Outer-Brace Stress Concentration Factors of Offshore Two-Planar Tubular DKT-Joints

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Abstract—In the present paper, a set of parametric FE stress analyses is carried out for two-planar welded tubular DKT-joints under two different axial load cases. Analysis results are used to present general remarks on the effect of geometrical parameters on the stress concentration factors (SCFs) at the inner saddle, outer saddle, toe, and heel positions on the main (outer) brace. Then a new set of SCF parametric equations is developed through nonlinear regression analysis for the fatigue design of two-planar DKT-joints. An assessment study of these equations is conducted against the experimental data; and the satisfaction of the criteria regarding the acceptance of parametric equations is checked. Significant effort has been devoted by researchers to the study of SCFs in various uni-planar tubular connections. Nevertheless, for multi-planar joints covering the majority of practical applications, very few investigations have been reported due to the complexity and high cost involved.

Keywords—Offshore jacket structure, Parametric equation, Stress concentration factor (SCF), Two-planar tubular KT-joint

I. INTRODUCTION

STEEL circular hollow sections (CHSs) are widely used in offshore structures due to their good resistances against bending, torsion and buckling, and a high strength-to-weight ratio. In a tubular joint, the members are connected by welding the prepared profiled end of the brace members onto the outer surface of the chord member. The fatigue design of such joints constitutes a critical factor towards safeguarding the integrity of tubular structures. The complex joint geometry causes significant stress concentrations at the vicinity of the welds. Under repeated loadings they result in the formation of cracks, which can grow to a size sufficient to cause joint failure. The location of maximum stress concentration is called ''hot-spot'' and the corresponding local stress is referred to as ''hot-spot stress'' (hss).

For fatigue design purposes, the ''hot-spot stress method'' has been quite efficient and popular. According to this method, the nominal stress range at the joint members is multiplied by an appropriate stress concentration factor (SCF) to provide the so-called ''geometric stress'' S' at a certain location. Hence, this design method relies on the accurate prediction of SCFs for tubular joints. The SCF is the ratio of the local surface stress to the nominal direct stress in the brace. The SCF value depends on joint geometry, loading type, weld size and type, and the location around the weld under consideration.

Geometric stresses S' are calculated at various locations around the welds and the maximum geometric stress is the hot-spot stress S. The fatigue life of the joint is estimated through an appropriate S–N fatigue curve, N being the number of load cycles.

Over the past thirty years, significant effort has been devoted to the study of SCFs in various uni-planar tubular joints (i.e. joints where the axes of the chord and the braces lay in the same plane). As a result, many parametric design equations (formulae) in terms of the joint geometrical parameters have been proposed, providing SCF values at certain locations adjacent to the weld for several loading conditions. Multi-planar joints are an intrinsic feature of offshore tubular structures. As can be seen in Fig. 1, right-angle 2-planar DKT-joints connecting the braces to the main legs are of the most critical tubular joints in a typical jacket structure. The multi-planar effect plays an important role in the stress distribution at the brace-to-chord intersection areas of the spatial tubular joints. For such multi-planar connections, the parametric stress formulae of simple uni-planar tubular joints are not applicable in SCF prediction. Nevertheless, for multi-planar joints which cover the majority of practical applications, very few investigations have been reported due to the complexity and high cost involved. The second section reviews the research works currently available in the literature.

The value of SCF along the weld toe of a tubular joint is mainly determined by the joint geometry under any specific loading condition. In order to study the behavior of tubular joints and to relate this behavior easily to the geometrical properties of the joint, a set of non-dimensional geometrical parameters has been defined. Fig. 2 shows a right-angle 2-planar tubular DKT-joint with the four commonly named locations along the brace-chord intersection of the outer brace: inner saddle, outer saddle, toe, and heel. Geometrical parameters (β, γ, τ, ζ, α, and α_B) respective to chord and brace diameters D and d, and the corresponding wall thicknesses T and t are also shown in Fig. 2.

In the present paper, parametric stress analysis has been carried out for 81 steel multi-planar (right-angle 2-planar) tubular DKT-joints under two different axial loading conditions. The analysis results are used to present general remarks on the effect of geometrical parameters including τ (brace to chord thickness ratio), γ (chord wall slenderness ratio), β (brace to chord diameter ratio) and θ (outer brace inclination angle) on the SCF values at the inner saddle, outer saddle, toe, and heel positions on the main (outer) brace. To study the multi-planar effect and to investigate the effect of loading condition, SCFs in multi-planar joints under two axial load cases are compared with the SCFs in a uni-planar KT-joint having the same geometrical properties.

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Based on the multi-planar DKT-joint FE models which are verified against both experimental results and the predictions of Lloyd's Register (LR) equations, a complete set of SCF database is constructed for two considered axial load cases at four weld toe locations: inner saddle, outer saddle, toe, and heel. The FE models cover a wide range of geometrical parameters. Through nonlinear regression analysis, a new set of SCF parametric equations is established for the fatigue design of multi-planar DKT-joints under axial loads. An assessment study of these equations is conducted against the experimental data and the satisfaction of the criteria regarding the acceptance of parametric equations is checked.
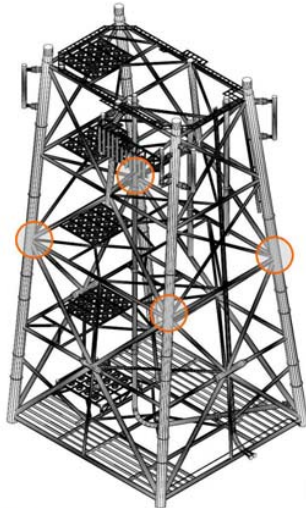

