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# Performance Analysis of the Time-Based and Periodogram-Based Energy Detector for Spectrum Sensing

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***Abstract*****—**Classically, an energy detector is implemented in time domain (TD). However, frequency domain (FD) based energy detector has demonstrated an improved performance. This paper presents a comparison between the two approaches as to analyze their pros and cons. A detailed performance analysis of the classical TD energy-detector and the periodogram based detector is performed. Exact and approximate mathematical expressions for probability of false alarm (Pf) and probability of detection (Pd) are derived for both approaches. The derived expressions naturally lead to an analytical as well as intuitive reasoning for the improved performance of (Pf) and (Pd) in different scenarios. Our analysis suggests the dependence improvement on buffer sizes. Pf is improved in FD, whereas Pd is enhanced in TD based energy detectors. Finally, Monte Carlo simulations results demonstrate the analysis reached by the derived expressions.

***Keywords*****—**Cognitive radio, energy detector, periodogram, spectrum sensing.

## I. Introduction

COGNITIVE Radio (CR) is a transceiver which senses the spectrum and utilizes the unused portions of the spectrum efficiently. It monitors the frequency bands, and whenever a vacant slot is detected, it is assigned to the unlicensed (secondary) user, without interfering with the authorized (primary) user [1], [2]. Multiple spectrum sensing techniques have been proposed, including energy detection (ED) [3], matched filtering detection (MF) [4], cyclostationary detection (CSD) [5], eigenvalue-based sensing [6], covariance-based sensing [7], etc. Three main spectrum sensing techniques vigorously used to determine spectrum holes are: MF, CSD, and ED. MF uses a known pattern to correlate the signals. This approach maximizes the received Signal to Noise ratio (SNR) and thus it is the optimum sensing method but it requires a-priori information about the signal waveforms, which at times is not available [8]-[10]. CSD exploits some periodic characteristics of the desired signal to perform detection. Noise power and source signal information is required by CSD. It works well over low SNR regime but requires complex calculations [5], [11].

ED method identifies the presence or absence of a signal based on energy estimation. In this detection technique, only noise power information is required. It is not efficient at low SNR, but is the simplest method to implement [8], [12]. ED is usually implemented in TD. Recently, some works demonstrated improved performance of ED by the use of periodogram technique [1], [13]. Although the improved performance for periodogram based ED is claimed by these authors, yet an analytical or intuitive reasoning for this enhanced performance is not available in literature [1], [13]. This work aims to fill this gap by derivations of exact and approximate expressions for $P_f$ and $P_d$, supported by extensive simulations using various performance parameters like SNR, buffer size and threshold to quantify the performance gap between TD and FD based energy detectors.

The rest of this paper is organized as follows. System model is described in Section II. Then, Section III presents the exact calculations. Next, Section IV gives the approximate calculations for both domains. Theoretical results verifications through Monte Carlo simulations are given in Section V. Finally, Section VI concludes the paper.

## II. System Model

Let y (n) be the sampled received signal at time 'n'. Samples at different time are supposed to be independent and identically distributed (i.i.d). The received signal is represented by the two hypotheses, indicating presence and absence of primary user:

$$y(n) = \begin{cases} w(n) & H_0 \\ s(n) + w(n) & H_1 \end{cases} \tag{1}$$

where w(n) ~ $\mathcal{CN}(0, \sigma_w^2)$ is an AWGN process, and s(n) is an i.i.d primary user's signal. Binary hypothesis in (1) is used to determine $P_f$, $P_d$ and Probability of missed detection ($P_m$) [1]:

$$P_f = Prob[Decision = H_1|H_0] \tag{2}$$

$$P_m = Prob[Decision = H_0|H_1] \tag{3}$$

$$P_d = Prob[Decision = H_1|H_1] \tag{4}$$

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## III. ED Performance Analysis via Exact Closed form Expressions

In the exact analysis, probability density function (PDF) and cumulative density function (CDF) expressions for central and non-central chi-square distribution [14], are used to derive closed form expressions for $P_f$ and $P_d$. Derivation for TD and FD based energy detectors are as under:

### *A. Energy Detector in TD*

In TD, the decision statistic of ED is expressed as

$$Y = \sum_{n=1}^{N} |y(n)|^2 \tag{5}$$

where y(n) are the samples of received signal as given in (1). *Y* has a non-central chi-square distribution with N degrees of freedom (DoF), under $H_1$. Otherwise, it has a central chi-square distribution with N DoF. $P_f$ and $P_d$, already available [15], [16], are given as

$$P_f = \frac{\Gamma(\frac{N}{2}, \frac{\gamma}{2})}{\Gamma(\frac{N}{2})} \tag{6}$$

$$P_d = Q_{N/2}(\sqrt{2\lambda}\ , \sqrt{\gamma}) \tag{7}$$

where $\Gamma(.)$ is gamma function, $\Gamma(\mathrm{a}, \mathrm{b})$ is incomplete gamma function, 'N' is number of samples, and '$\gamma$' is sensing threshold [17]. $Q_{N/2}(a, b)$ is the generalized Marcum Q-function with $\lambda = \sum_{i=1}^{N} \left(\frac{\mu_i^2}{\sigma_i^2}\right)$ [18].

### *B. Energy Detector in FD*

Here, the exact analysis is extended over FD and a periodogram based approach is presented. The decision statistic for this case is given as:

$$S(f) = \frac{1}{N} \left|\sum_{k=0}^{K-1} y(k) exp^{-jwk}\right|^2 = \frac{1}{N} |Y(f)|^2 \tag{8}$$

The decision statistic *S(f)* has exponential distribution and can be considered as chi-squared distributed with two DoF. Under $H_0$, *S(f)* has central chi-square distribution with two DoF. Else, it has non-central chi-square distribution with two DoF. CDF and PDF expressions given in [19] are used to derive new expressions for $P_f$ and $P_d$.

$$P_x(x) = \int_0^{\gamma} \frac{1}{2\sigma^2}\ exp^{\frac{-x}{2\sigma^2}}\, \mathrm{d}\gamma = 1\ - exp^{\frac{-\gamma}{2\sigma^2}} \tag{9}$$

$$P_f = exp^{\frac{-\gamma}{2\sigma^2}} \tag{10}$$

where $\gamma$ is the sensing threshold and $\sigma^2$ denotes the noise variance. A new, $P_d$ expression is derived using non-central chi-square distribution with two DoF as:

$$P_x(x) = \int_0^{y} \frac{1}{2\sigma^2}\ exp^{\frac{-(s^2+y)}{2\sigma^2}}\ I_0\left(\sqrt{y}\ \frac{s}{\sigma^2}\right) dy \tag{11}$$

$$P_d = 1 - Pm = Q_1(\sqrt{2\lambda}\ , \sqrt{\gamma}) \tag{12}$$

Marcum Q function is dependent on the modified Bessel function of first kind $I_{M-1}$ which in turn depends on the inverse gamma function having parameter N. Here, M=N/2 and N=DoF. In TD, N independent random variables in (5), show a DoF of N. Whereas in FD, exponential distribution in (8) presents a DoF of two. As the DoF increases in (7), the resultant Bessel function decreases, hence improving probability of detection. So, 'N' DoF in TD gives higher probability of detecting a primary signal as compared to two DoF for FD.

## IV. ED Performance Analysis Via Approximate Closed Form Expressions

In conventional TD ED, test statistic in (5) assumes the sum of independent random variables. The test statistics can be approximated by invoking central limit theorem (CLT) when buffer size (N) is large [20].

### *A. Energy Detector in TD*

The TD ED uses the same decision statistic as in (5). For the approximate analysis, Q-functions already available in [21], are used to calculate $P_f$ and $P_d$ as:

$$P_f = Q\left(\frac{\gamma - \mu o}{\sigma 0}\right) \tag{13}$$

$$P_d = Q\left(\frac{\gamma - \mu 1}{\sigma 1}\right) \tag{14}$$

where $\mu_o$ and $\sigma_o$ are the mean and standard deviation for $H_0$, and $\mu_1$ and $\sigma_1$ are the mean and standard deviation for $H_1$. The mean and standard deviation expressions are easily deduced using the properties of normal distribution [22] as:

$$\mu_0 = N(\mu_w^2 + \sigma_w^2) \tag{15}$$

$$\sigma_0 = \sqrt{2N}(2\mu_w^2\ \sigma_w^2 + \sigma_w^4) \tag{16}$$

$$\mu_1 = N[\sigma_s^2\ +\ \sigma_w^2 + (\mu_s + \mu_w)^2] \tag{17}$$

$$\sigma_1 = \sqrt{2N}[2(\mu_s + \mu_w)^2(\sigma_s^2\ +\ \sigma_w^2) + (\sigma_s^2\ + \sigma_w^2)^2] \tag{18}$$

