

# Performance Evaluation of Prioritized Limited Processor-Sharing System

Yoshiaki Shikata, Wataru Katagiri, Yoshitaka Takahashi

**Abstract**—We propose a novel prioritized limited processor-sharing (PS) rule and a simulation algorithm for the performance evaluation of this rule. The performance measures of practical interest are evaluated using this algorithm. Suppose that there are two classes and that an arriving (class-1 or class-2) request encounters  $n_1$  class-1 and  $n_2$  class-2 requests (including the arriving one) in a single-server system. According to the proposed rule, class-1 requests individually and simultaneously receive  $m / (m * n_1 + n_2)$  of the service-facility capacity, whereas class-2 requests receive  $1 / (m * n_1 + n_2)$  of it, if  $m * n_1 + n_2 \leq C$ . Otherwise ( $m * n_1 + n_2 > C$ ), the arriving request will be queued in the corresponding class waiting room or rejected. Here,  $m (\geq 1)$  denotes the priority ratio, and  $C (\leq \infty)$ , the service-facility capacity. In this rule, when a request arrives at [or departs from] the system, the extension [shortening] of the remaining sojourn time of each request receiving service can be calculated using the number of requests of each class and the priority ratio. Employing a simulation program to execute these events and calculations enables us to analyze the performance of the proposed prioritized limited PS rule, which is realistic in a time-sharing system (TSS) with a sufficiently small time slot. Moreover, this simulation algorithm is expanded for the evaluation of the prioritized limited PS system with  $N \geq 3$  priority classes.

**Keywords**—PS rule, priority ratio, service-facility capacity, simulation algorithm, sojourn time, performance measures

## I. INTRODUCTION

THE Processor-Sharing (PS) discipline has gained an important role in evaluating the performance of a variety of a resource allocation mechanism. Under processor-sharing (PS) discipline, if there are  $n (> 0)$  requests in a single server system, then each request receives  $1 / n$  of the service-facility capacity (called the service ratio for individual request). No arriving request has to wait for service because it will be served promptly, even if the service rate becomes slow [1]- [4]. The PS paradigm emerged as an idealization of Round-Robin scheduling algorithms in time-shared computer system. In such a PS paradigm, with an increase in the number of arriving requests, the service ratio for individual request decreases. Therefore, in theory, the sojourn time of each request increases to infinity with an increase in the number of arriving requests. In order to prevent an increase in the sojourn time of each request in such a PS paradigm and to realize a realistic model of sharing, a method for limiting the number of requests receiving service has been proposed [5]. In addition, a PS discipline with a priority structure has been proposed, wherein a larger service ratio is allocated to the high-priority request than that for a low-priority request [1].

In order to prevent excessive increase in the sojourn time of each request in such a PS discipline with a priority structure, we propose a prioritized limited PS system. In the proposed system, a high-priority request is allocated the service ratio that is  $m (\geq 1)$ , called the priority ratio) times that for a low-priority request. Moreover, the sum of the number of the requests receiving service is kept below a fixed value. The arriving request which cannot receive service will be queued or rejected. Performance measures of practical interest, e.g., the loss probability, waiting time in queue, and mean sojourn time in the server are evaluated using simulation programs. Under the PS rule, when a request either arrives at or departs from the system, the remaining sojourn time of the other requests will be extended or reduced, respectively. In our priority system, this extension or reduction of the sojourn time is calculated using the number of requests of each class and the priority ratio. Employing a simulation program to execute these events and calculations enables us to analyze the performance of the proposed prioritized limited PS rule, which is realistic in a time-sharing system (TSS) with a sufficiently small time slot. Moreover, this simulation algorithm is expanded for the analysis of the prioritized limited PS model with  $N \geq 3$  priority classes.

## II. PROCESSOR SHARING

### A. An approximate formula for the mean number of requests

An approximate formula for the mean number of requests in the GI/G/1 (PS) system has been obtained as shown below it [6]-[8].