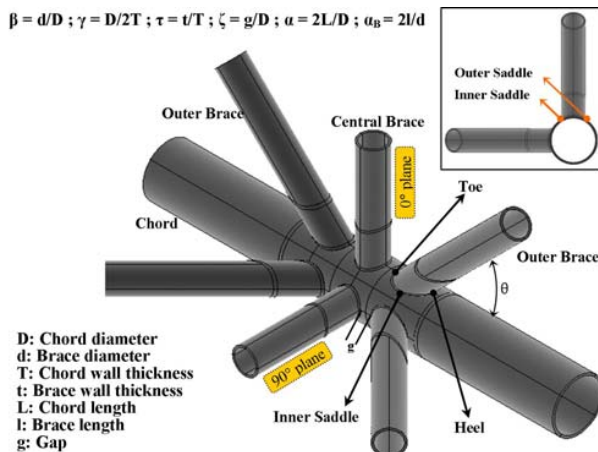Fig. 1 Two-planar DKT-joints in a typical jacket-type structure

Fig. 2 Geometrical notation for a 2-planar tubular DKT-joint

II. LITERATURE REVIEW

For the uni-planar tubular joints, the reader is referred for example to [1]–[3] (for SCF calculation at the saddle and crown positions of simple uni-planar T-, Y-, X-, K- and KT-joints), [4] (for SCF determination in uni-planar overlapped tubular joints), and [5]–[9] (for the study of SCF distribution along the weld toe of various uni-planar joints).

Following paragraph reviews the research works on the SCF calculation in the multi-planar tubular joints. Karamanos *et al.* [10] proposed a set of parametric equations to determine the SCFs for multi-planar welded CHS XX-connections. In this study, weld profile was modeled using 20-node solid elements while 8-node shell elements were used to model the chord and braces. This research covered the various loading modes including reference and carry-over loadings. Chiew *et al.* [11] studied the stress concentrations in DX-joints due to axial loads. Chiew *et al.* [12] developed a set of design formulae to determine the SCFs for multi-planar tubular XX-joints under axial, IPB and OPB loadings. Van Wingerde *et al.* [13] presented the equations and graphs to predict the SCFs for multi-planar KK-joints. The aim of this study was to simplify the equations for design purposes. Karamanos *et al.* [14] proposed SCF equations in multi-planar welded tubular DT-joints including bending effects. Woghiren and Brennan [15] developed a set of parametric formulae to predict the values of SCF in multi-planar rack-stiffened tubular KK-joints. An experimental database of SCFs for acrylic specimens of multi-planar K- and KT-joints has been presented In the HSE OTH 91 353 [16] prepared by Lloyd's Register. This report covers only the values of SCFs at the chord inner and outer saddle positions.

It can be seen that in the case of multi-planar joints, the studied connection types and load cases are very limited. Despite the frequent use of multi-planar CHS DKT-joints in the design of offshore jacket structures (see Fig. 1), no parametric equation is available to predict the SCF values in such tubular joints.

III. NUMERICAL SIMULATION OF TUBULAR JOINTS

Theoretical calculation of SCFs is difficult and the results from a strain gauged acrylic model test are not always reliable because the welding profile is not included in such specimens. The most accurate and reliable method for determining the SCFs is by testing strain gauged large scale or practical size steel joint specimens. However, due to its high cost and testing facility limitations, such a method is difficult to be used to study comprehensively the joints with various geometrical parameters and load conditions. Finite element method which has been used successfully to analyze the joints with various geometrical sizes and different load conditions is adopted in this study.

A. Geometrical Characteristics of the Models

To investigate the stress concentration in multi-planar tubular DKT-joints, 81 models are generated and analyzed using the multi-purpose FEM based software package, ANSYS [17]. The aim is to study the effect of dimensionless geometrical parameters on the SCF values at the inner saddle, outer saddle, toe, and heel positions on the outer brace. Different values assigned to each non-dimensional parameter are as follows: $\beta = 0.3, 0.4, 0.5$; $\gamma = 12, 18, 24$; $\tau = 0.3, 0.6, 0.9$; $\theta = 30°, 45°, 60°$. These values cover the practical range of the normalized parameters typically found in multi-planar tubular joints of offshore structures.

Geometrical characteristics of all braces are identical in each specific model. According to the values of γ, τ, and β in each joint, the values of diameter and the wall thickness of the braces are changed form one model to another. According to Lotfollahi-Yaghin and Ahmadi [9], the relative gap (ζ = g / D) has no considerable effect on the SCF values. Hence, a typical value of ζ = 0.2 is assigned for all joints. The values of α and $α_B$ which are fixed in all joints are 16 and 8, respectively. The reasons for choosing these specific values are given in sub-section III-C. The 81 generated models span the following ranges of the geometric parameters:

$$
\begin{aligned}
&0.3 \le \beta \le 0.5 \\
&12 \le \gamma \le 24 \\
&0.3 \le \tau \le 0.9 \\
&30° \le \theta \le 60°
\end{aligned}
\tag{1}
$$

B. Element Type and the Mesh Generation Method

The choice of element type for the analysis depends on the geometry of the joint and the purpose for which the results of the analysis are to be used. It has to be a compromise between the accuracy of representation and the computer time taken to analyze a particular model. The entire tubular joint can be modeled by 3D brick elements. Using this type of element, the weld profile is simulated as a sharp notch. This method will produce more accurate and detailed stress distribution near the intersection in comparison with a simple shell analysis. In the present study, ANSYS element type SOLID95 is used to model the chord, brace and weld profile. These elements have compatible displacements and are well suited to model curved boundaries. The element is defined by 20 nodes having three degrees of freedom per node. The element may have any spatial orientation.

