Parameters Affecting the Elasto-Plastic Behavior of Outrigger Braced Walls to Earthquakes

T. A. Sakr, Hanaa E. Abd-El- Mottaleb

Abstract—Outrigger-braced wall systems are commonly used to provide high rise buildings with the required lateral stiffness for wind and earthquake resistance. The existence of outriggers adds to the stiffness and strength of walls as reported by several studies. The effects of different parameters on the elasto-plastic dynamic behavior of outrigger-braced wall systems to earthquakes are investigated in this study. Parameters investigated include outrigger stiffness, concrete strength, and reinforcement arrangement as the main design parameters in wall design. In addition to being significantly affect the wall behavior, such parameters may lead to the change of failure mode and the delay of crack propagation and consequently failure as the wall is excited by earthquakes. Bi-linear stress-strain relation for concrete with limited tensile strength and truss members with bilinear stress-strain relation for reinforcement were used in the finite element analysis of the problem. The famous earthquake record, El-Centro, 1940 is used in the study. Emphasize was given to the lateral drift, normal stresses and crack pattern as behavior controlling determinants. Results indicated significant effect of the studied parameters such that stiffer outrigger, higher grade concrete and concentrating the reinforcement at wall edges enhance the behavior of the system. Concrete stresses and cracking behavior are too much enhanced while less drift improvements are observed.

Keywords—Structures, High rise, Outrigger, Shear Wall, Earthquake, Nonlinear.

I. INTRODUCTION

UTRIGGER-BRACED shear walls are considered now one of the most efficiently used systems for wind and earthquake resistance in high rise buildings. As the height of the building increases, separate cores or walls could not provide the building with adequate stiffness to keep the wind and earthquake drifts within the acceptable limits. Outriggers are deep, stiff beams which connect the central core or wall to the exterior most columns which restraint the rotation of the wall leading to the reduction of sway [12]. This system contains three main elements, i.e. deep outrigger beam, the core wall and the exterior column [2]. Outrigger-braced system helps in reducing the movement of the core and reduces the lateral drift at top [6], [8], [10] compared to the system with freely standing core without outriggers. For composite buildings subjected to wind loads, the existence of outriggers was also reported to reduce the top drift by 34, 42, and 51 percent for the cases studied by using a finite element

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model [4]. The same for case of lateral triangular loads was carried out [14] leading to an optimum location of outrigger 4 to 5 % higher than that for lateral uniform loads. Reference [13] studied the optimum location of outrigger wall systems up to 4 levels using multiple regression analysis which can be used for preliminary design. The same conclusions that the behavior of the structure can be significantly influenced by the location of the outrigger were also investigated [15]. It was also indicated that in most ordinary cases the best location of outriggers to minimize top drift is somewhere between 0.4 to 0.6 of the height of the structure. Increasing the rigidity of outriggers to very high vales which may result in high restraining moments leading to weak story have been studies [16]. They concluded that optimization analysis based on actual rigidity is very important and that infinite rigidity assumption affects the results. Other systems considered as "virtual" outriggers for tall buildings instead of conventional outriggers as belt trusses, façade riggers and basements had been also discussed [7], [9], [11]. As discussed almost all studies surveyed considered the linear analysis and optimization of outrigger braced systems subjected to lateral

In this paper, the elasto-plastic dynamic behavior of outrigger braced walls is investigated. Bi-linear material models are incorporated in a Dynamic Elasto-Plastic Finite element program (DEPF) especially developed [5]. Example wall of 40 story composed of rigid wall and external column with practical dimensions was prepared. One outrigger is added to mid-height of the wall which is, as reported, the location for optimized wall behavior [14], [15]. Time history analysis was carried out leading to detailed results of lateral drift, wall stresses and cracking patterns at each time step up to failure. Results were thoroughly investigated and conclusions were driven concerning the enhancements resulting for the addition of outrigger. Parameters investigated included outrigger stiffness, concrete strength, and reinforcement arrangement as the main important design parameters affecting the system behavior.

II. MODEL FOR DYNAMIC ELASTO-PLASTIC FINITE ELEMENT

A model was adopted for dynamic non-linear analysis composed of four node quadratic elements for concrete and two node truss element for reinforcement bars. Prior to its cracking or yielding, Concrete is assumed to be homogeneous and isotropic material in state of plane stress. The material model of concrete is based on bi-linear stress strain relationship with elasto-plastic behavior in compression and limited tensile strength considering the cracking process.