The request-arrival process forms a renewal process with independent and identically distributed (iid) inter-arrival time,  $A$ , with arrival rate  $\lambda$ , and squared coefficient of variation as

$$\begin{aligned} E(A) &= 1/\lambda, \\ C_A^2 &= V(A)/E(A)^2 \end{aligned} \quad (1)$$

The requested service time of an arriving request is iid with mean and squared coefficient of variation as

$$\begin{aligned} E(B) &= 1/\mu, \\ C_B^2 &= V(B)/E(B)^2 \end{aligned} \quad (2)$$

The traffic intensity,  $\rho$  is then given by

$$\rho = \gamma/\mu \quad (3)$$

which is assumed to be less than unity ( $\rho < 1$ ) for the system stability. An approximate formula for the mean number of requests,  $E(L)$ , in the GI/G/1 (PS) system is obtained by

$$E(L) = \{\rho^2 (C_A^2 + C_B^2)g(\rho, C_A^2, C_B^2) + \rho(1 - \rho)(1 + C_B^2)\} / \{(1 - \rho)(1 + C_B^2)\}$$

where  $g(\rho, x, y) = \exp \{-2(1 - \rho)(1 - x)^2/3\rho(x + y)\}$  if  $x \leq 1$

$$= \exp \{-(1 - \rho)(x - 1)/(x + 4y)\} \quad \text{if } x > 1 \quad (4)$$

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### B. Simulation algorithm

Under the PS rule, whenever a new request arrives or the requested service time of a request is over, the remaining sojourn time of each request receiving service is extended or reduced respectively. This extension or reduction of the remaining sojourn time can be calculated using the number of requests. By chasing these numerical changes in the remaining sojourn time in the simulation program (Fig.1), performance measures of practical interest, e.g., the mean sojourn time for a request, the mean number of requests in the system, and the maximum number of requests are evaluated.

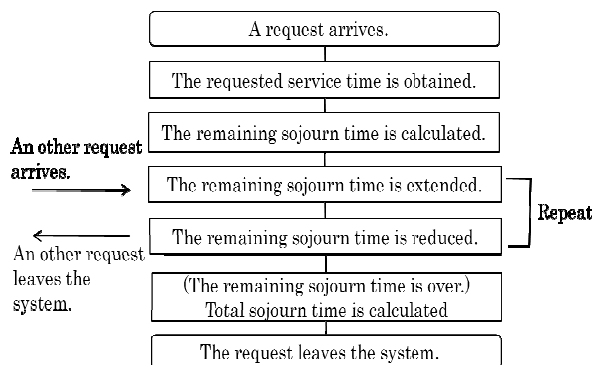


Fig.1 Processing flow of an arriving request in the simulation program

When  $n$  requests are being served, if a request arrives,  $1/(n+1)$  of the service-facility capacity will be given to this request from this time forward, until the arrival [departure] of the next request. The sojourn time of an arriving request  $S_a$  is then given by

$$S_a = S_r * (n + 1) \quad (5)$$

where  $S_r$  is the requested service time of an arriving request.

Moreover,  $1/n$  of the service-facility capacity is given to each request receiving service by this time, but from this time forward, till the arrival [departure] of the next request,  $1/(n+1)$  of the service-facility capacity will be given to each request. The remaining sojourn time of each request  $S_n$  is then extended as follows:

$$S_n = S_o * (n + 1)/n \quad (6)$$

where  $S_o$  is the remaining sojourn time of each request just before this request arrives.

In addition, at the end of the sojourn time of a request,  $S_n$  is reduced as follows:

$$S_n = S_o * (n - 1)/n \quad (7)$$

The mean number of requests obtained using the approximate formula mentioned in Sec.A, or the simulation program are compared in Table 1. This simulation program was developed employing the variable increment method, and written by C language. Both results almost agree. Therefore, it may be said that the approximate formula or the simulation program expresses the movement of the system precisely.

