International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:3, 2013

Periodic Solutions for a Food Chain System with Monod–Haldane Functional Response on Time Scales

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Abstract—In this paper, the three species food chain model on time scales is established. The Monod–Haldane functional response and time delay are considered. With the help of coincidence degree theory, existence of periodic solutions is investigated, which unifies the continuous and discrete analogies.

Keywords—Food chain system, periodic solution, time scales, coincidence degree.

I. INTRODUCTION

RESEARCH on food chain system has been a hotspot in population dynamics recently. Dynamical behavior of these models governed by differential equations and difference equations has been extensively studied in [1–5].

Motivated by the so–called Monod–Haldane function proposed in [6], we can get the following non–autonomous food chain model with time delays,

$$\begin{cases} \dot{u}_1(t) = u_1(t)[r_1(t) - d_1(t)u_1(t) - \frac{m_{12}(t)u_2(t)}{a_1(t)+b_1(t)u_1(t)+u_1^2(t)}], \\ \dot{u}_2(t) = u_2(t)[-r_2(t) + \frac{m_{21}u_1(t-\tau)}{a_1(t)+b_1(t)u_1(t-\tau)+u_1^2(t-\tau)} \\ \qquad -d_2(t)u_2(t) - \frac{m_{23}(t)u_3(t)}{a_2(t)+b_2(t)u_2(t)+u_2^2(t)}] \\ \dot{u}_3(t) = u_3(t)[-r_3(t) + \frac{m_{32}(t)u_2(t-\sigma)}{a_2(t)+b_2(t)u_2(t-\sigma)+u_2^2(t-\sigma)} \\ \qquad -d_3(t)u_3(t)], \end{cases} \tag{1}$$

where $u_1(t)$, $u_2(t)$ and $u_3(t)$ stand for the population density of prey, predator and top–predator at time t, respectively. All coefficients are positive continuous functions. $m_{i,i+1}(t)$ is the capture rate of the predator, $m_{i+1,i}(t)$ is a measure of the food quality that the prey provided for conversion into predator birth, where $i = 1, 2$.

However, if the populations have non–overlapping generations, the discrete model governed difference equations is more appropriate :

$$\begin{cases} u_1(n+1) = u_1(n)\exp[r_1(n) - d_1(n)u_1(n) \\ \qquad -\frac{m_{12}(n)u_2(n)}{a_1(n)+b_1(n)u_1(n)+u_1^2(n)}], \\ u_2(n+1) = u_2(n)\exp[\frac{m_{21}u_1(n-\tau)}{a_1(n)+b_1(n)u_1(n-\tau)+u_1^2(n-\tau)} \\ \qquad -r_2(n) - d_2(n)u_2(n) - \frac{m_{23}(n)u_3(n)}{a_2(n)+b_2(n)u_2(n)+u_2^2(n)}], \\ u_3(n+1) = u_3(n)\exp[\frac{m_{32}(n)u_2(n-\sigma)}{a_2(n)+b_2(n)u_2(n-\sigma)+u_2^2(n-\sigma)} \\ \qquad -r_3(n) - d_3(n)u_3(n)], \end{cases} \tag{2}$$

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where all the coefficients are positive periodic sequences.

Enlightened by the idea of Stefan Hilger [7], to unify the continuous and discrete dynamic systems, we consider the following dynamic system on time scales

$$\begin{cases} x^{\Delta}(t) = r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}, \\ y^{\Delta}(t) = -r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1e^{x(t-\tau)}+e^{2x(t-\tau)}} - d_2(t)e^{y(t)} \\ \qquad -\frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}, \\ z^{\Delta}(t) = -r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}} - d_3(t)e^{z(t)}, \end{cases} \tag{3}$$

where all coefficients are positive $\omega-$periodic functions. Set $u_1(t) = e^{x(t)}$, $u_2(t) = e^{y(t)}$, $u_3(t) = e^{z(t)}$, then (3) can be reduced to (1) and (2) when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, respectively.