A sub-zone mesh generation method is used during the FE modeling, in order to guarantee the mesh quality. In this method, the entire structure is divided into several different zones according to the computational requirements. The mesh of each zone is generated separately and then the mesh of entire structure is obtained by merging the meshes of all the sub-zones. This method can easily control the mesh quantity and quality and avoid badly distorted elements. The mesh generated by this method for a multi-planar right-angle tubular DKT-joint is shown in Fig. 3. It should be noted that in this study, the welding size along the brace-chord intersection satisfies the AWS specifications [18]. Modeling of the weld profile according to AWS [18] is extensively discussed in Lotfollahi-Yaghin and Ahmadi [9]. The models are meshed in such a way that leads to a compromise between the accuracy of results and the computer analyzing time, software generated file volume, etc. To verify the convergence of FE analysis, converging test is done and the meshes with different densities are used in this test, before generating the 81 models.
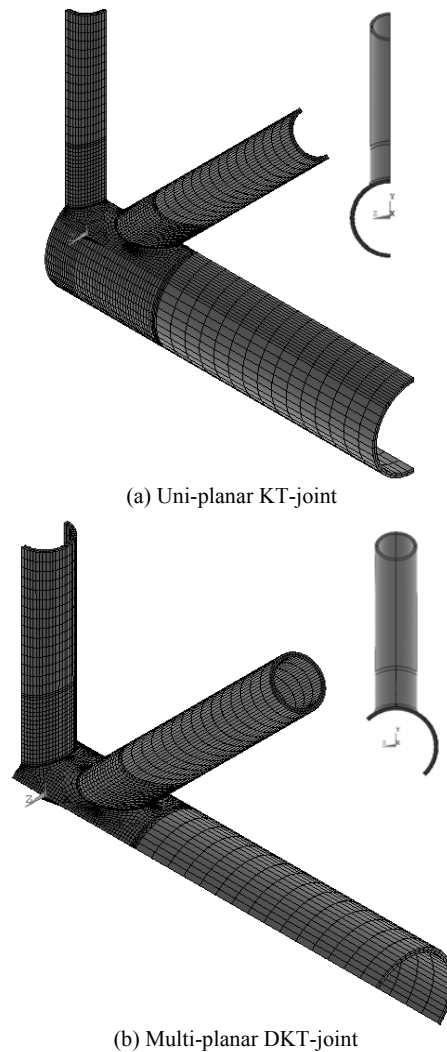

(a) Uni-planar KT-joint

(b) Multi-planar DKT-joint

Fig. 3 Generated mesh and view along the chord's longitudinal axis

C. Boundary Conditions

As shown in Fig. 3, Due to the symmetry in geometry of the connection and either symmetry or antisymmetry in loading conditions (Fig. 4), only one fourth of the entire multi-planar right-angle DKT-joint and equivalent uni-planar KT-connection are modeled. The chord end fixity conditions of tubular joints in offshore structures may range from "almost fixed" to "almost pinned" with generally being closer to "almost fixed" [2]. In practice, value of α in over 60% of tubular joints is in excess of 20 and is bigger than 40 in 35% of the joints [19]. According to Morgan and Lee [20], changing the end restraint from fixed to pinned results in a maximum increase of 15% in the SCF at crown for α = 6 joints, and this increase reduces to only 8% for α = 8. In view of the fact that the effect of chord end restraints is only significant for joints with α < 8 for high β and γ values, which do not commonly occur in practice, both chord ends are assumed to be fixed, with the corresponding nodes restrained.

Efthymiou [2] showed that sufficiently long chord greater than six chord diameters (i.e. $\alpha \geq 12$) must be used to ensure that the stresses at the brace-chord intersection are not affected by the end condition. Hence in this study, a realistic value of $\alpha = 16$ was assigned for all the models. The effect of brace length on SCF has been studied by Chang and Dover [5]. It was concluded that there is no effect when the ratio α_B is greater than the critical value. In the present study, in order to avoid the effect of short brace length, a realistic value of $\alpha_B = 8$ is selected for all joints.

D. Loading Conditions, Analysis Method, and SCF Extraction Procedure

Two different axial loading conditions are considered in the present research to study the SCFs in multi-planar DKT-joints. As shown in Fig. 4, in the 1st loading condition, all three braces located on the 0° plane are subjected to compressive loads while the ones on the 90° plane are under tensile loading. In the 2nd loading condition, tensile loads are applied to all six braces. Equivalent uni-planar KT-joints are subjected to tensile axial loads exerted on the central and outer braces. Static numerical calculations of the linear elastic type are appropriate to determine the SCFs in tubular joints [21]. This type of analysis is used in the present study. The Young's modulus and Poisson's ratio are taken to be 207 GPa and 0.3, respectively. The widely accepted conventional approach for fatigue strength assessment of tubular joints is to use the geometric stresses at the weld toe. According to IIW-XV-E [22], the peak stress is calculated from extrapolating the geometrical stresses at the two points in a linear way to the weld toe position. The minimum and maximum distances from the extrapolation region to the weld toe for chord member are $0.4T$ and $1.4T$ respectively; where T is the thickness of chord member. Therefore, the value of peak stress can be calculated as follows:

$$\sigma_{\text{weld toe}} = 1.4\sigma_1 - 0.4\sigma_2 \qquad (2)$$

where σ_1 and σ_2 are the von Mises stresses measured at the distance of $0.4T$ and $1.4T$ from the weld toe, respectively.

E. Verification of the finite element model

The accuracy of the FEA predictions should be verified against the experimental test results. As far as the authors are aware, there is no experimental database of SCFs for *steel* uni-planar and multi-planar tubular KT-joints currently available in the literature. In order to validate the finite element model, several related geometries including T-, Y- and K-joints are modeled and the FE results are validated against the LR equations [3] and test results published in HSE OTH 354 report [3].

The method of modeling the chord, the vertical brace, the inclined brace and the weld profile, and also the mesh generation procedure (including the selection of the element type) and the analysis method are identical for the validating models and the considered uni-planar and multi-planar KT-joints. Hence, the conclusion of the verification of the T-, Y- and K-joints with the experimental test results can be used to validate the generated uni-planar and multi-planar KT-joint models [9], [15].