TABLE 1

THE MEAN NUMBER OF REQUESTS FOR E2/H2/1 MODEL (MEAN SERVICE TIME: 1)

$\lambda$	0.1	0.3	0.5	0.7	0.9
Approximation	0.105	0.392	0.890	2.02	7.60
Simulation	0.104	0.378	0.853	1.95	7.52

*	0.0004	0.0017	0.0028	0.0087	0.114
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\*95% reliability section

### III. PRIORITIZED LIMITED PROCESSOR-SHARING

#### A. Evaluation Model

Suppose that there are two classes (class-1 or class-2) and that an arriving request (class-1 or class-2) encounters  $n_1$  class-1 and  $n_2$  class-2 requests (including the arriving one) in a single-server system. If  $m * n_1 + n_2 \leq C$ , class-1 requests individually and simultaneously receive  $m / (m * n_1 + n_2)$  of the service-facility capacity (called the service ratio for a class-1 request), while class-2 requests receive  $1 / (m * n_1 + n_2)$  of the service-facility capacity (called the service ratio for a class-2 request). Otherwise if  $(m * n_1 + n_2) > C$ , the arriving request will be queued in the corresponding class waiting room (called queuing system) or will be rejected (in what is called a loss system). Here,  $m (\geq 1)$  denotes the priority ratio and  $C (\leq \infty)$  denotes the service facility capacity. Our rule includes Kleinrock's priority PS rule [1] as a special case ( $C = \infty$ ) and the limited PS rule [5] as a special case ( $m = 1$ , and say  $n_2 \equiv 0$ ). In the queuing system, when the requested service time of a request is over and it leaves the system, first a class-1 request is taken from its queue and it begins to receive service. If no class-1 requests remain in the queue or if a class-1 request cannot receive service because of service-facility capacity restrictions, a class-2 request is taken from its queue and it begins to receive service. Approximate formulas for the performance measures in the proposed system have not been obtained.

#### B. Simulation algorithm

Under the prioritized limited PS rule, the extension or reduction of the remaining sojourn time of each request receiving service can also be calculated using the number of each class requests and the priority ratio. By chasing these numerical changes in the remaining sojourn time in the simulation program, the loss probability, mean waiting time in the queue, and mean sojourn time for an individual class request in the server are evaluated.

When  $n_1$  class-1 requests and  $n_2$  class-2 requests are being served, if a class-1 request arrives,  $m / \{m * (n_1 + 1) + n_2\}$  of the service-facility capacity will be given to this request from this time forward, until the arrival [departure] of the next request. The sojourn time of an arriving request  $S_a$  is then given by

$$S_a = S_r * \{m * (n_1 + 1) + n_2\} / m \quad (8)$$

where  $S_r$  is the requested service time of an arriving request. Moreover,  $m / (m * n_1 + n_2)$  (to a class-1 request) or  $1 / (m * n_1 + n_2)$  (to a class-2 request) of the service-facility capacity is given to each request receiving service by this time, but from this time forward, till the arrival [departure] of the next request,  $m / \{m * (n_1 + 1) + n_2\}$  (to a class-1 request) or  $1 / \{m * (n_1 + 1) + n_2\}$  (to a class-2 request) of the service-facility capacity will be given to each request. Therefore, the remaining sojourn time of each request after this request arrives  $S_{na}$  is then extended as follows:

$$S_{na} = S_{oa} * \{m * (n_1 + 1) + n_2\} / (m * n_1 + n_2) \quad (9)$$

where  $Soa$  is the remaining sojourn time of each request just before this request arrives. Similarly, at the arrival of a class-2 request, the sojourn time of an arriving request  $Sa$  is given by

$$Sa = Sr * (m * n_1 + n_2 + 1) \quad (10)$$

$Sna$  is extended as follows:

$$Sna = Soa * (m * n_1 + n_2 + 1) / (m * n_1 + n_2) \quad (11)$$

In addition, at the end of the sojourn time of a class-1 request, the remaining sojourn time of each request after this request departs from the system  $Sn1$  is reduced as follows:

$$Sn1 = Sol * \{m * (n_1 - 1) + n_2\} / (m * n_1 + n_2) \quad (12)$$

where  $Sol$  is the remaining sojourn time of each request just before this request departs from the system. At the conclusion of service of a class-2 request,  $Sn1$  is reduced as follows:

$$Sn1 = Sol * (m * n_1 + n_2 - 1) / (m * n_1 + n_2) \quad (13)$$

### C. Evaluation Results

In the evaluation, the priority ratio  $m$  is assumed to be 2, and the two-stage Erlang inter-arrival distribution and the two-stage hyper-exponential service time distribution are considered. The arrival rate and mean requested service time of each class request are assumed to have the same value. The simulation program for the performance evaluation of this rule was developed by modifying the program shown in Fig.1 so that there is the arrival of the requests consisting of two classes. Evaluation results are obtained as the average of ten simulation results.