The aim of this paper is to study the periodicity of three–species food chain system on time scales. We would like to mention that there are several papers on periodicity in dynamic systems on time scales by using the coincidence degree theory, see [8–11]. The remainder of the paper is organized as follows. In the following section, some preliminary results about calculus on time scales and the continuation theorem are stated. Next, the sufficient conditions for the existence of periodic solutions are explored.

II. PRELIMINARIES

For convenience, we first present the useful lemma about time scales and the continuation theorem of the coincidence degree theory; more details can be found in [12–14].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers $\mathbb{R}$. Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$ and $\bigcup_{k\in\mathbb{Z}}[2k, 2k+1]$. The following definitions and lemmas about time scales are from [12].

Lemma 2.1.([13]) Let t_1, $t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R} \in C_{rd}(\mathbb{T})$ is $\omega-$periodic, then

$$g(t) \le g(t_1) + \frac{1}{2}\int_k^{k+\omega} |g^{\Delta}(s)|\Delta s$$

and

$$g(t) \ge g(t_2) - \frac{1}{2}\int_k^{k+\omega} |g^{\Delta}(s)|\Delta s,$$

the constant factor $\frac{1}{2}$ is the best possible.

For simplicity, we use the following notations throughout this paper. Let $\mathbb{T}$ be ω-periodic, that is $t \in \mathbb{T}$ implies $t+\omega \in \mathbb{T}$,

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \quad I_\omega = [k, k+\omega] \cap \mathbb{T}, \quad g^L = \inf_{t \in \mathbb{T}} g(t),$$

$$g^M = \sup_{t \in \mathbb{T}} g(t), \quad \bar{g} = \frac{1}{\omega} \int_{I_\omega} g(s)\Delta s = \frac{1}{\omega} \int_k^{k+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, i.e., $g(t+\omega) = g(t)$ for all $t \in \mathbb{T}$.

Next, we state the Mawhin's continuation theorem, which is a main tool in the proof of our theorem.

Lemma 2.2. [14] (Continuation Theorem) Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose

(a) for each $\lambda \in (0,1)$, every solution u of $Lu = \lambda N u$ is such that $u \notin \partial\Omega$;

(b) $QNu \neq 0$ for each $u \in \partial\Omega \cap \ker L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then the operator equation $Lu = Nu$ has at least one solution lying in $\operatorname{Dom} L \cap \bar{\Omega}$.

III. Main Results

Theorem 3.1. If

$$m_{32}^L e^{L_2} > r_3^M (a_2^M + b_2^M e^{M_2} + e^{2M_2})$$

holds, where

$$L_2 = \ln \frac{a_2^L r_3^L}{m_{32}^M} - \frac{m_{21}^M \omega}{b_1^L}$$

and

$$M_2 = \ln \frac{r_1^M (a_1^M + \frac{b_1^M r_1^M}{d_1^L} + \frac{2 r_1^M}{d_1^L})}{m_{12}^L} + \frac{m_{21}^M \omega}{b_1^L},$$

then system (3) has at least one $\omega-$periodic solution.

Proof Let $X = Z = \{(x,y,z)^T \in C(\mathbb{T}, \mathbb{R}^3) : x(t+\omega) = x(t),\ y(t+\omega) = y(t), z(t+\omega) = z(t), \forall t \in \mathbb{T}\}$, $\|(x,y,z)^T\| = \max_{t \in I_\omega} |x(t)| + \max_{t \in I_\omega} |y(t)| + \max_{t \in I_\omega} |z(t)|, \quad (x,y,z)^T \in X$ (or in Z). Then X and Z are both Banach spaces when they are endowed with the above norm $\|\cdot\|$. Let

$$N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix},$$

where $N_1 = r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}$, $N_2 = -r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1 e^{x(t-\tau)}+e^{2x(t-\tau)}} - d_2(t)e^{y(t)} - \frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}$, $N_3 = -r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}} - d_3(t)e^{z(t)}$.

