

PAPR Reduction in OFDM Systems Using Orthogonal Eigenvector Matrix

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Abstract—OFDM systems are known to have a high PAPR (Peak-to-Average Power Ratio) compared with single-carrier systems. In fact, the high PAPR is one of the most detrimental aspects in the OFDM system, as it can cause power degradation (Inband distortion) and spectral spreading (Out-of-band radiation). In this paper, from the foundation of the PAPR analysis an effective method of PAPR reduction has been proposed based on Orthogonal Eigenvector Matrix (OEM) transform. Extensive computer simulations show that a PAPR reduction of up to 4.4 dB can be obtained without introducing in-band distortion or out-of-band radiation in the system.

Keywords—Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), Orthogonal Eigenvector Matrix (OEM).

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) is one of the most attractive multicarrier modulation schemes for high bandwidth efficiency and strong immunity to multipath fading [1]. OFDM offers a considerable high spectral efficiency, multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency [1], [2]. As a result, OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and worldwide interoperability for microwave access (mobile WiMAX) based on OFDM access technology [3].

One of the major challenges in OFDM is high PAPR of transmitted OFDM signals. Therefore, the OFDM receiver's detection efficiency is very sensitive to the nonlinear devices used in its signal processing loop, such as Digital-to-Analog Converter (DAC) and High Power Amplifier (HPA), which may severely impair system performance due to induced spectral regrowth and detection efficiency degradation. For example, most radio systems employ the HPA in the transmitter to obtain sufficient transmits power and the HPA is usually operated at or near the saturation region to achieve the maximum output power efficiency, and thus the memory-less nonlinear distortion due to high PAPR of the input signals will be introduced into the communication channels. If the HPA is not operated in linear region with large power back-off, it is impossible to keep the out-of-band power below the specified

limits. This situation leads to very inefficient amplification and expensive transmitters [4].

To overcome above mentioned serious drawbacks, various approaches have been proposed recently and some of the techniques have been summarized in [5] including clipping, filtering, coding schemes, phase optimization, nonlinear companding transforms, Tone Reservation (TR) and Tone Injection (TI), constellation shaping, Partial Transmission Sequence (PTS) and Selective Mapping (SLM). These reduction methods can be categorized into distortion and distortion-less techniques. Distortion techniques are considered to introduce spectral regrowth. They do not require any side information to be sent and they have low complexity compared to the distortion-less techniques. Here the simplest method is to clip the peak amplitude of the OFDM signal to some desired maximum which is an irreversible nonlinear process which surely degrades the system performance. Distortionless techniques on the other hand, do not suffer from spectral regrowth, but they do require sending side information to the receiver.

In this paper, an efficient PAPR reduction technique based on orthogonal eigenvector matrix method is proposed. This is a matrix transformation method to reduce the PAPR and outperforms the existing transformation techniques without introducing in-band distortion or out-of-band radiation in the system. The rest of the paper is organized as follows: A brief overview of PAPR in OFDM system, proposed orthogonal eigenvector matrix method along with existing transformation method is given in Section II. This section also finds an intimate relationship between autocorrelation function and peak power of the signal. Section III introduces the system model of the proposed method. Simulation results for the proposed method are shown in section IV. Finally, Section V concludes the article.

II. OVERVIEW OF OFDM AND PAPR PROBLEM

A. Peak-to-Average Power in OFDM Signals

In this section, we review the basic of OFDM transmitter and the PAPR definition. Consider an OFDM consisting of N subcarriers. Let a block of N symbols $\mathbf{X} = \{X_k, k = 0, 1, \dots, N-1\}$ is formed with each symbol modulating one of a set of subcarriers $\{f_k, k = 0, 1, \dots, N-1\}$. The N subcarriers are chosen to be orthogonal, i.e., $f_k = k\Delta f$, where $\Delta f = 1/(NT)$ and T is the original symbol period. Therefore, the complex baseband time domain OFDM signal can be written as

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$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j(2\pi f_k t)}, 0 \leq t \leq NT \quad (1)$$

where X_k is the symbol carried by the k_{th} sub-carrier, Δf is the frequency difference between sub-carriers. In the transmitter, the signal or sequence may be generated by the Inverse Fast Fourier Transform (IFFT) of the N -point $\{X_k\}$ sequence, and at the receiver, the Fast Fourier Transform (FFT) is employed to restore the signal.