#### 1. Loss system

Fig.2 shows the evaluation results for the relationship between the loss probability and the service-facility capacity. 95% reliability sections obtained from the ten simulation results are included in the range of markers. These evaluation results show that:

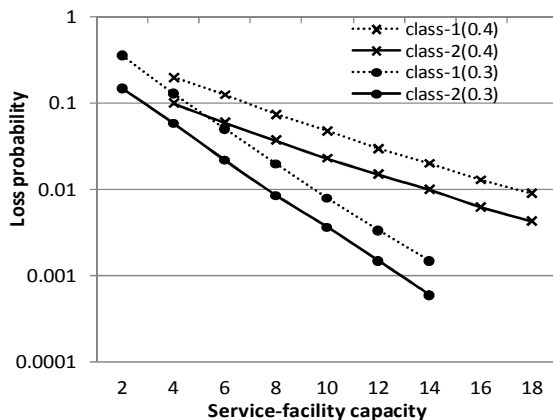


Fig. 2 Loss probability (Mean service time=1, Arrival rate =0.3 or 0.4)

(a) The logarithm of the loss probability decreases linearly with an increase in the service-facility capacity.

(b) The loss probability in the case of an arrival rate of 0.3 (indicated by the round markers) is more strongly affected by the decrease in the service-facility capacity than in the case of an arrival rate of 0.4 (indicated by the cross markers).

Fig.3 shows the evaluation results for the relationship between the mean sojourn time and the service-facility capacity for the loss system. The mean sojourn time of a class-1 request

(indicated by the dotted line) is smaller than that value for the class-2 request (shown as the solid line). In the case of 0.4 arrival rate, the difference of the mean sojourn time between two class requests becomes larger with increasing the service-facility capacity. On the other hands, in the case of 0.3 arrival rate, that difference is almost constant regardless of the service-facility capacity.

#### 2. Infinite queuing system

Fig.4 shows an evaluation example of the relationship between the mean sojourn time and the service-facility capacity for the queuing system, where an infinite waiting room is prepared. Both the mean waiting time in the queue (shown as the cross maker) and the mean sojourn time in the server (shown as the round maker) are evaluated. With the decrease of the service-facility capacity, the probability that the class-1 request enters the waiting room becomes higher than that value in the case of the class-2 request. Therefore, the mean waiting time of the class-1 class request becomes larger than that value for the class-2 class request. On the other hands, larger service-facility capacity is given to the class-1 request than that value in the case of the class-2 request. Therefore, the mean sojourn time of the class-1 request is smaller than that value of the class-2 request.

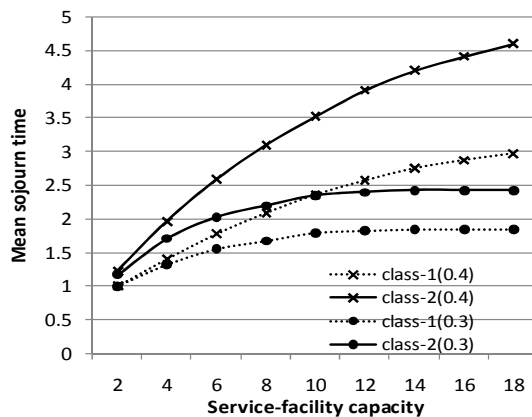


Fig. 3 Mean sojourn time (Mean service time=1, Arrival rate =0.3 or 0.4)