Simple bilinear stress-strain curve with strain-hardening is employed for steel reinforcement. The hysteretic response curves for the materials are presented in Fig. 1 [1]. Considering the above-mentioned material model, nonlinear time history analysis was carried out to solve the equation of motion of the system shown in (1) using the known implicit step-by-step, β-3 algorithm [3].

$$[M]_{n \times n} \{ \ddot{U} \}_{n \times 1} + [C]_{n \times n} \{ \dot{U} \}_{n \times 1} + [K]_{n \times n} \{ U \}_{n \times 1} = \{ R \}_{n \times 1}$$
 (1)

where N= number of degree of freedom; [K] = stiffness matrix; [C] = damping matrix; [M] = mass matrix; and $\{U\}$, $\{U^*\}$, $\{\ddot{U}\}$ nodal displacement, velocity and acceleration respectively.

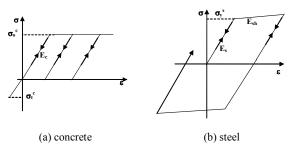


Fig. 1 Uni-axial stress- strain curves for cyclic loading [1]

III. EXAMPLE MODEL

A model for coupled wall-Column system with outrigger at mid-height was prepared for investigating the behavior of outrigger-braced system. This model is the same model as in [10]. The example system is 40 stories, 4 m height each with 500x8000 mm wall, 800x2000 mm column separated by 8000 mm. The outrigger was added in the 21st story full height with depth 4000 mm. for such system; several factors are assumed to affect its behavior under earthquake excitation. The effects of many parameters are included in this study to clarify how to adjust these parameters for enhancing the performance of outrigger braced wall system. Parameters considered are the outrigger stiffness, concrete strength, and distribution of wall reinforcement which are always altered for better behavior. The famous earthquake record (El-Centro, 1940) is used in the analysis. Horizontal components are only used after being normalized to 0.2 g which is the most common design PGA in the Middle East region. The finite element model of example system and the acceleration records of earthquakes used in the analysis are shown in Fig. 2.

IV. EFFECT OF OUTRIGGER STIFFNESS

Little is known quantitatively about the actual dynamic behavior of the coupled wall-column-outrigger and implications of outrigger size, strength, and ductility under earthquake motions. Thus, in this part, the effect of the outrigger stiffness is investigated by analyzing coupled wall-Column system with different outrigger thicknesses of 250, 400, 550 and 700 mm. Fig. 3 illustrates the effect of the stiffness of outrigger on the time history of lateral

displacement at the top of the coupled wall-Column system. As expected the case of 250 mm outrigger gives more lateral displacement than the reference case of 400 mm thickness outrigger. This can be simply attributed to the increase of the overall stiffness decrease in case of weak outrigger in addition to the early cracking of the outrigger itself. On the other hands, increasing the outrigger thickness to 550 and 700 mm has led to an adverse effect as the maximum lateral stiffness begun to increase over the reference case.

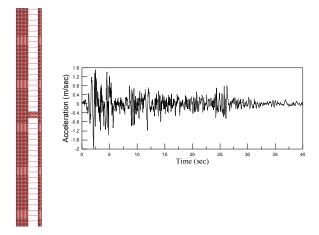


Fig. 2 Finite element model and earthquake records; El-Centro, 1940

The increase of maximum lateral stiffness is approximately 13.4% and 27.5% for the case of 550 mm and 700 mm outrigger, respectively, more than the reference case as shown in Fig. 4 which plots the maximum lateral top displacement against the outrigger thickness. Such increase of maximum lateral displacement is due to the excess cracking of walls near the outrigger for these cases, which lead to the reduction of outrigger coupling effect. The outrigger contribution to the wall stiffness is not as much as expected as illustrated in Table I which lists the natural periods and lateral top displacement as per the IBC code static provisions for walls without and with outrigger having different thicknesses. While the existence of outrigger adds much to the stiffness of walls, increasing the outrigger thickness contributes with very little values. The existence of outrigger reduces the wall natural period by about 41 percent of that without outrigger while increasing the outrigger thickness from 250 mm to 700 mm leads to a reduction with only 2.2 percent. The same behavior is obvious for the lateral displacement which falls from 1340 mm to 471 mm just when adding 250 mm outrigger and only to 450 mm when increasing the outrigger stiffness to 700 mm.

