

# Performance Analysis of a Combined Ordered Successive and Interference Cancellation Using Zero-Forcing Detection over Rayleigh Fading Channels in MIMO Systems

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**Abstract**—Multiple Input Multiple Output (MIMO) systems are wireless systems with multiple antenna elements at both ends of the link. Wireless communication systems demand high data rate and spectral efficiency with increased reliability. MIMO systems have been popular techniques to achieve these goals because increased data rate is possible through spatial multiplexing scheme and diversity. Spatial Multiplexing (SM) is used to achieve higher possible throughput than diversity. In this paper, we propose a Zero-Forcing (ZF) detection using a combination of Ordered Successive Interference Cancellation (OSIC) and Zero Forcing using Interference Cancellation (ZF-IC). The proposed method used an OSIC based on Signal to Noise Ratio (SNR) ordering to get the estimation of last symbol, then the estimated last symbol is considered to be an input to the ZF-IC. We analyze the Bit Error Rate (BER) performance of the proposed MIMO system over Rayleigh Fading Channel, using Binary Phase Shift Keying (BPSK) modulation scheme. The results show better performance than the previous methods.

**Keywords**—SNR, BER, BPSK, MIMO, Modulation, Zero forcing (ZF), OSIC, ZF-IC, Spatial Multiplexing (SM).

## I. INTRODUCTION

THE ever increasing demand for high performance wireless communications has spurred the development of various technologies such as Multiple Input Multiple Output (MIMO) systems. The reliability and capacity of a wireless communication can be significantly improved by the application of MIMO systems [1].

MIMO is a wireless technology which enables the use of multiple transmitting and receiving antennas to transfer more data in less time. MIMO system takes advantage of the spatial diversity that is obtained by spatially separated antennas in a dense multipath scattering environment [2].

A major drawback of wireless communication system is the effect of fading. Fading occurs due to multipath propagation and shadowing from obstacles affecting wave propagation. To overcome the detrimental effects of fading, multiple copies of data are transmitted from transmitter to receiver [3], [4].

Today's wireless communication systems demand high data rate and spectral efficiency with increased reliability.

MIMO systems have been popular techniques to achieve these goals through both spatial multiplexing and diversity schemes [2].

MIMO systems are wireless systems with multiple antenna elements are used at both the transmitter and receiver, and can be used to:

- 1- Increase the system reliability (decrease the bit or packet error rate).
- 2- Increase the achievable data rate and hence system capacity.
- 3- Increase the coverage area.
- 4- Decrease the required transmit power.

However, these four desirable attributes usually compete with one another; for example an increase in data rate often will require an increase in either the error rate or transmit power [5].

MIMO systems can be used for beamforming, diversity combining, or spatial multiplexing. The first two applications are the same as for the smart antennas, while Spatial Multiplexing (SM) is the transmission of multiple data streams on multiple antennas in parallel, leading to a substantial increase in capacity. MIMO technology and turbo coding are the two most prominent recent breakthroughs in wireless communication. MIMO technology promises a significant increase in capacity.

Depending on the availability of multiple antennas at the transmitter and/or the receiver, such techniques are classified as Single-Input Multiple-Output (SIMO), Multiple-Input Single-Output (MISO) or MIMO. When a multi-antenna terminal is involved, a full MIMO link may be obtained, although the term MIMO is sometimes also used in its widest sense, thus including SIMO and MISO as special cases [6].

The rest of the paper is structured as follows. In Section II, we discuss a background on the paper topic. In Section III, we explain the system description and also the performance results are discussed. Finally Section IV concludes the paper.

## II. BACKGROUND

The use of multiple antennas allows independent channels to be created in space and is one of the most interesting and promising areas of recent innovation in wireless communications. The spatial diversity, which can be created without using the additional bandwidth that time and

frequency diversity both required. In addition to providing spatial diversity, antenna arrays can be used to focus energy (beamforming) or create multiple parallel channels for carrying unique data streams (spatial multiplexing).