Assuming $\mu_w = \mu_s = 0$ for AWGN, $P_f$ and $P_d$ given in (13), (14) are evaluated using (15)-(18) as:

$$P_f = Q\left(\frac{\gamma - \mathrm{N}\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right) \tag{19}$$

$$P_d = Q\left(\frac{\gamma - \mathrm{N}(\sigma_w^2 + \sigma_s^2)}{\sqrt{2N}\ (\sigma_w^2 + \sigma_s^2\ )^2}\right) \tag{20}$$

where $\sigma_w^2$ is the noise variance, and $\sigma_s^2$ denotes the signal variance.

### *B. Energy Detector in FD*

TD calculations are extended for FD and new expressions for $P_f$ and $P_d$ are derived in this subsection. In FD, the ED is implemented using power spectral density (PSD) estimation.

The decision statistic in FD is same as in (8). Binary test hypothesis in (1) is used to determine $P_f$ and $P_d$. New expressions for mean and variance, under $H_0$ and $H_1$ are calculated as:

$$\mu_0 = P(e^{jw}) = \frac{1}{N}E\left\{\left|\sum_{k=0}^{N-1} w(k)exp^{-jwk}\right|^2\right\} = \sigma_w^2 \quad (21)$$

$$\sigma_0 = Cov[P(e^{jw1})P(e^{jw2})] = \sigma_w^4$$

$$\mu_1 = P_1(e^{jw}) = \frac{1}{N}E\{|\sum_{k=0}^{N-1}(w(k)+s(k))exp^{-jwk}|^2\} \quad (22)$$

$$\mu_1 = P_1(e^{jw}) = \sigma_w^2 + \sigma_s^2 \quad (23)$$

$$\sigma_1 = Cov[P_1(e^{jw1})P_1(e^{jw2})] = (\sigma_w^2 + \sigma_s^2)^2 \quad (24)$$

$P_f$ and $P_d$ are derived using (21-24) as:

$$P_f = Q\left(\frac{\gamma - \sigma_w^2}{\sigma_w^2}\right) \quad (25)$$

$$P_d = Q\left(\frac{\gamma - (\sigma_w^2+\sigma_s^2)}{(\sigma_w^2+\sigma_s^2)}\right) \quad (26)$$

## V. Simulation Results

The derived closed form expressions, both for exact and approximate analysis, are verified through Monte Carlo Simulations. A BPSK signal is passed over an AWGN channel. Neyman-Pearson criterion is used to determine the threshold [8]. Theoretical and simulated results can be evaluated for any given value of SNR and noise variance. However, for convenience, SNR=3 dB and $\sigma_w^2 = 1$ dB are frequently used, and results are estimated as following:

### *A. Probability of False Alarm for Variable N*

Fig. 1 shows improved $P_f$ when FD is considered. Exact and approximate expressions in (10) and (25) show that FD does not depend on buffer size N. The buffer size affects the noise variance. If N is increased or decreased, FD results are not affected. However, as N increases in TD, noise variance is enhanced, and performance starts to deteriorate in TD. Hence, FD outperforms TD in terms of $P_f$, as FD is independent of buffer length.

### *B. Probability of Detection for Variable Buffer Size*

Fig. 2 shows the theoretical and simulated results for $P_d$. When buffer size is increased in TD, signal variance gets improved. It is evident from (12) and (26) that FD does not depend on buffer length. As the length is increased, signal variance remains unaffected. It is observed that $P_d$ improves as the number of samples increases. Higher the buffer size, better the $P_d$. Hence, TD performs better than FD in terms of $P_d$.

### *C. ROC Analysis*

Approximate and exact results are evaluated for both TD and FD as shown in Figs. 3 and 4.