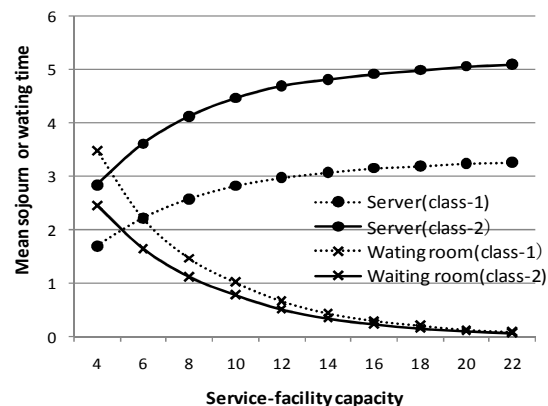


Fig. 4 Mean sojourn time and waiting time (Mean service time=1, Arrival rate=0.4)

### 3. Finite queuing system

Fig.5 shows the evaluation results of the relationship between the mean sojourn time and the service-facility capacity for the queuing system, where five waiting rooms are prepared. Even in the case of small service-facility capacity, the number of the class-1 requests entering into the queue is limited. Therefore, the mean waiting time in the queues of the class-1 request does not become large like as in the case of the infinite queuing system. Fig.6 shows the evaluation results of the loss probability. With decreasing the service-facility capacity, the loss probability of the class-1 class request becomes larger rapidly than that value of the class-2 class request. In the real system the number of waiting room and the service-facility capacity should be decided, considering a relation between the loss probability and the mean sojourn time.

### 4. Total mean sojourning time

With a decrease in the service-facility capacity, the probability that the request will enter the queue increases, and as a result, the mean waiting time in the queue increases. On the other hand, with a decrease in the service-facility capacity, the mean sojourn time in the server decreases. Therefore, under certain traffic conditions, there may be a service-facility capacity that minimizes the total mean sojourn time, which is the sum of the waiting time in the queue and the mean sojourn time in the server. Fig.7 shows the evaluation results for the total mean sojourn time in the case that infinite number of rooms or five waiting rooms is prepared. In the case of infinite number of waiting rooms, with a decrease in the service-facility capacity, the total mean sojourn time of the class-1 request increases rapidly.

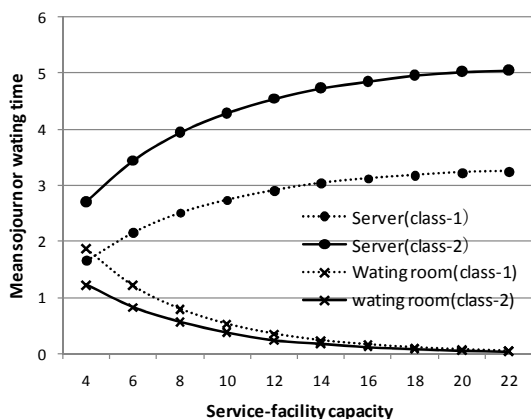


Fig. 5 Mean sojourn time and waiting time (Mean service time=1, Arrival rate=0.4)

On the other hand, the increment in the mean waiting time for a class-2 request is approximately the same as the decrement in the mean sojourn time in the server. Therefore, the total mean sojourn time is approximately the same, regardless of the service-facility capacity. In the case of five waiting rooms, the mean waiting time for each class request does not increase as it does in the case of infinite number of waiting rooms. Therefore, the total mean sojourn time level of a class-1 request becomes a minimum at a service-facility capacity of 10 and then almost becomes constant regardless of

the service-facility capacity. This value of a class-2 request decreases with a decrease in the service-facility capacity, as influenced by the decrease in the sojourn time in the server.

## IV. EXPANSION TO N PRIORITY CLASSES MODEL

### A. Evaluation model

The simulation algorithm mentioned in chapter III is expanded for the analysis of the prioritized limited PS model with  $N \geq 3$  priority classes. Suppose that there are  $N$  classes and that an arriving request encounters  $n_i$  class- $i$  ( $i: 1 - N$ ) requests (including the arriving one) in a single-server system. Class- $i$  request individually and simultaneously receives  $m_i / \sum_{i=1}^N m_i * n_i$  of the service capacity, if  $\sum_{i=1}^N m_i * n_i \leq C$ . Otherwise ( $\sum_{i=1}^N m_i * n_i > C$ ), the arriving request will be queued in the corresponding class waiting room or rejected. Here,  $m_i$  ( $\geq 1$ ,  $m_N = 1$ ) denotes the priority ratio of class- $i$  requests, and  $C$  ( $\leq \infty$ ) the service-facility capacity.