Consider the  $N_R \times N_T$  MIMO systems in Fig. 1. Let  $\mathbf{H}$  denote a channel matrix with  $(j,i)$  the entry  $h_{ji}$  for the channel gain between the  $i^{th}$  transmit antenna and the  $j^{th}$  receive antenna,  $j = 1, 2, \dots, N_R$  and  $i = 1, 2, \dots, N_T$ . The spatially-multiplexed user data and the corresponding received signals are represented by  $\mathbf{x} = [x_1, x_2, \dots, x_{N_T}]^T$  and  $\mathbf{y} = [y_1, y_2, \dots, y_{N_R}]^T$ , respectively, where  $x_i$  and  $y_j$  denote the transmit signal from the  $i^{th}$  transmit antenna and the received signal at the  $j^{th}$  receive antenna, respectively.

The MIMO channel ( $\mathbf{H}$ ) can be written in matrix form as:

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \dots & h_{N_R N_T} \end{bmatrix} \quad (1)$$

where the values  $h_{ji}$  are the channel response of the paths consisting of complex-Rayleigh elements [7], [8].

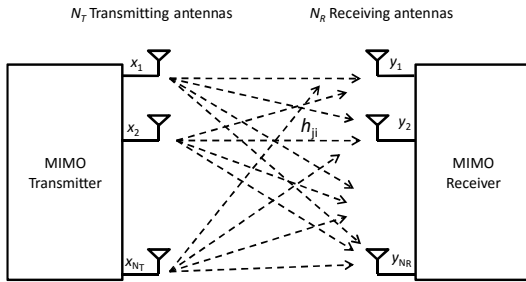


Fig. 1 Simplified transmitted model for MIMO system with  $N_T$  transmit antennas and  $N_R$  receive antennas, giving rise to an  $N_R \times N_T$  channel matrix,  $N_R \times N_T$  links

Let  $z_j$  denote the white Gaussian noise with a variance  $\sigma_z^2$  at the  $j^{th}$  receive antenna, and  $\mathbf{h}(\mathbf{i})$  denote the  $i^{th}$  column vector of the channel matrix  $\mathbf{H}$ . Now, the signal received at the receive antenna is as:

$$\mathbf{y}_{N_R \times 1} = \mathbf{H}_{N_R \times N_T} \mathbf{x}_{N_T \times 1} + \mathbf{z}_{N_R \times 1} \quad (2)$$

or,

$$\mathbf{y} = \mathbf{h}(1)x_1 + \mathbf{h}(2)x_2 + \dots + \mathbf{h}(N_T)x_{N_T} + \mathbf{z} \quad (3)$$

where,  $\mathbf{z} = [z_1, z_2, \dots, z_{N_R}]^T$

Sufficient antenna separation (typically half the carrier wavelength) makes elements of  $h_{ji}$  independent, zero-mean, complex Gaussian random variables (Rayleigh fading). However, at a given time,  $h_{ji}$  varies over frequency and time depending on multipath and Doppler spread respectively [7], [9].

#### A. Spatial Multiplexing

Spatial multiplexed (SM) MIMO systems can transmit data at higher speed than MIMO systems using antenna diversity

techniques. However, SM or signal detection at the receiver side is a challenging task for SM-MIMO systems.

One simple example of spatial multiplexing is when the input is demultiplexed into  $N_T$  separate streams, using serial-to-parallel converter, and each stream is transmitted from an independent antenna. As a result, the throughput is  $N_T$  symbols per channel use for a MIMO channel with  $N_T$  transmit antennas. This  $N_T$  fold increase in throughput will generally come at the cost of a lower diversity gain compared to space-time coding. Therefore, SM is a better choice for high rate systems operating at relatively high Signal to Noise Ratios (SNRs) while space-time coding is more appropriate for transmitting at relatively low rates and low SNRs. [8], [10].