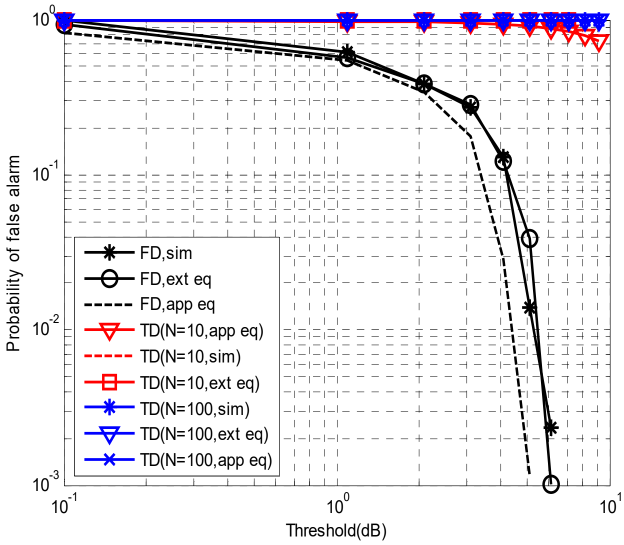


Fig. 1 $P_f$ vs. Threshold for Variable N

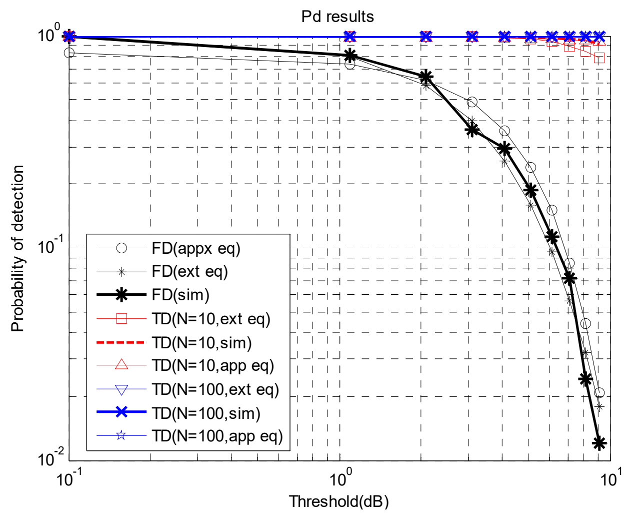


Fig. 2 $P_d$ vs. Threshold for Variable N

It is observed that as N increases, TD gives better $P_m$ than FD. TD is dependent on N as stated in (7) and (20), $P_m$ improves as buffer length increases and signal variance rises. The chance of detection gets enhanced as the number of samples increases. The theoretical and simulated results for both analyses are in accordance. Hence, $P_m$ improves in TD, by using Neyman-Pearson detector.

### *D. Probability of Missed Detection for Variable SNR*

Figs. 5 and 6 show the performance of $P_m$ over SNR regime for approximate and exact analysis respectively.

It is evident from (7) and (20) that $P_d$ for TD dependents on the buffer size N. Exact and approximate analysis show that $P_m$ improves as buffer size is increased. On the other hand, FD does not depend on N as given in (12) and (26). SNR improves with increasing N because the signal variance gets enhanced. Hence, TD performs better than FD.

