Performance Analysis of CATR Reflector with Super Hybrid Modulated Segmented Exponential Serrated Edges

T. Venkata Rama Krishna, P. Siddaiah, and B. Prabhakara Rao

Abstract**—**This paper presented a theoretical and numerical investigation of the Compact Antenna Test Range (CATR) equipped with Super Hybrid Modulated Segmented Exponential Serrations (SHMSES). The investigation was based on diffraction theory and, more specifically, the Fresnel diffraction formulation. The CATR provides uniform illumination within the Fresnel region to test antenna. Application of serrated edges has been shown to be a good method to control diffraction at the edges of the reflectors. However, in order to get some insight into the positive effect of serrated edges a less rigorous analysis technique known as Physical Optics (PO) may be used. Ripple free and enhanced quiet zone width are observed for specific values of width and height modulation factors per serrations. The performance of SHMSE serrated reflector is evaluated in order to observe the effects of edge diffraction on the test zone fields.

Keywords**—**Fresnel Region, Quiet Zone, Physical Optics, Ripples, Serrations.

I. INTRODUCTION

THE Compact Range techniques exploit the plane wave nature of the electromagnetic field in the vicinity of a ray collimating device to simulate a far field environment.

If your paper is intended for a *conference,* please contact your conference editor concerning acceptable word processor formats for your particular conference. According to Geometrical Optics (GO) theory, a singly or doubly curved parabolic reflector can be used to convert the diverging rays emanating from a cylindrical or spherical radiating source to a uniform plane wave which propagates in a direction parallel to the reflector axis. This wave is not strictly uniform or planar since the GO model is exact only in the limit that the reflector is infinite in extent and the wavelength of radiation is zero. When computing the near field of reflectors employed at microwave frequencies, edge diffraction effects must be included in the analysis. A very strong diffracted field emanates from the terminating edge. The diffracted signal interferes with the plane wave and causes amplitude and phase variations of the field that illuminates the test antenna. This diffracted field is one of the major contributions that limits the use of the compact antenna test ranges.

II. METHODOLOGY

The analysis of the Fresnel field of a square aperture with super hybrid segmented serrations using the method of physical optics (PO). In this paper four different shapes of serrations are used in four sides. This analysis is so general that it can be applied to any serration geometry. This paper presents a gist of the analysis of super hybrid segmented geometry shown in Fig. 1. The Fresnel diffraction formula which gives the x-polarized field over an arbitrary plane (z = constant) in the Fresnel region is [4]

$$E_x(x,y,z) = -\frac{jk}{2\pi z} e^{-jkz} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{ax}(x',y') e^{jk\left\{(x'-x)^2+(y'-y)^2\right\}/2z} dx'dy' \tag{1}$$

It will be a laborious task to find an analytical expression in a closed form for the Fresnel diffraction pattern of an aperture with these serrated edges. Hence, recourse is taken to decompose the aperture area S into three parts S_1, S_2 and S_3, such that $S=S_1+S_2-S_3$ (Fig. 2). A quasi-analytical expression can now be derived for the Fresnel field [1]-[10]. The super hybrid modulated segmented exponential serrations described by the boundary functions $h^+(x')$ and $g^+(y')$ are expressed as Fourier series of width modulated exponential with rate of rise 'a_i'. The serrated edges are described by the functions $h^+(x'), h^-(x')$, and $g^+(y')$ and $g^-(y')$ and $E_{ax}(x',y') = E_0$ for $(x',y') \in S.$ Now, equation (1) can be rearranged as

$$E_x(x,y,z) = \frac{-jE_0}{2} e^{-jkz}\left(I_1 + I_2 + I_3\right)$$

where

$$I_1 = \frac{k}{\pi z}\int_{-h=\frac{-b_0}{2}}^{h=\frac{b_0}{2}} e^{jk(y'-y)^2/2z} dy' \int_{g^-(y')}^{g^+(y')} e^{jk(x'-x)^2/2z} dx'$$