SM can be performed with or without channel knowledge at the transmitter. We consider the principle open-loop techniques; we always assume that the channel is known at the receiver. The open-loop technique for spatial multiplexing attempts to suppress the interference that result from all  $N_T$  streams being received by each of the  $N_R$  antennas [11], [12]

#### B. Linear Signal Detection

Linear signal detection method treats all transmitted signals as interferences except for the desired stream from the target transmit antenna. Therefore, interference signals from other transmit antennas are minimized or nullified in the course of detecting the desired signal from the target transmit antenna. To facilitate the detection of desired signals from each antenna, the effect of the channel is inverted by a weight matrix  $\mathbf{W}$  such that

$$\tilde{\mathbf{x}} = [\tilde{x}_1 \tilde{x}_2 \dots \tilde{x}_{N_T}]^T = \mathbf{W}\mathbf{y} \quad (4)$$

that is, detection of each symbol is given by a linear combination of the received signal.

Using basic linear algebra argument, it is straightforward to confirm that decoding  $N_T$  streams is theoretically possible when there exist at least  $N_T$  nonzero eigenvalues in the channel matrix, that is  $\text{rank}(\mathbf{H}) \geq N_T$  [13], [14].

#### C. Zero Forcing (ZF) Signal Detection

The zero-forcing (ZF) technique nullifies the interference by the following weight matrix:

$$\mathbf{W}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \quad (5)$$

where  $(\cdot)^H$  denotes the Hermitian transpose operation. In other words, it inverts the effect of channel as:

$$\tilde{\mathbf{x}}_{ZF} = \mathbf{W}_{ZF} \cdot \mathbf{y} \quad (6)$$

or,

$$\tilde{\mathbf{x}}_{ZF} = \mathbf{x} + (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{z} \quad (7)$$

or,

$$\tilde{\mathbf{x}}_{ZF} = \mathbf{x} + \tilde{\mathbf{z}}_{ZF} \quad (8)$$

where

$$\tilde{\mathbf{z}}_{ZF} = \mathbf{W}_{ZF} \cdot \mathbf{z} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{z} \quad (9)$$

Determining of  $\mathbf{W}_{ZF}$  might boost up the noise as bad subchannels that have lower eigenvalues are inverted. This can easily amplify the noise [7].

#### D. Zero Forcing Using Ordered Successive Interference Cancellation (OSIC) Signal Detection

In general, the performance of the linear detection methods is worse than that of other nonlinear receiver techniques. However, linear detection methods require a low complexity of hardware implementation. We can improve their performance without increasing the complexity significantly by an Ordered Successive Interference Cancellation (OSIC) method.

It is a bank of linear receivers, each of which detects one of the parallel data streams, with the detected signal components successively cancelled from the received signal at each stage. More specifically, the detected signal in each stage is subtracted from the received signal so that the remaining signal with the reduced interference can be used in the subsequent stage.

Fig. 2 illustrates the OSIC signal detection process. Due to the error propagation caused by erroneous decision in the previous stages, the order of detection has significant influence on the overall performance of OSIC detection. The SNR-Based ordering has been used in this paper. The algorithm includes three steps:

- Ordering;
- Interference cancellation;
- Interference nulling.

Symbols are detected one by one. The purpose of ordering step is to decide which transmitted symbol to detect at each stage of decoding. The symbol with highest SNR is the best pick in this step. The goal of the interference cancellation is to remove the interference from the already detected symbols in decoding the next symbol. Finally, interference nulling finds the best estimate of a symbol from the updated equations. This step is called interference nulling since it can be considered as removing the interference effects of undetected symbols from the one that is being decoded. If the interference cancellation step is removed, the algorithm is modified to one of the equalization methods. In this case, the codeword is detected using ZF for interference nulling [15], [16].

#### E. Zero-Forcing with Interference Cancellation (ZF-IC)

ZF-IC based on the properties of the QR factorization of  $\mathbf{H}$ . Recall from matrix theorem that any  $m \times n$  matrix ( $m \geq n$ ) can be decomposed into a product of two matrices,  $\mathbf{QR}$ , where  $\mathbf{Q}$  is unitary and dimensioned  $m \times m$ , and  $\mathbf{R}$  is upper triangular and dimensioned  $m \times n$ . It follows that the QR factorization of  $\mathbf{H}$  is given by:

$$\mathbf{H} = \mathbf{QR} \quad (10)$$

where  $\mathbf{Q}$  is dimensioned  $N_R \times N_R$  and  $\mathbf{R}$  is  $N_R \times N_T$ . In ZF-IC, the received matrix is pre-multiplied by  $\mathbf{Q}^H$ . If, we denote the matrix that results from the pre-multiplication step by  $\tilde{\mathbf{y}}$ , it follows that:

$$\tilde{\mathbf{y}} = \mathbf{Q}^H \mathbf{y} \quad (11)$$

or,

$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \mathbf{Q}^H \mathbf{z} \quad (12)$$