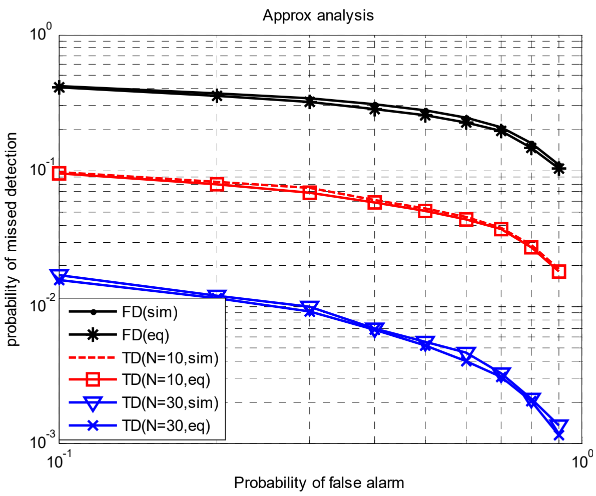


Fig. 3 $P_m$ vs. $P_f$ for approximate analysis

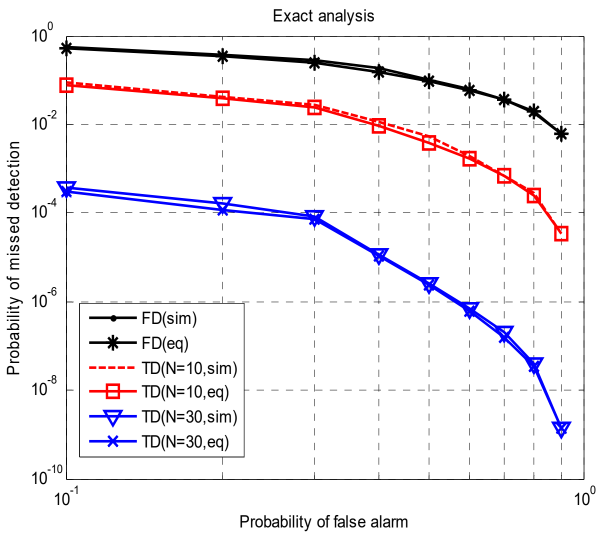


Fig. 4 $P_m$ vs. $P_f$ for exact analysis

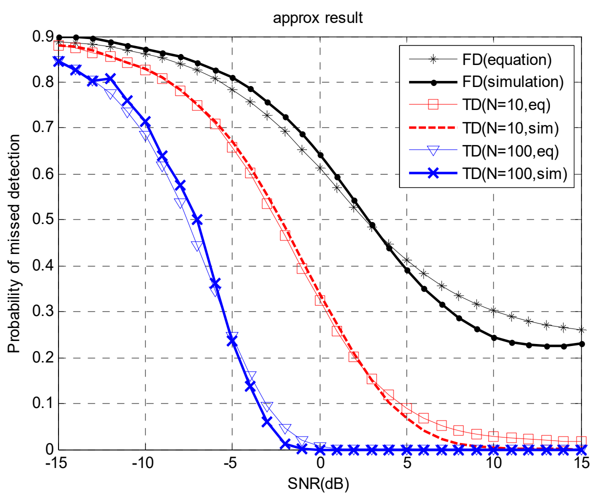


Fig. 5 $P_m$ vs. SNR for approximate analysis

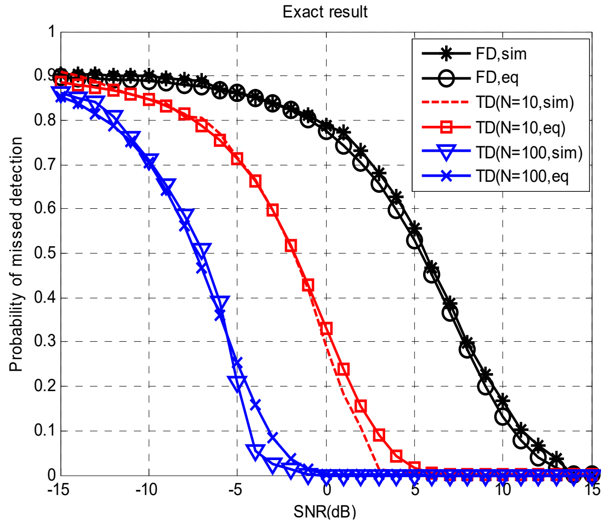


Fig. 6 $P_m$ vs. SNR for exact analysis

*E. Probability of False Alarm and Detection For Time Averaging*

Periodogram based detection performs averaging in FD, whereas the TD decision statistic does not perform averaging of energy samples as given in (5) and (8). Instead of using the simple energy decision statistic for TD, a new averaged TD energy detector is proposed. The decision statistic is given as

$$Y = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2 \tag{27}$$

The mean and variance of time averaged decision statistic is derived, and $P_f$ and $P_d$ are obtained using (6), (7) as

$$P_f = Q\left(\frac{\gamma - \sigma_w^2}{\sqrt{2}\sigma_w^2}\right) \tag{28}$$

$$P_d = Q\left(\frac{\gamma - (\sigma_w^2 + \sigma_s^2)}{\sqrt{2}\,(\sigma_w^2 + \sigma_s^2)^2}\right) \tag{29}$$