Verification results which are separately presented at saddle and crown positions are summarized in Table I. In this table, e1 denotes the percentage of relative difference between the predictions of LR equations and test results, and e2 denotes the percentage of relative difference between the results of FE model and experimental results. Hence, |e1|–|e2| indicates the difference between the accuracy of LR equations and FE model. It can be concluded from the comparison of the FE results with experimental data and the values predicted by LR equations that the finite element model is considered to be adequate to produce valid results.
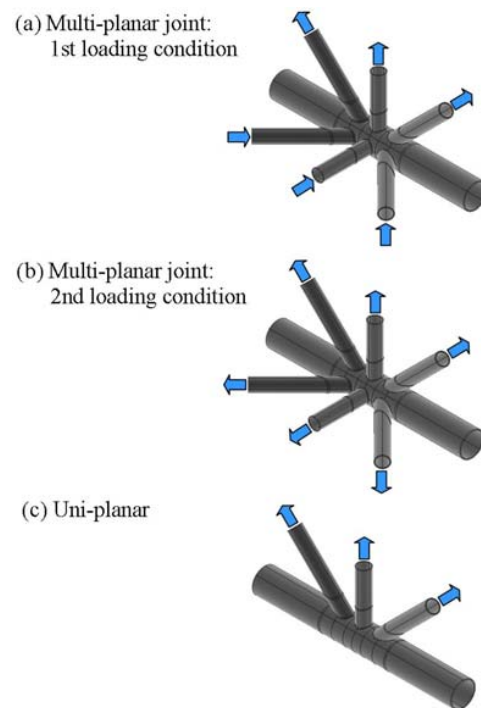

(a) Multi-planar joint: 1st loading condition

(b) Multi-planar joint: 2nd loading condition

(c) Uni-planar

Fig. 4 Studied loading conditions

TABLE I
VERIFICATION OF THE FEA RESULTS USING THE EXPERIMENTAL DATA [3] AND PREDICTIONS OF LR EQUATIONS [3]

Joint Type [a]	D (mm)	θ	α	τ	γ	β	Position	Test [3]	LR Eqs.	FEA	e1 [c] (%)	e2 [c] (%)	|e1|-|e2| (%)
T	508	90	6.2	0.99	20.3	0.8	Saddle	11.4	10.54	11.26	8	1	+7
							Crown	5.4	3.92	4.6	27	15	+12
Y	508	45	6.2	1.05	20.3	0.8	Saddle	8.3	5.48	5.46	32	34	-2
							Crown	4.7	3.5	4.7	25	0	+25
K [b]	508	45	12.6	1.0	20.3	0.5	Saddle	6.8	4.8	6.76	29.5	0.5	+29
							Crown	4.6	4.56	4.8	1	-4	-3

[a] Project reference: JISSP; [b] ζ = 0.15; [c] e1= (Test-LR Eqs.) / Test, e2= (Test-FE) / Test

IV. Results of Numerical Parametric Study

This section presents the results of numerical parametric study carried out to investigate the effect of non-dimensional geometrical parameters including β, γ, τ, and θ on the stress concentrations at the inner saddle, outer saddle, toe, and heel positions on the outer brace of the 2-planar right-angle DKT-joints.

A. Effect of Brace-to-Chord Diameter Ratio (β) on the SCFs

The parameter β is the ratio of brace diameter to chord diameter. Hence, increase of the β in the models having constant value of chord diameter leads to increase of brace diameter. This sub-section presents the results of investigating the effect of β on the SCFs. In this study, the influence of the parameters τ and γ over the effect of β on stress concentration is also investigated. For example, six diagrams are presented in Fig. 5 showing the change of SCFs due to the change in the value of β and the interaction of this parameter with the γ. Corresponding geometrical parameters, the position for the extraction of SCF, and the considered loading condition are given in the legend of each diagram. A total of 72 comparative diagrams were used to study the effect of the β and only 6 of them are presented here for the sake of brevity.

The general remarks which are concluded through investigating the effect of β on the stress concentration can be summarized as follows:

- Under the 1st loading condition, for small values of the γ and τ (say γ = 12 and τ = 0.3), increasing the β from 0.3 to 0.5 leads to decrease of SCFs at both inner and outer saddle positions in the joints with small values of θ (say θ = 30°). However, such increase in the β results in the increase of SCFs at these positions in the joints having big θ values (say θ = 60°). For intermediate values of θ (say θ = 45°), SCF change in these two positions due to the increase of the β follows this pattern: $SCF_{\beta=0.4} > SCF_{\beta=0.3}$; $SCF_{\beta=0.5} < SCF_{\beta=0.4}$.
- Under the 1st loading condition, in the joints with the intermediate and bigger values of γ and τ (say γ = 18, 24; τ = 0.6, 0.9), the maximum SCF at the inner saddle position always occurs in the joints having the intermediate value of the β (say β = 0.4).
- Under the 1st loading condition, the change of the SCFs at the toe and heel positions due to the increase of the β does not follow a regular pattern for different geometrical parameters. However, magnitude of these changes in the SCFs is not considerable.
- Under the 2nd loading condition, increase of the β leads to decrease of SCFs at the inner and outer saddles but increase of SCF values at the toe and heel positions. For example, in a joint having the following geometrical parameters: γ = 24, τ = 0.9, θ = 60°, $SCF_{\beta=0.5} / SCF_{\beta=0.3}$ ratio is 0.44, 0.76, 1.41, and 1.58 for inner saddle, outer saddle, toe, and heel positions, respectively.
- Under the 2nd loading condition, at the toe and heel position, increase of the τ leads to the increase in the magnitude of SCF growth due to the increase of the β. on the contrary, the magnitude of changing the SCF values

due to the increase of β follows an decreasing pattern at the inner and outer saddles as the τ takes bigger values.