TABLE I CHARACTERISTICS FOR DIFFERENT OUTRIGGER THICKNESSES

Outrigger	No	250	400	550	700
	Out.	mm	mm	mm	mm
Natural Period	9.84	5.82	5.74	5.71	5.69
IBC Max. Drift	1340	471	460	454	450

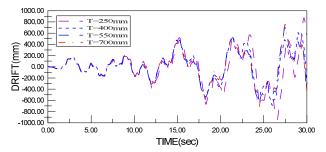


Fig. 3 Time history of lateral top displacement of the coupled system for different outrigger thickness

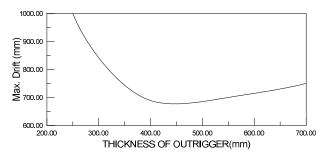


Fig. 4 The Relation between the maximum lateral top displacements against the outrigger thickness

Another controlling parameter is investigated through Figs. 5, 6. These figures show the plot of the crack pattern obtained from nonlinear analysis at critical times for the coupled wall-Column system with different outrigger thicknesses. After 5 seconds as shown in Fig. 5, the effect of outrigger thickness is very clear as shear cracks propagates more in the outrigger with smaller thickness. As can be easily observed, cracks for 250 mm thickness outrigger are more than that developed in the 400 and 550 mm thickness, while the initial cracks for the case of thick outrigger are not found in the outrigger like other cases but they are found in the wall beside the outrigger. This behavior is attributed to the higher resistance of thicker outriggers that lead to the delay of crack propagation inside it. At this stage, no cracks were observed in the wall or column, except in case of thicker outrigger for which cracks were developed in the wall near the outrigger. This may explain the stress concentration in case of extremely stiff outrigger discussed in previous work [16]. As excitation continues and time passes, cracks began to propagate in the coupled wall-Column system due to the flexure failure in addition to the increase of shear cracks in the coupling outrigger. After 20 seconds, as shown in Fig. 6, almost all the outrigger elements were indicated as cracked in case of the 250 mm thickness outrigger. For the 400 mm and 550 mm thickness, most of elements are cracked across the overall section of the outrigger indicating shear and flexure failure of outrigger elements. The 700 mm thick outrigger is different such that end parts are only cracked as result of flexure-shear stress interaction. The middle part of the outrigger which is subjected to pure shear has limited cracks as a result of the outrigger resistance enhancement. Wall cracks are observed at this time for all cases of outrigger thickness. It is clear that as cracks propagate

and cover the entire outrigger, the outrigger behaves as link member, losing the system coupling and coupled wall-Column system with outrigger behaves like separated wall and column. It is observed from the extents of cracking in walls that the outrigger protect the wall below its level from cracking especially for relatively thick outrigger such that no cracks in wall below the outrigger level in case of thickness of 550 and 700 mm except the wall bottom cracks while cracks propagates below the outrigger level for the other cases. Cracks at wall bottom are slightly affected by the outrigger thickness such that more outrigger thicknesses produce fewer cracks.

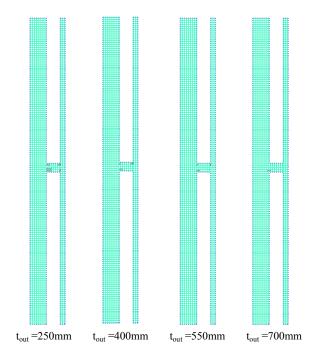


Fig. 5 Crack patterns for different outrigger thickness at 5 sec

The stress behavior of wall outrigger system is investigated through Fig. 7 that plots the time history of outrigger shear stress against excitation time. The outrigger element selected is that element attached to the wall side of the outrigger. While small shear stresses were observed in the element at the beginning of excitation during the first 15 seconds. At these stages, no significant difference was observed between different outrigger thickness cases. Later seconds of excitation produce more values of shear stresses and more significant difference between different cases which may be attributed to the development of cracks with different extents. The value of maximum shear stress is clearly decreased for thicker outriggers such that increasing the outrigger thickness to 400, 550, and 700 mm reduce the maximum shear stress by 43%, 58.3% and 60.4%, respectively, with respect to the case of 250 mm thickness.