Finally,

$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{z}} \quad (13)$$

where

$$\tilde{\mathbf{z}} = \mathbf{Q}^H \mathbf{z} \quad (14)$$

where use of the fact that  $\mathbf{Q}^{-1} = \mathbf{Q}^H$  has been made as the fact that  $\mathbf{Q}$  is unitary. Since (13) has been reduced to an upper triangular system, which is very easy to solve with back substitution,  $\mathbf{x}$  can now be decoded on a symbol-by-symbol basis starting with the last symbol [17].

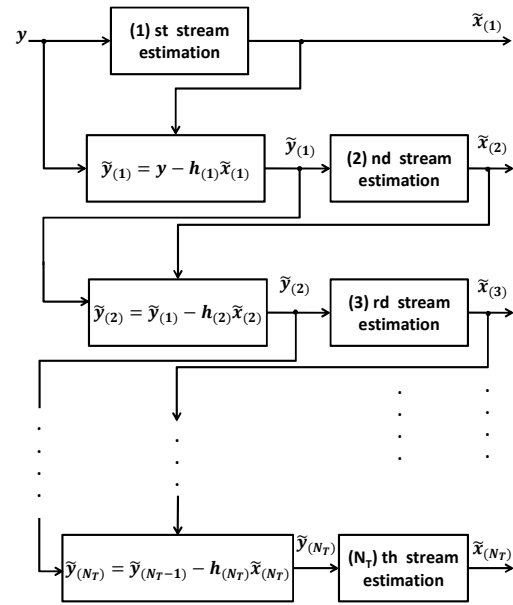


Fig. 2 Illustration of OSIC signal detection for  $N_T$  spatial streams.

### III. SYSTEM DESCRIPTION AND PERFORMANCE RESULTS

We consider a MIMO system with  $N_T$  transmit antennas and  $N_R$  receive antennas. We assume that  $N_R = N_T = N$ . The block diagram of the proposed system is shown in Fig. 3. The system consists of two stages, the first stage is an OSIC, the ordering is based on SNR. The OSIC system is used to estimate the last symbol,  $\tilde{x}_{N_T}$ . The estimated symbol  $\tilde{x}_{N_T}$  is fed to the ZF-IC system. Finally, the ZF-IC system estimates the rest of symbols,  $x_1, x_2, \dots, x_{N_T-1}$ . The output that results from the first stage is considered to be the input of the next stage which is ZF-IC.

The modulation scheme used is Binary Phase Shift Keying (BPSK). We have performed these simulations using MATLAB. In this model we use three different values of streams ( $N$ ), (i.e. 2, 3, and 4).

The Bit Error Rate (BER) was used to analyze the system performance. The BER is an important parameter which is

used to analyze the transmission impairments like noise, jitter and interference in wireless communication systems. The bit error rate is the probability that any given bit of the received data will be in error. A bit error rate of  $10^{-6}$  means that one bit in  $10^6$  will be in error. The simulation results of the proposed system using BPSK modulation scheme over Rayleigh fading channels has been obtained by plotting the BER versus the Signal to Noise Ratio (SNR) [18].