B. Effect of Chord Wall Slenderness Ratio (γ) on the SCFs

The parameter γ is the ratio of radius to thickness of the chord. Hence, increase of the γ in the models having constant value of chord diameter leads to decrease of chord thickness. This sub-section presents the results of investigating the effect of γ on the SCFs. In this study, the influence of the parameters β and τ over the effect of β on stress concentration is also investigated. A total of 72 comparative diagrams were used to study the effect of the γ and only 4 of them are presented in Fig. 6, for the sake of brevity. This figure shows the change of SCFs due to the change in the value of γ and the interaction of this parameter with the τ. All four diagrams are results of the joints under the 1st loading condition.

Through investigating the effect of the γ on the SCFs, it can be concluded that:

- Under both loading conditions, increase of the γ results in increase of SCF values at the inner and outer saddle positions. Magnitude of the increase in these SCFs becomes larger as the τ increases. For example, under the 1st loading condition, in the joint having the following geometrical parameters: β = 0.4, θ = 45°, τ = 0.3, 0.6, 0.9; the increase of the SCF at the inner saddle position due to the change of the γ form 12 to 24 is 192%, 257%, and 374%, respectively. In the other words, the SCF has respectively increased by a factor of 2.92, 3.57, and 3.74.
- Under the 1st loading condition, the change of the SCFs at the toe and heel positions due to the increase of the γ does not follow a regular pattern for different geometrical parameters. On the contrary, increase of γ leads to increase in the SCFs at the toe and heel positions under the 2nd loading condition. Under both loading conditions, magnitude of the SCF change at the toe and heel positions is less than corresponding value at the saddle positions.

C. Effect of Brace-to-Chord Thickness Ratio (τ) on the SCFs

The parameter τ is the ratio of brace thickness to chord thickness and γ is the ratio of radius to thickness of the chord. Hence, increase of τ in the models having constant value of γ leads to increase of brace thickness. This sub-section presents the results of investigating the effect of τ on the SCFs. In this study, the influence of the parameters β and γ over the effect of τ on stress concentration is also investigated. For example, four diagrams are presented in Fig. 7 showing the change of SCFs due to the change in the value of τ and the interaction of this parameter with the β.

All four diagrams are results of the joints under the 1st loading condition. A total of 72 comparative diagrams were used to study the effect of the τ and only 4 of them are presented here for the sake of brevity.

Main conclusions of investigating the effect of the τ on the SCF values are summarized as follows:

- Under both loading conditions, increase of the τ results in increase of SCF values at all four considered positions: inner saddle, outer saddle, toe, and heel.

- At the inner and outer saddle positions, magnitude of SCF growth due to the increase of the τ is larger under the 1st loading condition compared to the 2nd one. For example, on the outer saddle of a joint having the following geometrical parameters: $\beta = 0.5$, $\gamma = 24$, $\theta = 45˚$, $SCF_{\tau = 0.9}$ / $SCF_{\tau = 0.3}$ ratio is 4.59 and 4.12 under the 1st and 2nd loading conditions, respectively.

- The magnitude of increase in the SCF values due to the increase of the τ is highly remarkable in comparison with the other geometrical parameters. For example, as can be seen in Fig. 7, due to the change of the τ from 0.3 to 0.9 in a joint with $\beta = 0.3$, the SCFs have increased by a factor of 2.48, 2.64, 4.34, and 4.63 at the heel, toe, inner saddle, and outer saddle positions, respectively. It can also be seen that magnitude of the SCF changes at the toe and heel positions is less than corresponding values at the saddle positions.

D. Effect of Outer Brace Inclination Angle (θ) on the SCFs

This sub-section presents the results of studying the effect of outer brace inclination angle θ on the SCFs at different positions and its interaction with the other dimensionless geometrical parameters. A total of 72 comparative diagrams were used to study the effect of the θ and its interaction with the other dimensionless geometrical parameters.

Through investigating the effect of the θ on the SCFs, it can be concluded that:

- Under both loading conditions, increase of the θ from 30˚ to 60˚ leads to increase of SCF values at inner and outer saddle positions.

- Under the 1st loading condition, the change of the SCFs at the toe and heel positions due to the increase of the θ does not follow a regular pattern for different geometrical parameters. On the contrary, under the 2nd loading condition, increase of the θ always results in increase of SCF value at the toe and heel positions.

- Under both loading conditions, magnitude of effect of different geometrical parameters on the SCFs follows the below order: Effect of τ > Effect of γ > Effect of θ > Effect of β

- Magnitude of the SCF changes due to the increase of the θ is smaller at the toe and heel positions in comparison with the corresponding values at the inner and outer saddle positions.

E. Comparison of the SCFs at Different Positions

As can be seen in Fig. 8, the maximum stress concentration under the 1st loading condition always occurs at the inner saddle position. While the minimum stress concentrations always occur at the heel position. In the other words:

$$SCF_{inner\ saddle} > SCF_{outer\ saddle} > SCF_{toe} > SCF_{heel} \qquad (3)$$
$$(1st\ loading\ condition)$$

On the contrary, under the 2nd loading condition, the order of SCFs at the four studied positions does not follow a regular pattern in joints having different geometrical parameters. For example, as can be seen in Fig. 8b, the order is $SCF_{outer\ saddle} > SCF_{toe} > SCF_{inner\ saddle} > SCF_{heel}$ for three joints having the following geometrical parameters: $\theta = 45˚$, $\gamma = 18$, $\beta = 0.4$, $\tau = 0.3, 0.6, 0.9$. However, in the joints with the big values of τ and θ (say $\tau = 0.9$, $\theta = 60˚$), the order is always as follows:

$$SCF_{outer\ saddle} > SCF_{inner\ saddle} > SCF_{toe} > SCF_{heel} \qquad (4)$$
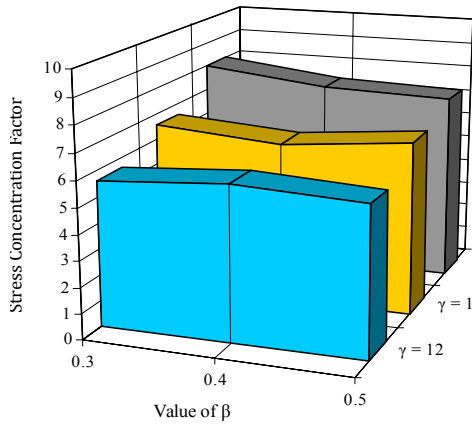$$(2nd\ loading\ condition)$$