$$L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x^\Delta \\ y^\Delta \\ z^\Delta \end{bmatrix}, \quad P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega} x(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} t(t)\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega} z(t)\Delta t \end{bmatrix}.$$

Obviously,$\ker L = \mathbb{R}^3$, $\operatorname{Im} L = \{(x,y,z)^T \in Z : \bar{x} = \bar{y} = \bar{z} = 0, t \in \mathbb{T}\}$, $\dim \ker L = 3 = \operatorname{codim} \operatorname{Im} L$. Since $\operatorname{Im} L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projections such that $\operatorname{Im} P = \ker L$ and $\operatorname{Im} L = \ker Q = \operatorname{Im}(I-Q)$. Furthermore, the generalized inverse (of L) $K_P : \operatorname{Im} L \to \ker P \cap \operatorname{Dom} L$ exists and is given by

$$K_P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \int_k^t x(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x(s)\Delta s \Delta t \\ \int_k^t y(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t y(s)\Delta s \Delta t \\ \int_k^t z(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t z(s)\Delta s \Delta t \end{bmatrix}.$$

Thus

$$QN \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_k^{k+\omega}(r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}})\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega}(-r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1 e^{x(t-\tau)}+e^{2x(t-\tau)}} \\ -d_2(t)e^{y(t)} - \frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}})\Delta t \\ \frac{1}{\omega}\int_k^{k+\omega}(-r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}} \\ -d_3(t)e^{z(t)})\Delta t \end{bmatrix},$$

and

$$K_P(I-Q)N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \int_k^t x(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t x(s)\Delta s\Delta t \\ -\left(t - k - \frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{x} \\ \int_k^t y(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t y(s)\Delta s\Delta t \\ -\left(t - k - \frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{y} \\ \int_k^t z(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t z(s)\Delta s\Delta t- \\ \left(t - k - \frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t\right)\bar{z} \end{bmatrix}.$$

Clearly, QN and $K_P(I-Q)N$ are continuous. According to the Arzela-Ascoli theorem, it is not difficulty to show that $K_P(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega \subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$.

Now, we shall search an appropriate open bounded subset Ω for the application of the continuation theorem, Lemma 2.2. For the operator equation $Lu = \lambda N u$, where $\lambda \in (0,1)$, we have

$$\begin{cases} u_1^\Delta(t) = \lambda(r_1(t) - d_1(t)e^{x(t)} - \frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}), \\ u_2^\Delta(t) = \lambda(-r_2(t) + \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1 e^{x(t-\tau)}+e^{2x(t-\tau)}} \\ \qquad -d_2(t)e^{y(t)} - \frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}), \\ u_3^\Delta(t) = \lambda(-r_3(t) + \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}} \\ \qquad -d_3(t)e^{z(t)}). \end{cases} \tag{4}$$

Assume that $(u_1, u_2, u_3)^T \in X$ is a solution of system (4) for a certain $\lambda \in (0,1)$. Integrating (4) on both sides from k to $k+\omega$, we obtain

$$\begin{cases} \int_k^{k+\omega}[d_1(t)e^{x(t)} + \frac{m_{12}(t)e^{y(t)}}{a_1(t)+b_1(t)e^{x(t)}+e^{2x(t)}}]\Delta t = \bar{r}_1\omega, \\ \int_k^{k+\omega}[r_2(t) + d_2(t)e^{y(t)} + \frac{m_{23}(t)e^{z(t)}}{a_2(t)+b_2(t)e^{y(t)}+e^{2y(t)}}]\Delta t \\ \qquad = \int_k^{k+\omega} \frac{m_{21}(t)e^{x(t-\tau)}}{a_1(t)+b_1 e^{x(t-\tau)}+e^{2x(t-\tau)}}\Delta t, \\ \int_k^{k+\omega}[r_3(t) + d_3(t)e^{z(t)}]\Delta t \\ \qquad = \int_k^{k+\omega} \frac{m_{32}(t)e^{y(t-\sigma)}}{a_2(t)+b_2(t)e^{y(t-\sigma)}+e^{2y(t-\sigma)}}\Delta t. \end{cases} \tag{5}$$