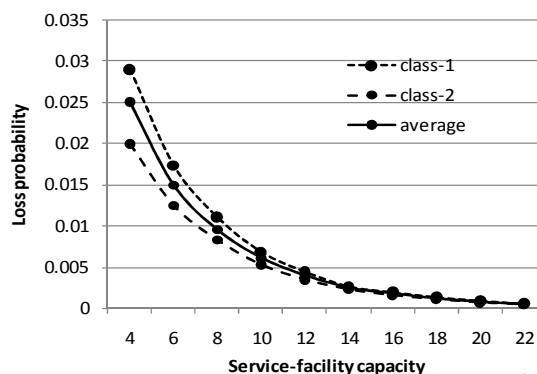


Fig. 6 Loss probability for finite queuing system (Mean service time=1, Arrival rate = 0.4)

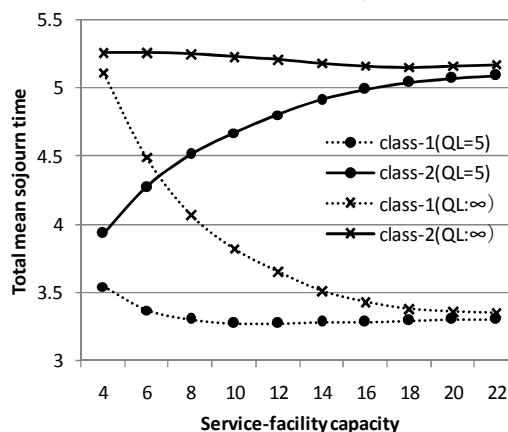


Fig. 7 Total mean sojourn time (Mean service time=1, Arrival rate=0.4)

### B. Simulation algorithm

At the arrival of a request, or at the end of a sojourn time of a request, a remaining sojourn time for each request receiving service is extended or reduced. When  $n_i$  class- $i$  requests are being served, if a class- $k$  request arrives,

$m_k / \{ \sum_{i=1}^{k-1} m_i * n_i + m_k * (n_k + 1) + \sum_{i=k+1}^N m_i * n_i \}$  of the service-facility capacity will be given to this request from this time forward, until the arrival [departure] of the next request. The sojourn time of an arriving request  $S_a$  is then given by

$$S_a = S_r * \{ \sum_{i=1}^{k-1} m_i * n_i + m_k * (n_k + 1) + \sum_{i=k+1}^N m_i * n_i \} / m_k \quad (14)$$

where  $S_r$  is the requested service time of an arriving request.

Moreover,  $m_i / (\sum_{i=1}^N m_i * n_i)$  of the service-facility capacity is given to  $n_i$  class request receiving service by this time, but from this time forward, till the arrival [departure] of the next request,

$m_i / \{ \sum_{i=1}^{k-1} m_i * n_i + m_k * (n_k + 1) + \sum_{i=k+1}^N m_i * n_i \}$  will be given to  $n_i$  class request. Therefore, the remaining sojourn time of each request after this request arrives  $S_{na}$  is then extended as follows:

$$S_n = S_o *$$

$$\frac{\{ \sum_{i=1}^{k-1} m_i * n_i + m_k * (n_k + 1) + \sum_{i=k+1}^N m_i * n_i \}}{(\sum_{i=1}^N m_i * n_i)} \quad (15)$$

where  $S_o$  is the remaining sojourn time of each request just before that this class-k request arrives.

Similarly, at the end of sojourn time of a class-k request,  $S_n$  is reduced as follows:

$$S_n = S_o *$$

$$\frac{\{ \sum_{i=1}^{k-1} m_i * n_i + m_k * (n_k - 1) + \sum_{i=k+1}^N m_i * n_i \}}{(\sum_{i=1}^N m_i * n_i)} \quad (16)$$

When the number of requests being served in the system exceeds the fixed value ( $\sum_{i=1}^N m_i * n_i > C$ ), the arriving request will be queued in the corresponding class waiting room (called queuing system) or rejected (called loss system). In the queuing system, when the sojourn time of a request is over and it leaves the system, the first priority request is picked up from its queue, and start receiving service. If no first priority requests remain in the queue or if these requests cannot receive service because of service-facility capacity restrictions, the second priority request is picked up from its queue, and start receiving service. Then, a lower priority request is picked up from its queue, and start receiving service one after another with the order of priority.