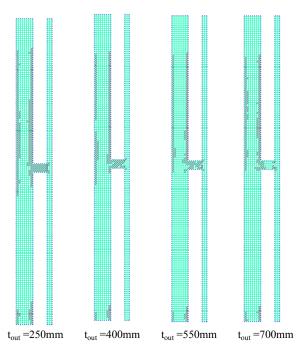


Fig. 6 Crack patterns for different outrigger thickness at 20 sec

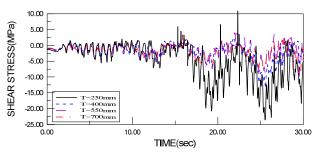


Fig. 7 Time history of the shear stress at the shown element for different outrigger thickness

V.EFFECT OF CONCRETE STRENGTH

One of the important parameters that are willing to affect the outrigger-braced system behavior is to change the grade of concrete. Different values of compressive strength of concrete are considered to investigate their effect on the system behavior. Four common values of the compressive strength (40, 50, 60 and 70 MPa) are considered. The time history responses of the lateral top displacement for walls with different concrete strengths are shown in Fig. 8. The results demonstrated that by increasing the value of the compressive strength of concrete, the lateral displacement decreases across the excitation time. Such improvement in displacement can be attributed to the slight increase of elastic modulus of concrete for higher concrete grades. In addition, especially in later times of excitation, more cracking are propagated for low strength concrete which leads to more drift. By investigating the values of maximum lateral drift, increasing the concrete strength to 50, 60, 70 MPa reduced the lateral top drift by 8.7, 30.1, and 37.4 percent, respectively, less than that of the case of 40 MPa concrete.

On the other hands, Fig. 9 shows the crack pattern of the system for different concrete strengths at different times. The cases of 40 and 70 Mpa are only demonstrated as the extreme values and as other cases are located in between. The crack pattern after 5 seconds illustrates how the cracks begin in the wall at the outrigger elements attached to walls but the difference between both cases is that in the case of high concrete strength, very few elements were cracked compared to system with low concrete strength which is expected due to the direct relation in the analysis model between the concrete compressive strength and the tensile strength which lead to the cracking phenomena. As excitation extends more time and after 10 seconds, cracks are extended to walls in regions above the outrigger level and near the wall base in case of lower concrete strength in addition being propagated in the outrigger itself. In case of higher strength concrete, 70 Mpa, cracks are observed to be very limited in the outrigger while for wall elements too little cracks began at the wall base. Near the end of earthquake time and after 30 seconds, the difference is clear between the shown cases. Outrigger elements for the case of low concrete strength are almost all cracked while the higher concrete can resist such that most of its outrigger elements still un-cracked. The coupling that the outrigger offer to the system is thus expected to be no longer exists for concrete with low strength which may also demonstrates the relatively large difference between drift for the two cases during the later times. Wall elements verify the same behavior of walls as cracks are propagated through the lower strength concrete across wide portions of the wall. This can also demonstrates the loss of coupling resulting from the spread of outrigger cracks and leading to more stresses in walls. For higher concrete strength, wall elements are almost clear of cracks verifying the existence of coupling as the outrigger still works and having few cracks. Thorough analysis of such investigations illustrates the great effect of increasing the concrete strength on the stability and resistance of wall outrigger systems.

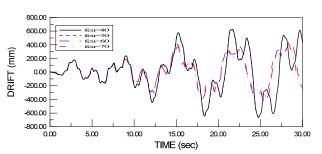


Fig. 8 Time history of lateral displacement at the top of the coupled wall-Column system for different concrete Grades

It is also so important to study the effect of changing the concrete strength on the stresses in wall and outrigger. Fig. 10 shows time history of the normal stress at wall element at the bottom of wall for the four cases of concrete strength. As shown in the plot, the first part and at early times, all cases behave in similar manner as the wall still un-cracked and the linear part of the stress strain diagram still dominates. Slightly

before 10 seconds, different behavior is observed as cracks are expected to spread on the outrigger and then in the wall. It can be observed that tensile strength governs the system behavior rather than compressive strength such that in case of higher concrete strength, the compressive stresses are below the extreme values that mean that tension cracks are most frequent. The big difference of behavior at later times demonstrates the effect of difference in crack pattern between different cases as discussed before. Little increase in tensile

stresses for higher grade concrete also verifies the little cracks observed in such cases. At later times and slightly before the end of earthquake, compressive stresses in walls for 60 and 70 MPa concrete are observed to be in very low levels which can be attributed to the coupling effect resulting from the delay of outrigger cracks. In all situations, it is clear that the increase of concrete strength lead to great improvement of the stress behavior of the wall-outrigger system.