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$$= \frac{k}{\pi z}\left[F\left(t_+\right)-F\left(t_-\right)\right]\left[F\left(s_+^{'}\right)-F\left(s_-^{'}\right)\right] \tag{2a}$$

$$I_2 = \frac{k}{\pi z}\int_{-w=\frac{-a_0}{2}}^{w=\frac{a_0}{2}} e^{jk\left(x'-x\right)^2/2z}dx'\int_{h^-(x')}^{h^+(x')} e^{jk\left(y'-y\right)^2/2z}dy'$$

$$= \frac{k}{\pi z}\left[F\left(s_+\right)-F\left(s_-\right)\right]\left[F\left(t_+^{'}\right)-F\left(t_-^{'}\right)\right] \tag{2b}$$

$$I_3 = \frac{k}{\pi z}\int_{-w=\frac{-a_0}{2}}^{w=\frac{a_0}{2}} e^{jk\left(x'-x\right)^2/2z}dx'\int_{-h=\frac{-b_0}{2}}^{h=\frac{b_0}{2}} e^{jk\left(y'-y\right)^2/2z}dy'$$

$$= \frac{k}{\pi z}\left[F\left(s_+\right)-F\left(s_-\right)\right]\left[F\left(t_+\right)-F\left(t_-\right)\right] \tag{2c}$$

$$t_\pm = \sqrt{\frac{k}{\pi z}}\left(\pm h - y\right), s_+^{'} = \sqrt{\frac{k}{\pi z}}\left(-g^-\left(y'\right)-x\right)$$

$$s_-^{'} = \sqrt{\frac{k}{\pi z}}\left(-g^+\left(y'\right)-x\right)$$

$$s_\pm = \sqrt{\frac{k}{\pi z}}\left(\pm w - x\right), t_+^{'} = \sqrt{\frac{k}{\pi z}}\left(-h^-\left(x'\right)-y\right)$$

and

$$t_-^{'} = \sqrt{\frac{k}{\pi z}}\left(-h^+\left(x'\right)-y\right), F(s) = \int_0^s e^{-j\pi r^2/2}dr$$

= the complex form of the Fresnel integral

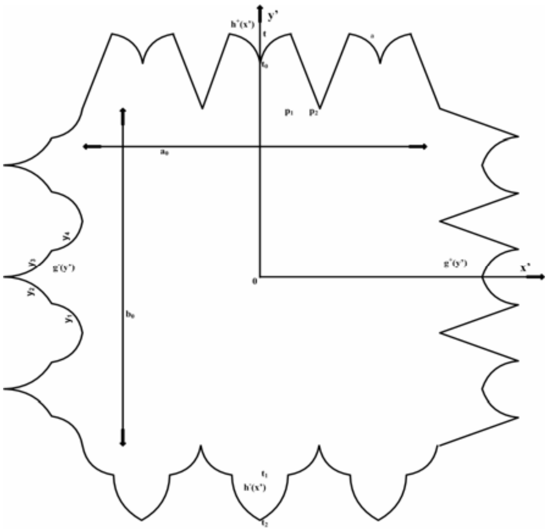


Fig. 1 SHMSE Serrated CATR

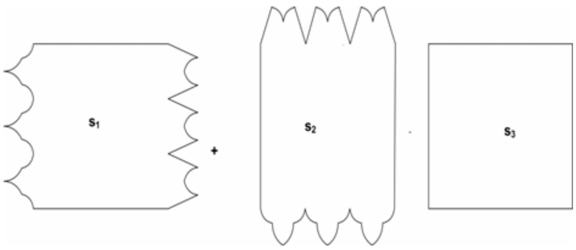


Fig. 2 Decomposition of Serrated Aperture

A) Fourier Series of Width & Height Modulated Identical Segmented Convex Serrations

The serrations described by the boundary functions $h^-\left(x'\right)$ is expressed as a Fourier series of width and height modulated identical segmented convex function. The Fourier series expansion is $h^-\left(x'\right)$ is given by

$$h^-(x') = C_{01} + \sum_{n=-\infty}^{\infty} C_{n1}e^{\frac{jn\omega}{T}t}$$

$$C_{01} = \frac{1}{T}\Bigg[2t_1p_1 + 2t_2p_1\left(1-e^{-ap_1}\right)$$

$$+2t_1\left(p_2-p_1\right)\left(1-e^{-a\left(p_2-p_1\right)}\right)\Bigg]$$

$$C_{n1} = \frac{1}{T}\Bigg[\frac{2t_1}{n\omega}e^{-a\left(p_2-p_1\right)}\left(\sin\left(n\omega p_2\right)-\sin\left(n\omega p_1\right)\right)$$