### C. Evaluation results

Using a simulation program, the loss probability in the loss system, and the mean sojourn time in the loss or queuing system, where three priority classes ( $m_1=4, m_2=2, m_3=1$ ) are considered, are evaluated. The 2-stage Erlang inter-arrival distribution and the 2-stage hyper-exponential service time distribution are considered. The arrival rate, and the mean requested service time of each class request are assumed to be the same value.

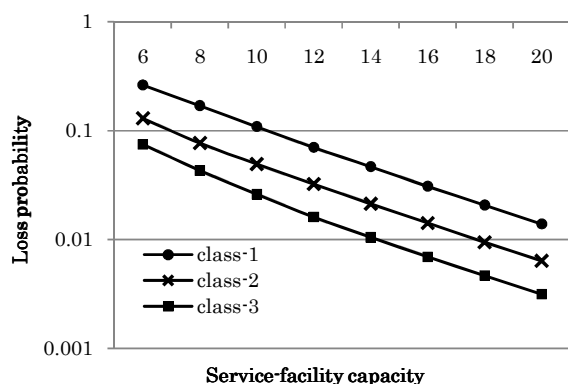


Fig.8 Loss probability (Mean service time=0.6, Arrival rate=0.4)

### 1. Loss system

Fig.8 shows the evaluation results of the relationship between the loss probability and the service-facility capacity for the loss system. These evaluation results show that:

- (a) Logarithm of the loss probability decreases linearly with increasing service-facility capacity.
- (b) Logarithms of the loss probability of each priority class decrease by approximately the same proportion.

Fig.9 shows the evaluation results of the relationship between the mean sojourn time and the service-facility capacity for the loss system. The mean sojourn time of the class-1 request becomes large more rapidly than that value for the class-2, 3 requests with increasing the service-facility capacity.

### 2. Infinite Queuing system

Fig. 10 shows an evaluation example of the relationship between the total mean sojourn time and the service-facility capacity for the queuing system, where an infinite waiting room is prepared. With the decrease of the service-facility capacity, the total mean sojourn time of the class-1 requests becomes large like as in the case of two request classes (Fig.7). On the other hands, the total mean sojourn time of the class-3 request is strongly affected by the mean sojourn time in the server. Therefore, the total mean sojourn time of the class-3 request increases with increasing the service-facility capacity, which is contrary to the case of the class-1 request. In the case of the class-2 request, the total mean sojourn time is hardly affected by the service-facility capacity. In this traffic model, when the service-facility capacity exceed 20, the mean sojourn time of becomes approximately constant. Therefore, it may be said that it is not necessary to prepare the service-facility capacity more than 20.

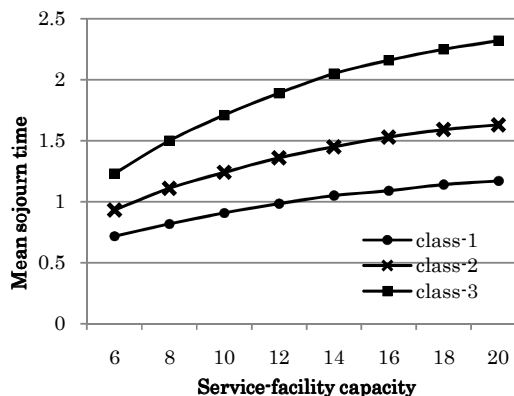


Fig. 9 Mean sojourn time (Mean service time=0.6, Arrival rate=0.4)

### 3. Finite queuing system

Fig.11 shows the evaluation results of the relationship between the total mean sojourn time and the service-facility capacity for the queuing system, where five waiting rooms are prepared. Even in the case the service-facility capacity is small, the number of the class-1 request entering into the waiting room is limited. Therefore, the total mean sojourn time of the class-1 request does not become large like as in the case of infinite queuing system (Fig.10). The total mean sojourn time of the class-3 request is strongly affected by the mean sojourn time in the server, and becomes large rapidly with increasing the service-facility capacity. On the other hands, the total mean sojourn time of the class-2 request increases with increasing the service-facility capacity, which is contrary to the case of the infinite queuing system (Fig.10). Fig.12 shows the evaluation results of the loss probability. The difference between the loss probability of the class-2 request and that of the class-3 request becomes smaller than that in the case of the infinite queuing system.