Since $(x, y, z)^T \in X$, there exist $\xi_i, \eta_i \in I_\omega$, $i = 1, 2, 3$, such that

$$\begin{array}{ll} x(\xi_1) = \min_{t\in I_\omega}\{x(t)\}, & x(\eta_1) = \max_{t\in I_\omega}\{x(t)\}, \\ y(\xi_2) = \min_{t\in I_\omega}\{y(t)\}, & y(\eta_2) = \max_{t\in I_\omega}\{y(t)\}, \\ z(\xi_3) = \min_{t\in I_\omega}\{z(t)\}, & z(\eta_3) = \max_{t\in I_\omega}\{z(t)\}. \end{array} \quad (6)$$

From (4) and (5), we have

$$\int_k^{k+\omega} \left|x^\Delta(t)\right| \Delta t \le 2\bar{r}_1\omega,$$

$$\int_k^{k+\omega} \left|y^\Delta(t)\right| \Delta t \le 2\frac{m_{21}^M\omega}{b_1^L},$$

$$\int_k^{k+\omega} \left|z^\Delta(t)\right| \Delta t \le 2\frac{m_{32}^M\omega}{b_2^L}.$$

By the first equation of (5) and (6),

$$d_1(\xi_1)e^{x(\xi_1)} < r_1(\xi_1),$$

that is

$$x(\xi_1) < \ln\frac{r_1^M}{d_1^L}.$$

From the second equation of (5), we have

$$r_2(\eta_2) < \frac{m_{21}(\eta_2)e^{x(\eta_2-\tau)}}{a_1(\eta_2)}$$

and

$$x(\eta_1) \ge x(\eta_2 - \tau) > \ln\frac{r_2^L a_1^L}{m_{21}^M}.$$

According to Lemma 2.1, we have

$$\begin{aligned} x(t) &\le x(\xi_1) + \frac{1}{2}\int_k^{k+\omega}\left|x^\Delta(t)\right|\Delta t \\ &\le \ln\frac{r_1^M}{d_1^L} + \bar{r}_1\omega \triangleq M_1, \end{aligned}$$

$$\begin{aligned} x(t) &\ge x(\eta_1) - \frac{1}{2}\int_k^{k+\omega}\left|x^\Delta(t)\right|\Delta t \\ &\ge \ln\frac{r_2^L a_1^L}{m_{21}^M} - \bar{r}_1\omega \triangleq L_1. \end{aligned}$$

From the first equation of (5) and (6), we can obtain

$$\frac{m_{12}(\xi_1)e^{y(\xi_1)}}{a(\xi_1) + b_1(\xi_1)e^{x(\xi_1)} + e^{2x(\xi_1)}} < r_1(\xi_1),$$

and

$$y(\xi_2) < y(\xi_1) < \ln\frac{r_1^M\left(a_1^M + \frac{b_1^M r_1^M}{d_1^L} + \frac{2r_1^M}{d_1^L}\right)}{m_{12}^L},$$

then

$$\begin{aligned} y(t) &\le y(\xi_2) + \frac{1}{2}\int_k^{k+\omega}\left|y^\Delta(t)\right|\Delta t \\ &< \ln\frac{r_1^M\left(a_1^M + \frac{b_1^M r_1^M}{d_1^L} + \frac{2r_1^M}{d_1^L}\right)}{m_{12}^L} + \frac{m_{21}^M\omega}{b_1^L} \\ &\triangleq M_2. \end{aligned}$$

From the third equation of (5), we have

$$\frac{m_{32}(\xi_3)}{a_2(\xi_3)}e^{y(\xi_3-\sigma)} > r_3(t),$$

this reduces to

$$y(\eta_2) \ge y(\xi_3 - \sigma) > \ln\frac{a_2^L r_3^L}{m_{32}^M},$$

then

$$\begin{aligned} y(t) &\ge y(\eta_2) - \frac{1}{2}\int_k^{k+\omega}\left|y^\Delta(t)\right|\Delta t \\ &\ge \ln\frac{a_2^L r_3^L}{m_{32}^M} - \frac{m_{21}^M\omega}{b_1^L} \triangleq L_2. \end{aligned}$$