An OFDM signal consists of an N number of independently modulated Sub carriers, which can give a large peak-to-average power (PAP) ratio when added up coherently, they produce a peak power that is N times the average power. PAPR is the ratio between the maximum power and the average power of the complex signal. The PAPR can be expressed for the time domain OFDM signal $x(t)$ as

$$PAPR = 10 \log_{10} \left[\frac{\max |x(t)|^2}{E\{|x(t)|^2\}} \right] \quad (2)$$

where $E\{\cdot\}$ denotes the expectation operation. As more subcarriers are added, higher peak values may occur, hence the PAPR increases proportionally with the number of subcarriers. Reducing $\max |x(t)|$ is the principle goal of PAPR reduction techniques. Complementary Cumulative Distribution Function (CCDF) is the most common way to evaluate the statistic properties of PAPR by estimating the probability of PAPR when it exceeds a certain level $PAPR_0$. When the number of the subcarriers N is relatively small, the CCDF expression of the PAPR of OFDM signals can be written as [5]

$$CCDF = P(PAPR > PAPR_0) = 1 - (1 - \exp(-PAPR_0))^N$$

This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$, sometimes referred as symbol clip probability.

B. Existing Transforms to Reduce PAPR

1) Hadamard Transform

Hadamard matrices are simple in structures, they are square, and have elements +1 or -1. Yet they have been actively studied for over 145 years and still have more secrets to be discovered. Hadamard transform may reduce the occurrence of the high peaks comparing the original OFDM system. Hadamard matrix have mutually orthogonal row vectors and orthogonal column vectors, means every two different rows in this matrix represent two perpendicular vectors. The idea to use the orthogonal rows of or columns to reduce the autocorrelation of the input sequence and it requires no side information as well. The hadamard matrix of $2N$ orders may be constructed by

$$\mathbf{H}_{2N} = \frac{1}{\sqrt{2N}} \begin{bmatrix} \mathbf{H}_N & \mathbf{H}_N \\ \mathbf{H}_N & -\mathbf{H}_N \end{bmatrix} \quad (3)$$

where $-\mathbf{H}_N$ is the complementary of \mathbf{H}_N . Hadamard matrix satisfy the relation $\mathbf{H}_{2N} \mathbf{H}_{2N}^T = \mathbf{H}_{2N}^T \mathbf{H}_{2N} = \mathbf{I}_{2N}$

2) Discrete Cosine Transform

Like Fourier transform, discrete cosine transform (DCT) express a signal in terms of a sum of sinusoids with different frequencies and amplitudes. A one-dimensional DCT of length N is given by:

$$X_T(k) = \alpha(k) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{\pi(2n+1)k}{2N} \right] \quad (4)$$

Equation (3) can be represented in vector as $\mathbf{X}_T = \mathbf{H}_N \mathbf{x}$, where \mathbf{H}_N is a DCT matrix of dimension $N \times N$; \mathbf{X}_T and \mathbf{x} are the vector representation of $X_T(k)$ and $x(n)$ respectively. The rows (or columns) of the DCT matrix \mathbf{H}_N are orthogonal matrix vector in nature, using this property it is possible to reduce the autocorrelation of the sequence and thus peak power of OFDM signals. The parameter $\alpha(k)$ is defined as

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{N}}, & \text{for } k = 0 \\ \frac{2}{\sqrt{N}}, & \text{for } k \neq 0 \end{cases} \quad (5)$$

For $k = 0, 1, \dots, N-1$ and the inverse transform is defined as

$$x(n) = \sum_{k=0}^{N-1} \alpha(k) X_T(k) \cos \left[\frac{\pi(2n+1)k}{2N} \right] \quad (6)$$

For $n = 0, 1, \dots, N-1$

C. Proposed Symmetric, Orthogonal Eigenvector Matrix

A symmetric matrix is a square matrix that is equal to its transpose $\mathbf{A} = \mathbf{A}^T$, where T represents the transpose of the matrix. The entries of a symmetric matrix are symmetric with respect to the main diagonal. So if the entries are written as $\mathbf{A} = (a_{ij})$, then $a_{ij} = a_{ji}$, where i, j being the coordinates.

In linear algebra, *symmetry* means real eigenvalues and perpendicular eigenvectors. A matrix with these properties must be symmetric; every symmetric matrix has these properties. Diagonal matrices are included as a special case, they are obviously symmetric. Their eigenvalues are on the main diagonal and their eigenvectors lie along the coordinate axes. These directions are certainly orthogonal. For other symmetric matrices the eigenvectors point in other directions, but the key property remains true: the eigenvectors are *perpendicular*, [7, p. 60].