Fig. 8 also shows that considerable difference exists between the saddle and toe/heel SCFs. It can also be seen that under the 2nd loading condition, the difference between the $SCF_{inner\ saddle}$ and $SCF_{outer\ saddle}$ is much larger than this difference under the 1st loading condition. These two latest observations highlight the necessity of proposing eight individual parametric equations for the calculation of SCFs at four studied positions on the outer brace under two considered loading conditions.
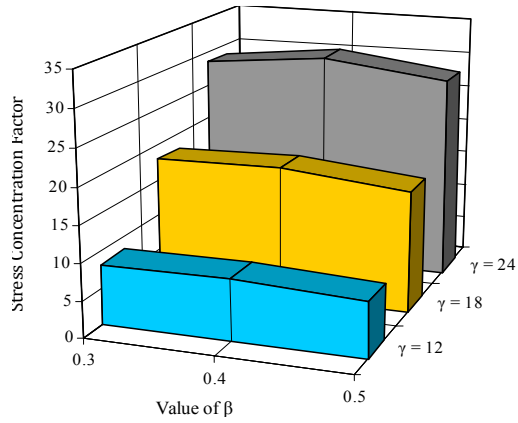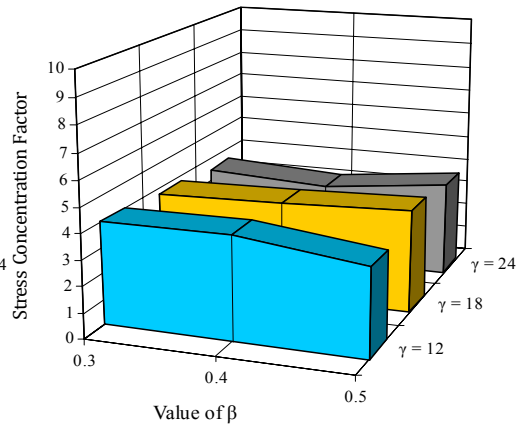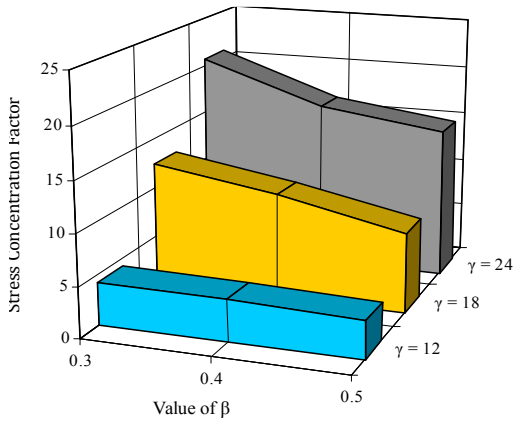
(a) θ = 45˚, τ = 0.6: Inner Saddle 1st Loading Condition
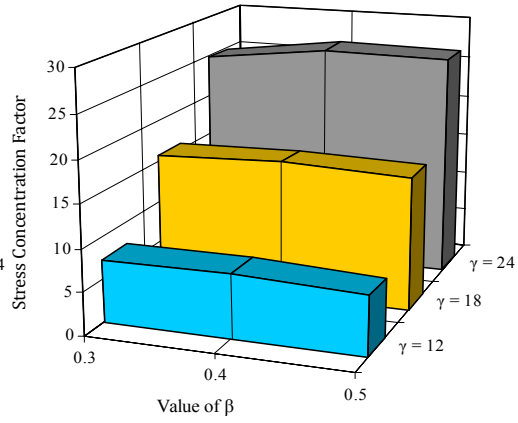
(b) θ = 45˚, τ = 0.6: Outer Saddle 1st Loading Condition

(c) θ = 45˚, τ = 0.6: Toe 1st Loading Condition

(d) θ = 45˚, τ = 0.6: Heel 1st Loading Condition

(e) θ = 45˚, τ = 0.9: Outer Saddle 2nd Loading Condition
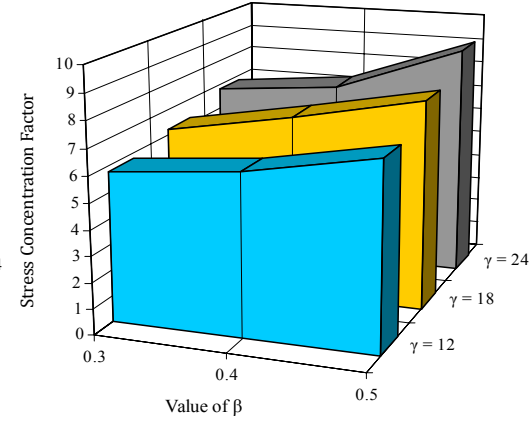
(f) θ = 45˚, τ = 0.9: Toe 2nd Loading Condition

Fig. 5 Effect of β on the SCFs at the different positions on the outer brace

F. Comparison of the SCFs in Uni- and Multi-Planar Joints

As can be seen in Fig. 9, highly remarkable differences exist between the SCF values in a multi-planar DKT-joint and the corresponding SCFs in an equivalent uni-planar KT-joint having the same geometrical properties. It can be clearly concluded from this observation that using the equations proposed for uni-planar KT-connections to compute the SCFs in multi-planar DKT-joints will lead to considerably either under-predicting or over-predicting results. Hence it is necessary to develop SCF formulae specially designed for multi-planar DKT-joints.