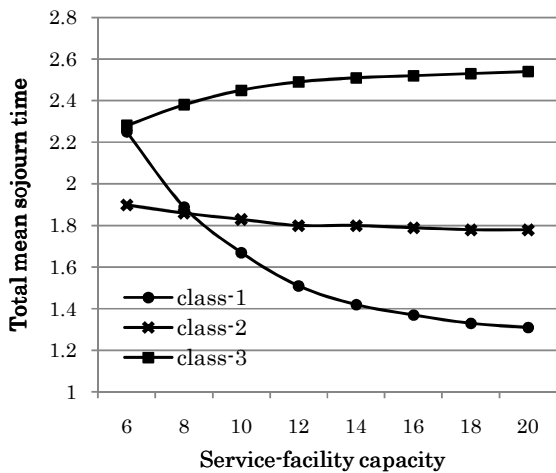


Fig. 10 Total mean sojourn time (Mean service time=0.6, Arrival rate =0.4)

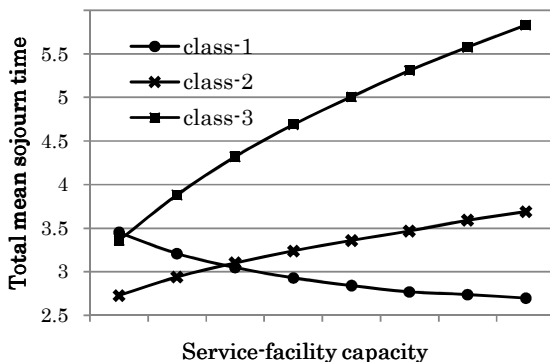


Fig. 11 Total mean sojourn time (Mean service time=0.6, Arrival rate =0.5)

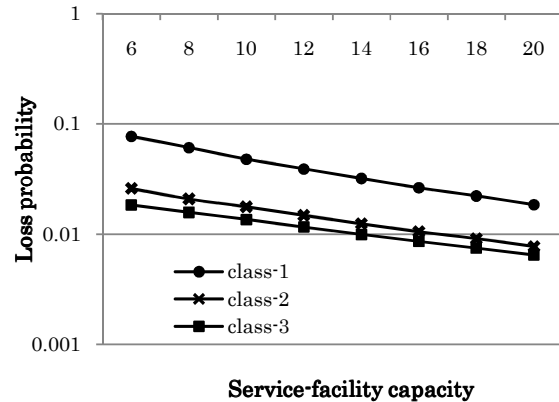


Fig. 12 Loss probability (Mean service time=0.6, Arrival rate=0.5)

### V. CONCLUSION

We proposed a novel prioritized limited processor sharing rule, and the simulation algorithm for the performance evaluation of this rule. In this simulation algorithm, at the arrival or the departure of a request the remaining sojourn time of all requests receiving service are reevaluated. Using simulation programs, performance measures of practical interest, such as the loss probability, mean waiting time in the queue, and mean sojourn time in the server, were clarified. In the case of the 2 classes model, it have shown that the service-facility capacity that minimizes the total mean sojourn time, which is the sum of the waiting time in the queue and mean sojourn time in the server, can be obtained. In the case of the 3 classes model with the infinite waiting rooms, the total mean sojourn time of the request with the highest priority is affected by the service facility capacity greatly. On the other hands, in the case of the 3 classes model with finite waiting rooms, that value of the request with the lowest priority is affected by the service facility capacity greatly. In the future, we intend to study the prioritized, limited PS model considering the quantum size (ex. time slot length in the TSS system).

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