$$+\frac{2t_1}{n\omega}\sin\left(n\omega p_2\right)+\frac{2t_2}{n\omega}\sin\left(n\omega p_1\right)\left(1+e^{-ap_1}\right)$$

$$+\frac{2t_2e^{-ap_1}}{a^2+n^2\omega^2}\left[a-e^{ap_1}\left(a\cos\left(n\omega p_1\right)-n\omega\sin\left(n\omega p_1\right)\right)\right]$$

$$+\frac{2t_2e^{-ap_2}}{a^2+n^2\omega^2}\Big[e^{ap_1}\left(a\cos\left(n\omega p_1\right)+n\omega\sin\left(n\omega p_1\right)\right)$$

$$-e^{ap_2}\left(a\cos\left(n\omega p_2\right)+n\omega\sin\left(n\omega p_2\right)\right)\Big]\Bigg]$$

B) Fourier Series of Width & Height Modulated Non-Identical Segmented Convex Serrations

The serrations described by the boundary functions $h^+\left(x'\right)$ is expressed as a Fourier series of width & height modulated Non-identical segmented convex function. The Fourier series expansion of $h^+\left(x'\right)$ is given by

$$h^+(x') = C_{02} + \sum_{n=-\infty}^{\infty} C_{n2}e^{\frac{jn\omega}{T}t}$$

$$C_{02}=\frac{1}{T}\left[2p_1\left(t_0+t\right)+\frac{t}{a}\left(1-e^{-ap_1}\right)+t\left(p_2-p_1\right)\right]$$

$$C_{n2}=\frac{1}{T}\left[-\frac{2ta}{a^2+n^2\omega^2}+\frac{2\left(t_0+t\right)}{n\omega}\sin\left(n\omega p_1\right)\right.$$

$$+\frac{2t_1}{\left(p_2-p_1\right)}\left(\frac{p_1}{n\omega}\sin\left(n\omega p_1\right)+\frac{1}{n^2\omega^2}\left(\cos\left(n\omega p_1\right)-\cos\left(n\omega p_2\right)\right)\right.$$

$$\left.\left.-\frac{p_2}{n\omega}\sin\left(n\omega p_1\right)\right)+\frac{2te^{-ap_1}}{a^2+n^2\omega^2}\left(a\cos\left(n\omega p_1\right)-n\omega\sin\left(n\omega p_1\right)\right)\right]$$

C) Fourier Series of Width & Height Modulated Identical Segmented Concave Serrations

The serrations described by the boundary functions $g^-\left(y'\right)$ is expressed as a Fourier series of width and height modulated identical segmented Concave function. The Fourier series expansion of $g^-\left(y'\right)$ is given by

$$g^-(y')=C_{03}+\sum_{n=-\infty}^{\infty}C_{n3}e^{\frac{jn\omega}{T}t}$$

$$C_{03}=\frac{1}{T}\left[2t_1p_1+\frac{2t_2}{a}\left(1-e^{-ap_1}\right)+\frac{2t_1e^{ap_1}}{a}\left(e^{-ap_1}-e^{-ap_2}\right)\right]$$

$$C_{n3}=\frac{1}{T}\left\{\frac{2t_1}{n\omega}\sin\left(n\omega p_1\right)+\frac{2t_2}{a^2+n^2\omega^2}\left[a\right.\right.$$

$$\left.-ae^{-p_1a}\left(\cos\left(n\omega p_1\right)-n\omega\sin\left(n\omega p_1\right)\right)\right]$$

$$+\frac{2t_1e^{ap_1}}{a^2+n^2\omega^2}\left[e^{-p_1a}\left(a\cos\left(n\omega p_1\right)\right)-n\omega\sin\left(n\omega p_1\right)\right.$$

$$\left.\left.-e^{-p_2a}\left(a\cos\left(n\omega p_2\right)\right)-n\omega\sin\left(n\omega p_2\right)\right]\right\}$$

D) Fourier Series of Width & Height Modulated Non-Identical Segmented Concave Serrations