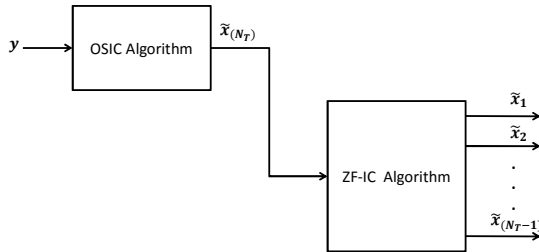


Fig. 3 Illustration of OSIC signal detection for  $N_T$  spatial streams

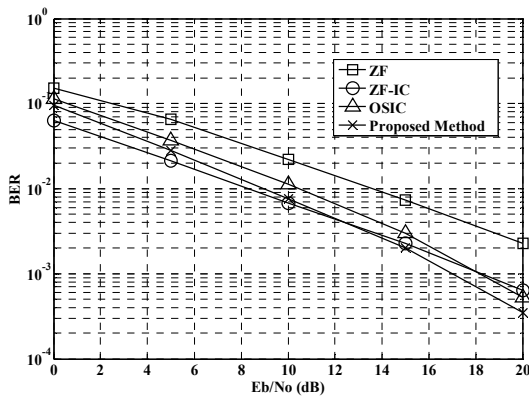


Fig. 4 The simulation results for  $N = 2$

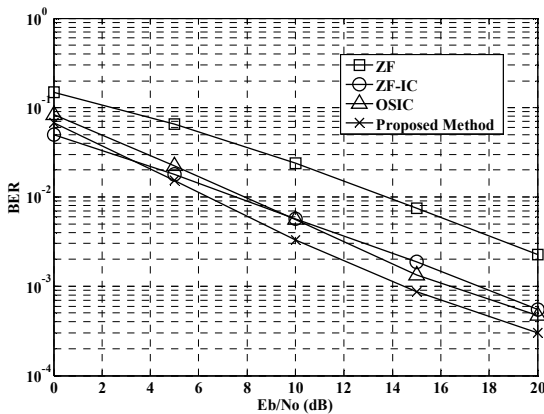


Fig. 5 The simulation results for  $N = 3$

For the comparison purposes, we run the simulation program for three different values of  $N$ ; the first value ( $N = 1$ ) reflects a little number of streams. The third value ( $N = 4$ ) reflects bigger value of streams. Figs. 4-6 show the comparison of results between the proposed method and ZF, ZF-IC and OSIC algorithms for different values of number of

streams ( $N$ ). It can be shown from the three figures that the proposed algorithm gives better performance than others. By increasing the number of streams ( $N$ ), The BER performance is improved.

Fig. 7 shows a comparison of results of the proposed method between different values of number of streams ( $N$ ). Fig. 7 shows that the performance is enhanced when increasing the number of spatial streams ( $N$ ) due to the interference cancellation.

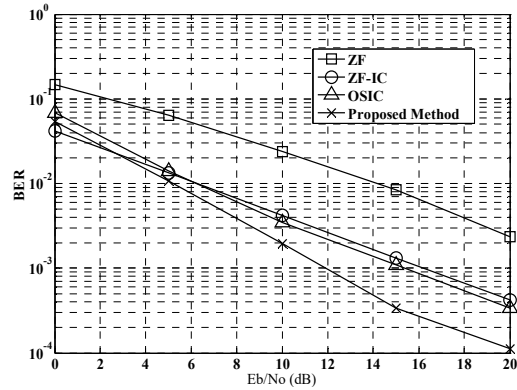


Fig. 6 The simulation results for  $N = 4$

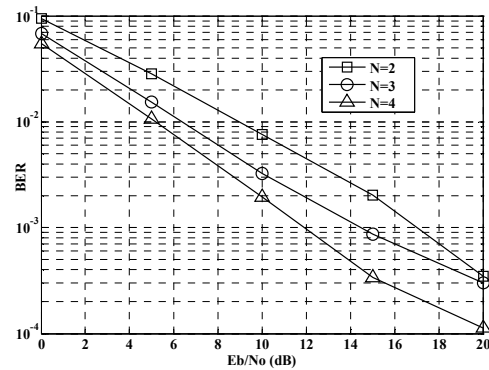


Fig. 7 Comparison of results between different values of streams

#### IV. CONCLUSION

In this paper, we propose a linear signal technique for detection a spatial multiplexing. This technique is based on a combination of an OSIC method and ZF-IC method. The Signal to Noise Ratio (SNR) versus Bit Error Rate (BER) was evaluated under BPSK modulation scheme for MIMO system over Rayleigh fading channel. Increasing the number of spatial streams ( $N$ ), the BER performance of the proposed scheme becomes higher. The results show that our scheme outperforms the other methods used for signal detection.

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