As shown in Fig. 9a, the SCF value at the inner saddle position on the outer brace of a multi-planar DKT-joint under the 1st loading condition can be 2.27 times bigger than the corresponding SCF value in the equivalent uni-planar KT-joint. However, this uni-planar SCF is 2.42 times bigger than the corresponding SCF in the multi-planar joint under the 2nd loading condition. Such observations highlight the necessity of proposing individual parametric equations for each loading condition. It can also be concluded from Fig. 9 that under both loading conditions, the maximum difference between the SCFs in uni- and multi-planar joints always occurs at the inner saddle position while the minimum difference will always be at the toe position.
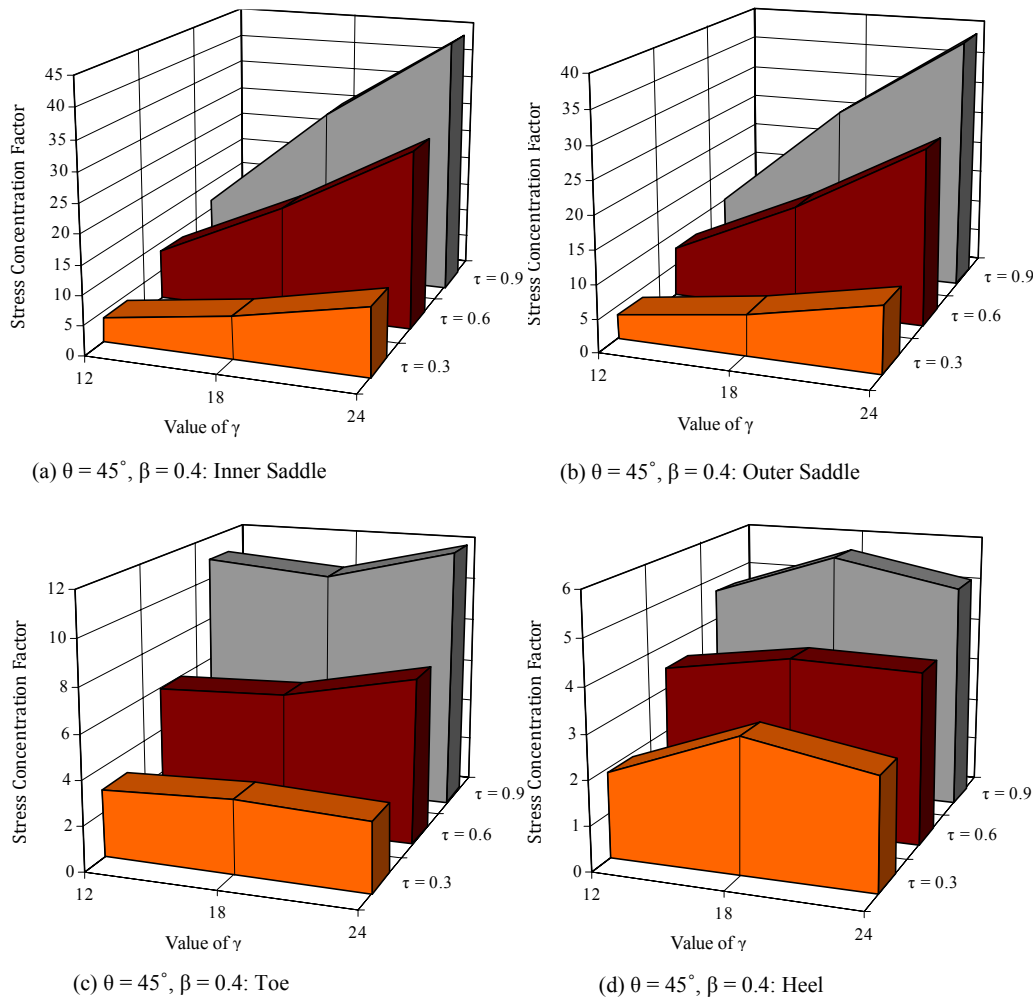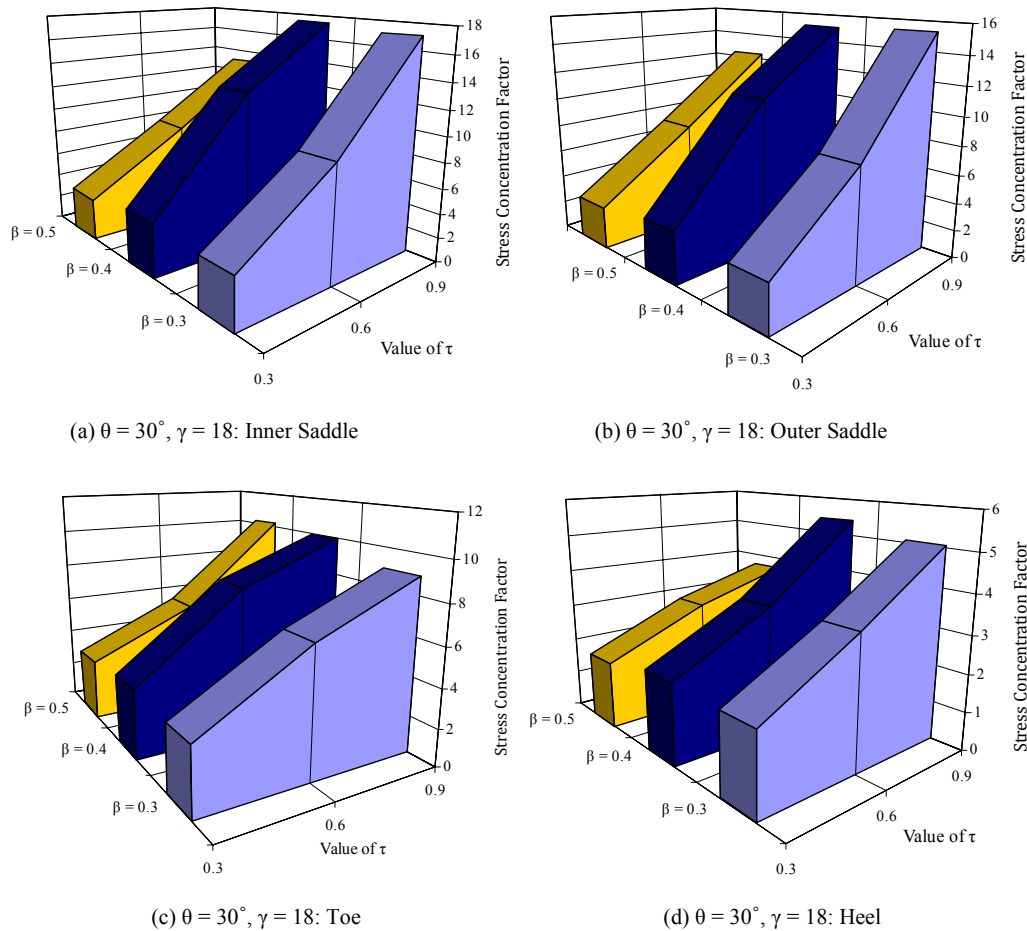