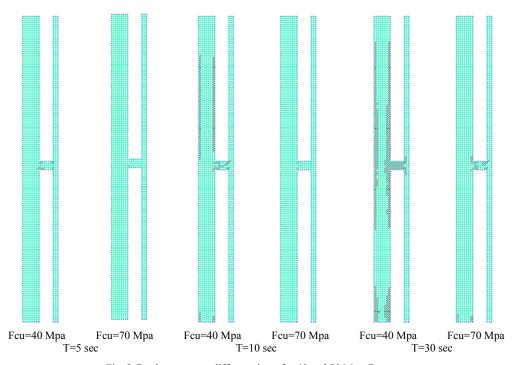


Fig. 9 Crack patterns at different times for 40 and 70 Mpa Concrete

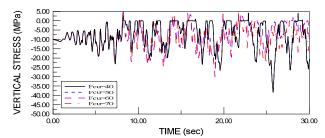


Fig. 10 Time History for wall base Normal stresses for different Concrete Grades

Shear stress behavior of the system is also plotted in Fig. 11 at which time history for the shear stresses in an element of the outrigger attached to the wall is plotted. As shown, the behavior is very similar at the beginning of excitation time. After short time and as outrigger elements began to crack especially for walls with lower concrete strength, difference began to be observed. The lower values of shear stresses in cases of 60, 70 MPa concrete till the end of excitation illustrates that the system keeps its coupling and that the outrigger works for the overall earthquake time. Relatively

high values of shear stresses in the outrigger element in cases of 40, 50 MPa concrete reflects the cracking of outrigger elements and the vanishing of system coupling.

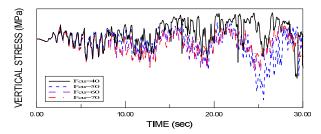


Fig. 11 Time History for Outrigger Shear stresses for different Concrete Grades

VI. EFFECT OF REINFORCEMENT ARRANGEMENT

There is an open discussion about the arrangement of reinforcement in walls and columns subjected to lateral loads. While uniform distribution is frequently used, the concentration of extra bars is recommended for flexure resistance. To investigate the effect of reinforcement

distribution on wall-outrigger behavior, two models are considered. One with uniform reinforcement through wall and column and the other has extra bars on the wall and column edge represents 0.15 percent of the concrete section as recommended by most codes. Fig. 12 shows the time history of the lateral top displacement in the wall for the systems with different reinforcement arrangement. This plot demonstrates that, at the first stage of the earthquake at which the system still elastic, the value of the lateral top displacement is very close for the two cases. The difference is observed to be significant at later times after cracks began to propagate in the system such that the value of maximum lateral top displacement increased about 21.7% in case of uniform reinforcement over that having extra concentrated reinforcement that indicates that cracks are more for uniform reinforcement case. The increase of vibration period for the uniformly reinforced system also verified the decrease of its stiffness as a result of cracking. This means that, the response of the overall lateral displacement in the wall-column system is improved due to increase the stiffness of the system by arranging extra edge steel reinforcement.

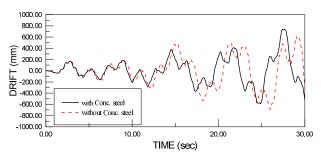


Fig. 12 Time history of the lateral top displacement in the wall for the systems with different reinforcement arrangement

The better behavior for the case of concentrated reinforcement can be also verified by investigating the stresses in wall base and the base of column as shown in Figs. 13, 14 respectively. These figures show time history for the normal stresses in wall element at wall side attached to the base and the base of column. It can be easily observed that at the beginning and prior to the earthquake, initial stresses at the selected points are less in case of wall with concentrated reinforcement than that in case of uniform reinforcement. The same indication is observed at the element in the column base. This static observation demonstrates the important share of load supported by the extra reinforcement provided at wall and column edge. During the entire time history, the normal stresses at wall and column base are observed to be higher for the case of uniform reinforcement which indicates that the existence of wall and column concentrated reinforcement produces better performance through two effects. The first is by delaying the cracking of wall or columns and the second by directly share the load with concrete element leading to reduction of its stresses.

