}|y(t)-\hat{y}(t)|\le V(T)e^{-\lambda t}$$

for $t\ge T$. So, it follows from the decreasing of $V(t)$ that there exists a sufficient large $T_0>T$ such that

$$\begin{aligned}
&|x(t)-\hat{x}(t)|\le\alpha^{-1}M_1V(T)e^{-\lambda t},\\
&|y(t)-\hat{y}(t)|\le M_2V(T)e^{-\lambda t}.
\end{aligned} \tag{10}$$

for $t \geq T_0$. It follows from Definition 2.2 that system (1) is exponential stable. Meanwhile, from (10) we have

$$\lim_{t\to+\infty} |x(t) - \hat{x}(t)| = 0 \text{ and } \lim_{t\to+\infty} |y(t) - \hat{y}(t)| = 0.$$

Thus system (1) is global attractive. Theorem 2.2 is proved.

Proof of Theorem 2.3.
The proof of Theorem 2.3 is similar to that of Theorem 2.2, it is only needed to let $\lambda = 0$ in (6), so we omit it here.

IV. Simulation

In order to verify the feasibility of our results, we give an example.

$$\begin{cases} x' = x[6.5 + 0.01\sin(t) - 6x(t-0.01)] - \dfrac{0.5x^2y}{2x^2+y}, \\ y' = -1.3y + \dfrac{6x^2y}{2x^2+y}, \end{cases} \tag{11}$$

According to model (1), we have $a(t) = 6.5 + 0.01\sin(t)$, $b(t) = 6$, $c(t) = 0.5$, $d(t) = 1.3$, $e(t) = 6$, $k = 2$, $\tau = 0.01$, $\sigma = 0.5$. By simple calculating we get

$$M_1 = 1.157983334, \quad M_2 = 6.188886471,$$
$$m_1 = 0.9767528522, \; m_2 = 1.081252285,$$
$$\underline{H} = 5.921008333, \quad \underline{F} = 1.700000000$$

and

$$\liminf_{t\to+\infty} G1 \approx 1.000000000 \times 10^{-9},$$
$$\liminf_{t\to+\infty} G2 \approx 0.004707144943,$$

where $\alpha = 0.23121257$, $\lambda = 0.06700037647$. Moreover, we get that the interval of the exponent, which guarantees the exponential stability of system (11), is $I = (0, 0.06700037648)$ (i.e., the maximum of the exponent λ is less than 0.06700037648). Thus, by Theorems 2.1 and 2.2, we know that system (11) is permanent and exponential stable, which is showing in Fig.1.

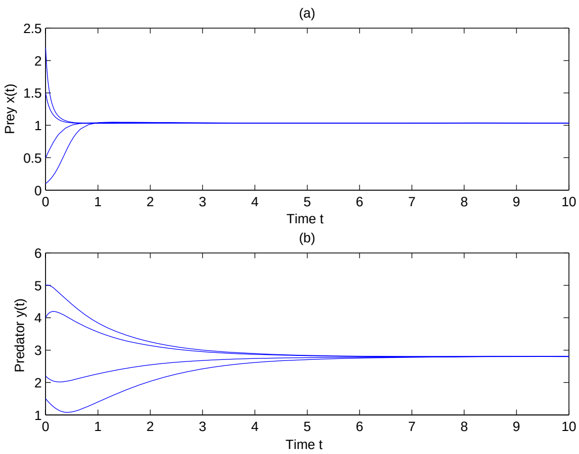

Fig.1. The integral curves of the solution of system (11) with initial values: $(x(t), y(t)) = (1.5, 5), (2.2, 4), (0.5, 2.2), (0.1, 1.5)$ for $t \in [-0.01, 0]$.

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References

[1] A. Lotka, The Elements of Physical Biology, Williams and Wilkins, Baltimore, 1925.
[2] V. Volterra, Variazioni e fluttuazioni del numero di individui in specie animali conviventi, Mem. Accd. Lincei. 2 (1926) 31-113.
[3] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, Academic Press, New York, 1993.
[4] C. Chen and F.D. Chen, Conditions for global attractivity of multispecies ecological competition-predator system with Holling III type functional response. J. Biomath. 19 (2004) 136-140.
[5] X.Z. He, Stability and delays in a predator-prey system. J. Math. Anal. Appl. 198 (1996) 355-370.
[6] F.D. Chen, Permanence and global stability of nonautonomous Lotka-Volterra system with predator-prey and deviating arguments. Appl. Math. Comput. 173 (2006) 1082-1100.
[7] M. Fan, Global existence of positive periodic solution of predator-prey system with deviating arguments. Acta Math. Appl. Sin. 23 (2000) 557-561.
[8] J.D. Zhao and W.C. Chen, Global asymptotic stability of a periodic ecological model. Appl. Math. Comput. 147 (2004) 881-892.
[9] F.D. Chen, The permanence and global attractivity of Lotka-Volterra competition system with feedback controls. Nonlinear Anal. Real World Appl. 7 (2006) 133-143.
[10] Y.H. Xia, J.D. Cao and S.S. Cheng, Periodic solutions for a Lotka-Volterra mutualism system with several delays. Appl. Math. Modelling 31 (2007) 1960-1969.
[11] C. Cosner, D.L. DeAngelis, J.S. Ault and D.B. Olson, Effect of spatial grouping on the functional response of predators, Theor. Pop. Biol. 56 (1999) 65-75.
[12] C.S. Holling, The components of predation as revealed by a study of small mammal predation of the European pine sawfly, Canad. Entomological Soc. 91 (1959) 293-320.
[13] C.S. Holling, Some characteristics of simple types of predation and parasitism, Canad. Entomologist 91 (1959) 385-395.
[14] J.R. Beddington, Mutual interference between parasites or predators and its effect on searching efficiency, J. Animal Ecology 44 (1975) 331-341.
[15] D.L. DeAngelis, Goldsten RA and Neill R, A model for trophic interaction, Ecology 56 (1975) 881-892.
[16] P.H. Crowley and E.K. Martin, Functional response and interference within and between year classes of a dragonfly population. J. North American Benthological 8 (1989) 211-21.
[17] M.P. Hassell and G.C. Varley, New inductive population model for insect parasites and its bearing on biological control, Nature 223 (1969) 1133-1137.
[18] S.-B. Hsu, T.-W. Hwang and Y. Kuang, Global dynamics of a predatorCprey model with Hassell-Varley type functional response, Discrete Contin. Dyn. Syst. Ser. B 10 (2008) 857-871.
[19] K. Wang, Permanence and global asymptotic stability of a delayed predator-prey model with Hassell-Varley type functional response, Bullet. Iranian Math. Soci. 37 (2011) 203-220.
[20] D. Schenk, L. Bersier and S. Bacher, An experimental test of the nature of predation: neither prey- nor ratio-dependent, J. Animal Ecology 74 (2005) 86-91.
[21] X.X. Liu and Y.J. Lou, Global dynamics of a predator-prey model, J. Math. Anal. Appl. 371 (2010) 323-340.