A isan orthogonal matrix if its transpose is equal to its inverse, i.e., $\mathbf{A}^T = \mathbf{A}^{-1}$ which entails $\mathbf{A}^T \mathbf{A} = \mathbf{A} \mathbf{A}^T = \mathbf{I}$ where \mathbf{I} is the identity matrix. An orthogonal matrix \mathbf{A} is necessarily invertible ($\mathbf{A}^{-1} = \mathbf{A}^T$), unitary ($\mathbf{A}^{-1} = \mathbf{A}^*$), and normal $\mathbf{A}^* \mathbf{A} = \mathbf{A} \mathbf{A}^*$, where $*$ represents the conjugate of the matrix.

According to the Theorem 2.1 in [6, p. 157], such a symmetric orthogonal, eigenvector matrix can be given as

$$S = \sqrt{\frac{2}{n+1}} \left(\left(\sin \frac{ij\pi}{n+1} \right) \right)_{i,j=1}^n, \rho_n(S) \leq \frac{n+1}{2} \quad (7)$$

S is a symmetric, orthogonal eigenvector matrix for the second difference matrix and ρ_n is the *growth factor* [6]. For our

analysis we define this Symmetric, orthogonal eigenvector matrix \mathbf{H} as an $N \times N$ square matrix:

$$\mathbf{H}_N(i, j) = \sqrt{\frac{2}{N+1}} \left(\sin \frac{ij\pi}{N+1} \right) \quad (8)$$

Mathematically I can be proved that $\mathbf{H} = \mathbf{H}^T = \mathbf{H}^{-1}$ and $\mathbf{H}^T \mathbf{H} = \mathbf{H} \mathbf{H}^T = \mathbf{I}$, which proves that the matrix is symmetric as well as orthogonal.

D. PAPR Related to Autocorrelation

Composite OFDM signals can exhibit a very high PAPR when the input sequences are highly correlated. The idea of PAPR reduction using orthogonal eigenvector matrix transform is to reduce the autocorrelation of the input sequence before the IFFT operation and it requires no side information to be transmitted to the receiver. A close relation between the PAPR of an OFDM signal and the periodic autocorrelation function (ACF) of an input vector can be found in [8]. Assume $\rho(i)$ is the ACF of a signal X , then

$$\rho(i) = \sum_{k=0}^{N-1-i} X_{k+i} X_k^* \text{ for } i = 0, 1, \dots, N-1 \quad (9)$$

where $*$ denotes the complex conjugate. Then the PAPR of the transformed signal is bounded by

$$\text{PAPR} \leq 1 + \frac{2}{N} \sum_{i=1}^{N-1} |\rho(i)| \quad (10)$$

A proof of (10) can be found in [8]. Letting $\lambda = \sum_{i=1}^{N-1} |\rho(i)|$, we found that an input signal with a lower λ yields a signal with a lower PAPR in OFDM systems. It is shown in this paper, that if the input vector is transformed by OEM before IFFT operation, the $\rho(i)$ and thus PAPR can be reduced.

III. SYSTEM MODEL OF PROPOSED METHOD

The block diagram of the transmitter is shown in Fig. 1 for the proposed method. Firstly, the serial data sequence is divided into block of length 'N' using serial to parallel (S/P) converter, then mapped to constellation points by PSK to produce the modulated symbols m_0, m_1, \dots, m_{N-1} . Then each block of modulated symbols is multiplied by OEM matrix. Considering an $N \times N$ OEM matrix we get the transformed OFDM signal as $\mathbf{x}_N = \text{IFFT}[\mathbf{X}_N \mathbf{H}_N]$ from (1). Therefore, the OEM transformed signals at the transmitter can be recovered correctly through the corresponding OEM inverse-transform at the receiver by \mathbf{H}_N^{-1} as $\tilde{\mathbf{m}}_N = \tilde{\mathbf{X}}_N \tilde{\mathbf{H}}_N \mathbf{H}_N^{-1}$. In general, \mathbf{H}_N is a transform matrix used to reduce the PAPR and can be Hadamard, DCT or OEM transform using (3), (4) or (8) respectively.

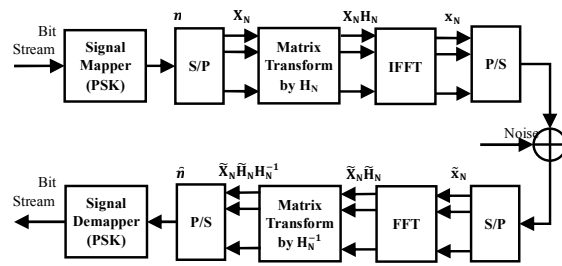


Fig. 1 OFDM transmitter with OEM transformation

IV. SIMULATION

In this section, we present simulations for a complex baseband OFDM system with $N = 64$ number of subcarrier employing a BPSK modulation by using 10^7 randomly generated OFDM symbols.