The serrations described by the boundary functions $g^+\left(y'\right)$ is expressed as a Fourier series of width and height modulated Non-identical segmented concave function. The Fourier series expansion of $g^+\left(y'\right)$ is given by

$$g^+(y')=C_{04}+\sum_{n=-\infty}^{\infty}C_{n4}e^{\frac{jn\omega}{T}t}$$

$$C_{04}=\frac{1}{T}\left[t\left(p_2-p_1\right)+\frac{2t_0}{a}\left(e^{ap_1}-1\right)\right]$$

$$C_{n4}=\frac{1}{T}\left[-\frac{2ta}{a^2+n^2\omega^2}++\frac{2t_1}{\left(p_2-p_1\right)}\left(\frac{p_1}{n\omega}\sin\left(n\omega p_1\right)\right.\right.$$

$$\left.+\frac{1}{n^2\omega^2}\left(\cos\left(n\omega p_1\right)-\cos\left(n\omega p_2\right)\right)-\frac{p_2}{n\omega}\sin\left(n\omega p_1\right)\right)$$

$$\left.+\frac{2t_0e^{ap_1}}{a^2+n^2\omega^2}\left(a\cos\left(n\omega p_1\right)-n\omega\sin\left(n\omega p_1\right)\right)\right]$$

III. RESULTS AND DISCUSSION

The technique presented here is best suited to the analysis of serrated reflectors commonly employed in compact range systems for reduced edge diffraction. A square reflector of aperture dimensions 45λ×45λ is considered to be equipped with SHMSES equations in Tables III & IV have been used in conjunction with equations(2a-2c)to evaluate the Fresnel field at a transverse distance in wavelengths at a distance of z=64 λ from the reflector aperture plane over the line y= 0, 0<x< 45λ. An integration step size of 0.25λ has been used. The Fresnel integral were simulated using Matlab7.2. The Fresnel field is computed for the different values of width and height modulation factors indicated in Tables I & II. The relative power in dB vs. transverse distance in wavelengths with the space constant a_i=0.6 for exponential serrations is presented for different cases in Figs. 3 to 5. This implies fewer ripples at the centre of the quiet zone which is indicative of better cancellation of diffraction effects.

TABLE I
HEIGHT MODULATION FACTOR FOR SHMSES

t	t_1/ t	t_2/ t
1λ	0.5	1

TABLE II
WIDTH MODULATION FACTORS FOR SHMSES

CASE	P	P_1/P	P_2/P	Number of Serrations
1	(a_0/2)/112.5	7.5	22.5	1
2	(a_0/2)/56.25	3.75	11.25	2
3	(a_0/2)/37.50	2.5	7.5	3
4	(a_0/2)/28.125	2.25	5.625	4
5	(a_0/2)/22.50	1.5	4.5	5
6	(a_0/2)/18.75	1.25	3.75	6
7	(a_0/2)/16.07	1.0714	3.2143	7
8	(a_0/2)/14.0625	0.938	2.8125	8
9	(a_0/2)/12.50	0.8333	2.5	9

TABLE III
NON-IDENTICAL EXPONENTIAL SERRATION FUNCTIONS

	NON-IDENTICAL SEGMENTED CONCAVE	NON-IDENTICAL SEGMENTED CONVEX
Defining Equation	$f\left(x^{'}\right)=y_1+y_2+y_3+y_4$ $y_1=\frac{t}{p_2-p_1}\left(x^{'}+p_2\right)$ $-p_2<x^{'}<-p_1$ $y_2=t_0e^{-ax^{'}}$ $-p_1<x^{'}<0$ $y_3=t_0e^{ax^{'}}$ $0<x^{'}<p_1$ $y_4=\frac{t}{p_1-p_2}\left(x^{'}-p_2\right)$ $p_1<x^{'}<p_2$ where $t_0=te^{-ap_1}$	$f\left(x^{'}\right)=y_1+y_2+y_3+y_4$ $y_1=\frac{t}{p_2-p_1}\left(x^{'}+p_2\right)$ $-p_2<x^{'}<-p_1$ $y_2=t_0+t\left(1-e^{ax^{'}}\right)$ $-p_1<x^{'}<0$ $y_3=t_0+t\left(1-e^{-ax^{'}}\right)$ $0<x^{'}<p_1$ $y_4=\frac{t}{p_1-p_2}\left(x^{'}-p_2\right)$ $p_1<x^{'}<p_2$ where $t_0=te^{-ap_1}$