Further observation for the behavior of wall-outrigger system with different reinforcement arrangement can be carried out by investigating their cracking pattern, as shown in Fig. 15. Crack pattern is plotted for both cases at earlier and later times of 5 and 30 seconds of ground motion. At 5 seconds, the same crack pattern is observed for both cases such that the same cracks are formed. Identical behavior can be attributed to the existence of cracks in the outrigger edge which is not affected by the arrangement of wall or column reinforcement. Later at 30 seconds, pronounced difference is observed between the studies cases. The system with uniform reinforcement generates too many cracks in the wall in the upper and lower part in addition to the outrigger cracks. In case of additional concentrated reinforcement, cracks in the upper part are lesser than that in the uniform reinforcement case and no cracks are observed in the lower part of the wall except near the fixed base. This reflects the lower stresses in the case of concentrated reinforcement case which delay the propagation of cracks. It is observed also that the outrigger in case of concentrated reinforcement is almost totally cracked while in case of uniform reinforcement there are few elements not cracked yet. This is also attributed to the increased resistance of walls that leading to more stiff system and causing more cracks in the outrigger.

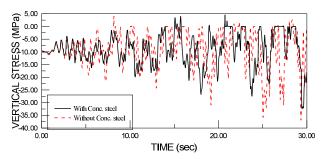


Fig. 13 Time history of the normal stress at the element in the wall base for the systems with different reinforcement arrangement

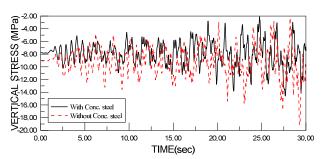


Fig. 14 Time history of normal stress at the element in the column base for the systems with different reinforcement arrangement

VII. SUMMARY AND CONCLUSIONS

The use of outrigger braced systems in high rise buildings for lateral load resistance is increased nowadays due to the large spread of skyscrapers all over the world. The present study investigated the effects of important design parameters on the behavior of outrigger braced walls. Finite element model is justified considering bi-linear elasto-plastic stress-strain relationship for concrete and bilinear behavior of

reinforcement bars with strain hardening. The main findings of the study are as follows

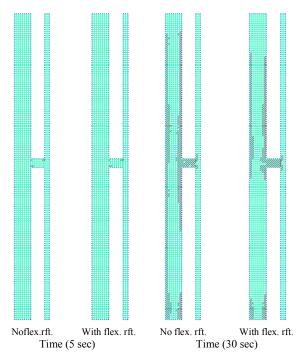


Fig. 15 Crack patterns for the systems with different reinforcement arrangement

- While the existence of outrigger adds too much to the overall stiffness of the system by reducing its drift, increasing the outrigger stiffness has little effect on the drift behavior. Increasing the outrigger thickness more contribute to the wall stresses and cracking behavior such that it delay the propagation of cracks in walls especially in the lower part of the wall.
- Increasing the concrete strength is observed to positively add to the behavior of outrigger-wall systems especially the stresses and cracking behavior. The drift behavior is also affected by increasing the concrete strength as a result of delaying the cracks and extending the elastic behavior more time.
- The distribution of reinforcement in walls verified more influence in the behavior of wall-outrigger systems. The provision of extra edge reinforcement for flexure in walls reduces the lateral drift and delay the cracking of walls in a manner better than the use of uniform reinforcement. The stresses in walls especially in the lower parts of walls are also enhanced as a result of reinforcement concentration.

It is concluded that the existence of outrigger enhances significantly the drift behavior and improve its overall behavior. The increase of outrigger stiffness and concrete strength makes little improvements to the drift behavior but enhance in significant manner the stress and cracking behavior. It is also recommended to provide flexure reinforcement to walls in addition to the uniform reinforcement as this arrangement enhances more the drift, stress and cracking behavior of the system.

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