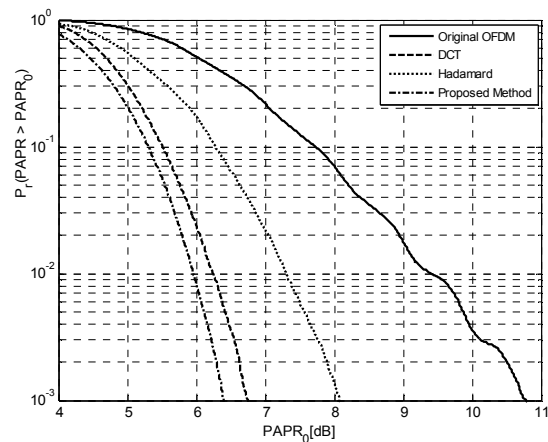


Fig. 2 Comparison of the CCDFs of different PAPR reduction schemes and the proposed method (OEM)

Fig. 2 shows the CCDF comparison of some existing powerful transforms and the proposed method based on OEM transform. With this method, at $\text{CCDF} = 10^{-3}$ the peak power is reduced by 4.4dB when compared with the case of original OFDM system, where DCT and Hadamard transform can reduce 4.1 dB and 2.8 dB respectively. This improvement can be achieved without sending any side information to the receiver.

Original OFDM signals have a very sharp, power spectrum as shown in Fig. 3. This property can be affected by the PAPR reduction schemes, e.g. slower spectrum roll-off, spectrum side-lobes, and adjacent channel interference. For example, clipping method of PAPR reduction causes in-band signal distortion and out-of-band radiation. It is also shown that the proposed method does not increase out-of-band distortion and the PSD is same as Original OFDM system, DCT or Hadamard transform. The PSD plot in Fig. 3 supports this statement.

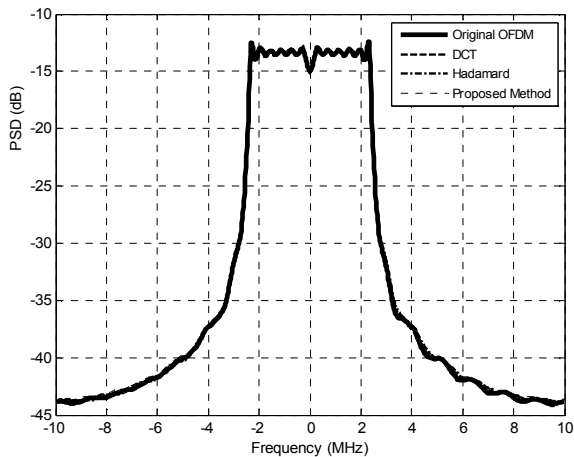


Fig. 3 PSD Comparison of different PAPR reduction schemes and the proposed method (OEM)

Computational Complexity of the proposed method is the complexity of matrix multiplication of \mathbf{X}_N and \mathbf{H}_N . According to *schoolbook multiplication*, the complexity of multiplying $\mathbf{X} \in \mathbb{R}^{1 \times N}$ by $\mathbf{H} \in \mathbb{R}^{N \times N}$ is $O(N^3)$, where \mathbb{R} denotes the set of real numbers. Complexity can be further reduced to $O(N^{2.81})$ using *Strassen's algorithm*. Direct application of DCT on the other hand, would require $O(N^2)$ operations. However, like FFT there are many fast methods to compute DCT as proposed in [9], where it is possible to compute DCT with only $O(N \log N)$ complexity by factorizing the computation [9].

V. CONCLUSION

OFDM is a very attractive technique for communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high peak power when the input sequences are highly correlated. In this paper, a new method of PAPR reduction technique has been proposed based on orthogonal eigenvector matrix transformation to reduce the autocorrelation coefficients of IFFT input. Performances of the proposed method are evaluated by MATLAB simulation. It has been shown, that the proposed method shows better CCDF performance than existing transformations. Without sending any side information or degrading the power spectrum, the proposed method reduces the PAPR by 4.4 dB over the original system where conventional transformations fail to achieve this. Mathematical analysis, including the distribution of the PAPR, PSD of the proposed method has been provided to support the statement.

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