The $P_f$ and $P_d$ expressions are evaluated against variable noise variance and SNR values. Fig. 7 shows that $P_f$ improves as noise variance is reduced. Noise variance is low for FD as evident from (25). However, noise variance gets enhanced in case of time averaged ED as given in (28). Higher noise variance deteriorates the performance of $P_f$. Hence, lower the noise variance, better the $P_f$. FD ED performs better than averaged TD detector.

It is evident from Fig. 8 that, higher the SNR, better the $P_d$. Noise variance is reduced by increasing SNR in TD, as shown in (26). However, in FD, noise variance gets enhanced by increasing SNR, and $P_d$ deteriorates as given in (29). Hence, TD gives better $P_d$.

The theoretical and simulated results discussed in Figs. 1-8 are in accordance. It is evident from the analytical and simulated analysis that $P_f$ gets improved in FD. Whereas, $P_d$ is enhanced in TD.

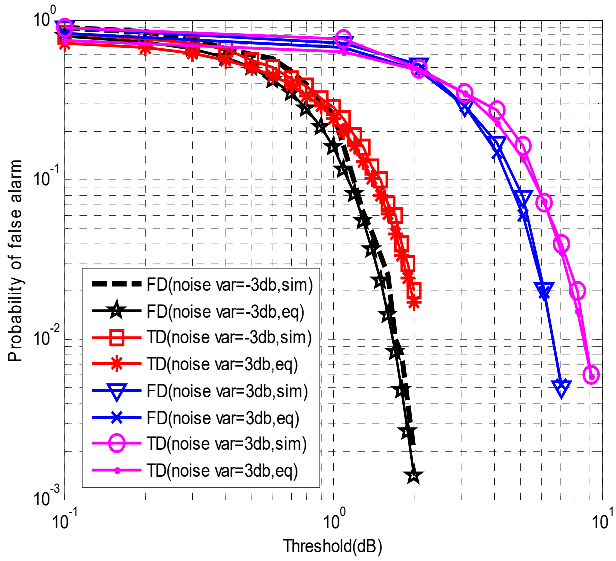


Fig. 7 $P_f$ vs. Threshold for time averaging

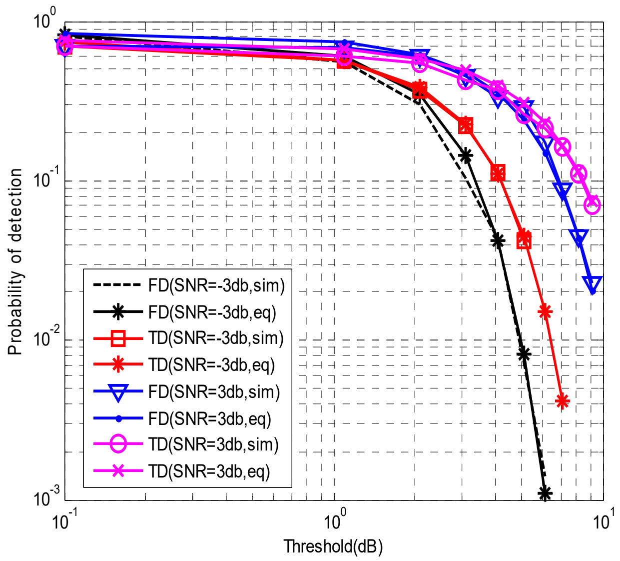


Fig. 8 $P_d$ vs. Threshold for time averaging

## VI. Conclusion

An analytical or intuitive reasoning behind improved performance of FD and TD based energy detectors is provided in this paper. Mathematical analysis and simulations are performed for both TD and FD energy detectors over AWGN channel. It is observed that FD gives better $P_f$ when buffer size is increased. TD gives improved $P_d$ when observation length is enhanced, Neyman-Pearson detector is used, and SNR is varied. It is also observed that time averaged ED does not bring any improvement over the classical TD ED. Further analysis in fading channels will be carried out as future work to ascertain the results.

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