TABLE IV
IDENTICAL EXPONENTIAL SERRATION FUNCTIONS

	IDENTICAL SEGMENTED CONCAVE	IDENTICAL SEGMENTED CONVEX
Defining Equation	$f\left(x^{'}\right)=y_1+y_2+y_3+y_4$ $y_1=t_1e^{a\left(x^{'}+p_1\right)}$ $-p_2<x^{'}<-p_1$ $y_2=t_2e^{ax^{'}}+t_1$ $-p_1<x^{'}<0$ $y_3=t_2e^{-ax^{'}}+t_1$ $0<x^{'}<p_1$ $y_4=t_1e^{-a\left(x^{'}-p_1\right)}$ $p_1<x^{'}<p_2$	$f\left(x^{'}\right)=y_1+y_2+y_3+y_4$ $y_1=t_1e^{-a\left(p_2-p_1\right)}+t_1\left(1-e^{-a\left(p_2+x^{'}\right)}\right)$ $-p_2<x^{'}<-p_1$ $y_2=t_2e^{-ap_1}+t_2\left(1-e^{-a\left(p_1+x^{'}\right)}\right)+t_1$ $-p_1<x^{'}<0$ $y_3=t_2e^{-ap_1}+t_2\left(1-e^{-a\left(p_1-x^{'}\right)}\right)+t_1$ $0<x^{'}<p_1$ $y_4=t_1e^{-a\left(p_2-p_1\right)}+t_1\left(1-e^{-a\left(p_2-x^{'}\right)}\right)$ $p_1<x^{'}<p_2$

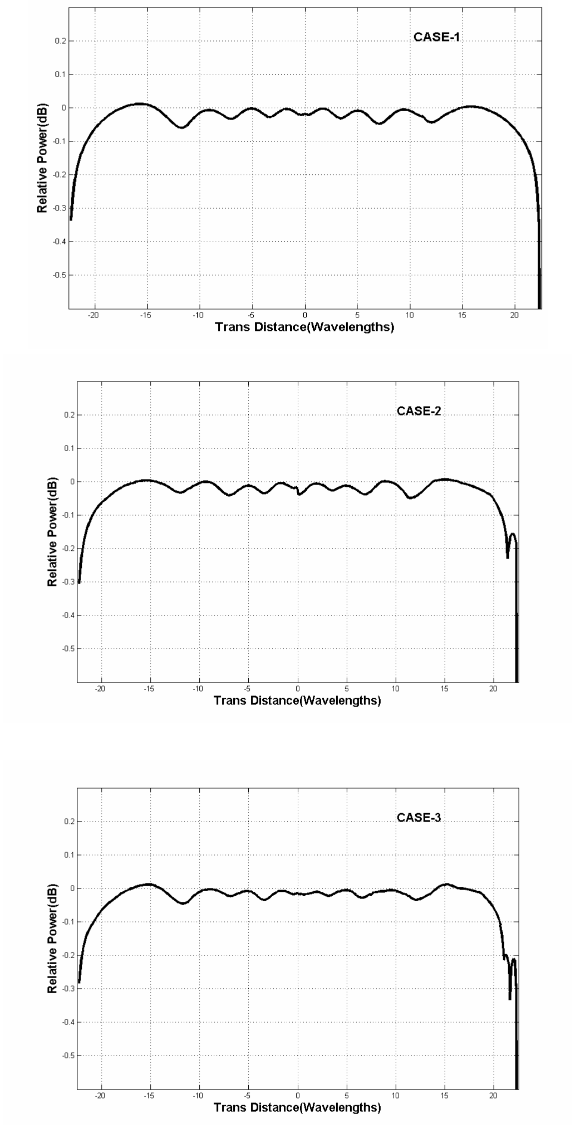
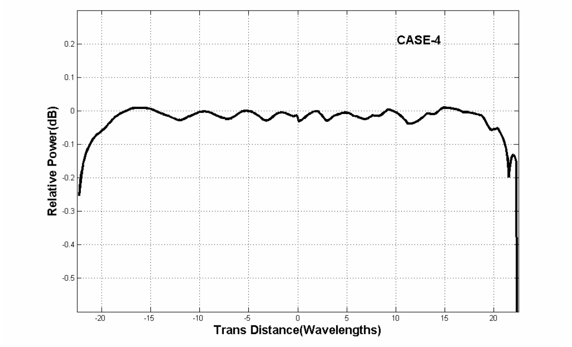