According to the first equation of (5), we have

$$\frac{m_{32}(\xi_3)}{b_2(\xi_3)} > d_3(\xi_3)e^{z(\xi_3)},$$

then

$$\begin{aligned} z(t) &\le z(\xi_3) + \frac{1}{2}\int_k^{k+\omega}\left|z^\Delta(t)\right|\Delta t \\ &\le \ln\frac{m_{32}^M}{b_2^L d_3^L} + \frac{m_{32}^M\omega}{b_2^L} \triangleq M_3. \end{aligned}$$

Besides, we have

$$d_3^M e^{z(\eta_3)} > \frac{m_{32}^L e^{L_2}}{a_2^M + b_2^M e^{M_2} + e^{2M_2}} - r_3^M,$$

then

$$\begin{aligned} z(t) &\ge z(\eta_3) - \frac{1}{2}\int_k^{k+\omega}\left|z^\Delta(t)\right|\Delta t \\ &\ge \ln\frac{\frac{m_{32}^L e^{L_2}}{a_2^M + b_2^M e^{M_2} + e^{2M_2}} - r_3^M}{d_3^M} - \frac{m_{32}^M\omega}{b_2^L} \\ &\triangleq L_3. \end{aligned}$$

Therefore, we have

$$\begin{aligned} \max_{t\in[k,k+\omega]}|x(t)| &\le \max\{|M_1|, |L_1|\} \triangleq R_1, \\ \max_{t\in[k,k+\omega]}|y(t)| &\le \max\{|M_2|, |L_2|\} \triangleq R_2, \\ \max_{t\in[k,k+\omega]}|z(t)| &\le \max\{|M_3|, |L_3|\} \triangleq R_3. \end{aligned}$$

Clearly, R_1, R_2 and R_3 are independent of λ. Let $R = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that for for the following algebraic equations:

$$\begin{cases} \bar{r}_1 - \bar{d}_1 e^x - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{m_{12}(t)e^y}{a_1(t)+b_1(t)e^x+e^{2x}}\Delta t = 0, \\ -\bar{r}_2 + \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{m_{21}(t)e^x}{a_1(t)+b_1(t)e^x+e^{2x}}\Delta t - \bar{d}_2 e^y \\ \qquad -\frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{\bar{m}_{23}e^z}{a_2(t)+b_2(t)e^y+e^{2y}}\Delta t = 0, \\ -\bar{r}_3 + \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\frac{m_{32}(t)e^y}{a_2(t)+b_2(t)e^y+e^{2y}}\Delta t - \bar{d}_3 e^z = 0, \end{cases} \quad (7)$$

every solution $(x^*, y^*, z^*)^T$ of (7) satisfies $\|(x^*, y^*, z^*)^T\| < R$. Now, we define $\Omega = \{(x, y, z)^T \in X : \|(x, y, z)^T\| < R\}$. Then it is clear that Ω verifies the requirement (a) of Lemma 2.2. If $(u_1, u_2, u_3)^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^3$, then $(x, y, z)^T$ is a constant vector in $\mathbb{R}^3$ with $\|(x, y, z)^T\| = |x|+|y|+|z| = R$, so we have

$$QN \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

By the assumption in Theorem 3.1 and the definition of topological degree, a direct calculation yields $\deg(JQN, \Omega \cap \ker L, 0) \neq 0$. We have verified that Ω satisfies all requirements of Lemma 2.2; therefore, system (3) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$. This completes the proof.

Acknowledgment

This work was supported by the Anhui Provincial Natural Science Foundation under Grant 1208085QA11, Natural Science Foundation of the Higher Education Institutions of Anhui Province under Grant KJ2012B002 and KJ2012B004.

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