Fig. 3 Fresnel zone field for 45λ×45λ SHMSE serrated CATR for cases 1, 2, 3



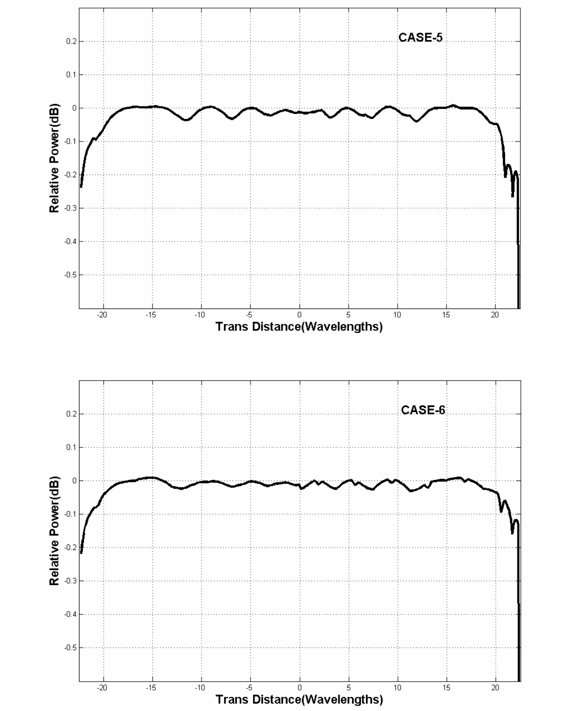


Fig. 4 Fresnel zone field for 45λ×45λ SHMSE serrated CATR for cases 4, 5, 6

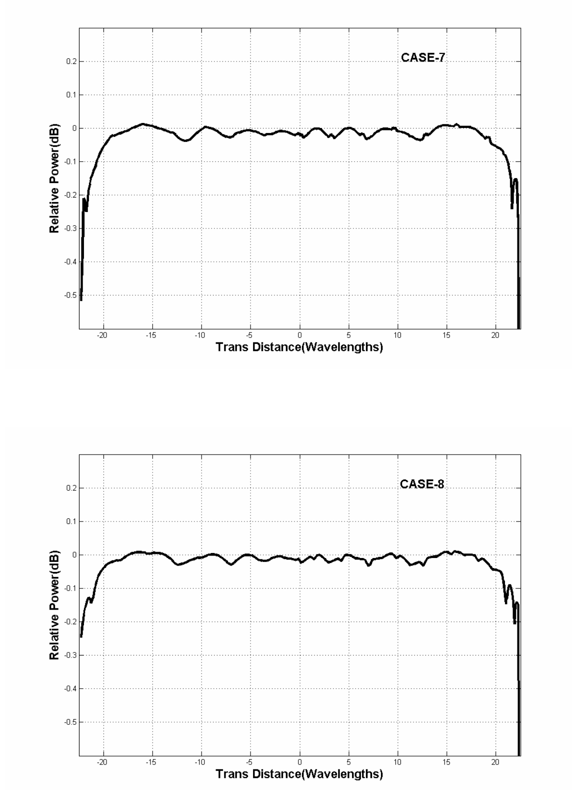


Fig. 5 Fresnel zone field for 45λ×45λ SHMSE serrated CATR for cases 7, 8, 9

IV. CONCLUSION

This paper presented a performance evaluation of the SHMSE serrated edge reflector with rectangular aperture for different values of width and height modulation factors. The quiet zone field of a 45λ×45λ is assessed for different cases as illustrated in Tables I & II. From the graphs, it is observed that less ripple and enhanced quiet zone width are observed in this super hybrid segmented exponential serrated CATR than identical serrated CATRs. Cases 3 & 5 give very superior performance than the remaining cases. It is concluded that, SHMSE serrated CATRs gives better performance.

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