(a) $\theta = 45°$, $\beta = 0.4$: Inner Saddle

(b) $\theta = 45°$, $\beta = 0.4$: Outer Saddle

(c) $\theta = 45°$, $\beta = 0.4$: Toe

(d) $\theta = 45°$, $\beta = 0.4$: Heel

Fig. 6 Effect of γ on the SCFs at the different positions on the outer brace

(a) θ = 30˚, γ = 18: Inner Saddle

(b) θ = 30˚, γ = 18: Outer Saddle

(c) θ = 30˚, γ = 18: Toe

(d) θ = 30˚, γ = 18: Heel

Fig. 7 Effect of τ on the SCFs at the different positions on the outer brace

V. DEVELOPMENT OF PARAMETRIC EQUATIONS FOR THE OUTER BRACE

Although the FEM has been successfully utilized to analyze the tubular joints, the extensive use of such a numerical method is not feasible in a normal day-to-day design office operation. Instead, parametric design equations expressed in the form of the non-dimensional geometrical parameters are useful and desirable for fatigue design. In the present study, eight individual parametric equations are proposed for the calculation of the SCFs at the inner saddle, outer saddle, toe, and heel positions on the outer brace of a right-angle 2-planar DKT-joint subjected to two considered axial loading conditions.

A. Nonlinear Regression Analysis

The parametric equations are derived based on multiple nonlinear regression analyses performed by the statistical software package, SPSS. Values of dependent variable (i.e. SCF) and independent variables (β, γ, τ, and θ) constitute the input data which is imported as a matrix. Each row of this matrix involves the information about the value of SCF at a certain position in a multi-planar tubular DKT-joint having specific geometrical properties.

The number of rows and columns of input matrix for each equation are 81 (number of the joints) and 5 (number of variables), respectively. Hence the whole FEM SCF database is arranged as eight 81×5 input matrices.

When the dependent (i.e. SCF) and independent (i.e. β, γ, τ, and θ) variables are defined, a model expression must be built with defined parameters. The parameters of the model expression are unknown coefficients and exponents. The researcher must specify a starting value for each parameter, preferably as close as possible to the expected final solution. Poor starting values can result in failure to converge or in convergence on a solution that is local (rather than global) or is physically impossible. Various model expressions must be built to derive a parametric equation having a high coefficient of correlation.

After performing nonlinear analyses, the following parametric equations are proposed for predicting the SCF values at the inner saddle, outer saddle, toe, and heel positions on the outer brace of a right-angle 2-planar DKT-joint under two considered axial loading conditions:

Inner saddle:

$$SCF = 0.278\ \beta^{0.038}\ \gamma^{1.734}\ \tau^{1.179}\ \theta^{1.228}$$
(1st loading condition) $R^2 = 0.988$ (5)

$$SCF = 0.027\ \beta^{-1.350}\ \gamma^{1.598}\ \tau^{1.146}\ \theta^{1.381}$$
(2nd loading condition) $R^2 = 0.970$ (6)

Outer saddle:

$$SCF = 0.274\ \beta^{0.155}\ \gamma^{1.745}\ \tau^{1.203}\ \theta^{1.251}$$
(1st loading condition) $R^2 = 0.989$ (7)

$$SCF = 0.047\ \beta^{-0.539}\ \gamma^{1.838}\ \tau^{1.154}\ \theta^{1.454}$$
(2nd loading condition) $R^2 = 0.989$ (8)

Toe:

$$SCF = 3.458\ \beta^{0.003}\ \gamma^{0.395}\ \tau^{1.026}\ \theta^{-0.123}$$
(1st loading condition) $R^2 = 0.944$ (9)

$$SCF = 4.930\ \beta^{0.528}\ \gamma^{0.336}\ \tau^{0.980}\ \theta^{0.247}$$
(2nd loading condition) $R^2 = 0.973$ (10)

Heel:

$$SCF = 3.458\ \beta^{0.003}\ \gamma^{0.395}\ \tau^{1.026}\ \theta^{-0.123}$$
(1st loading condition) $R^2 = 0.944$ (11)

$$SCF = 2.108\ \beta^{-0.114}\ \gamma^{0.327}\ \tau^{0.790}\ \theta^{0.185}$$
(2nd loading condition) $R^2 = 0.840$ (12)

In the above equations, R^2 denotes the coefficient of determination and θ should be inserted in radians.

B. Assessment According to UK DoE [23] Acceptance Criteria

The UK Department of Energy (UK DoE) [23] recommends the following assessment criteria regarding the applicability of the commonly used SCF parametric equations (P/R stands for the ratio of the *predicted* SCF from a given equation to the *recorded* SCF from test or analysis):

- For a given dataset, if % SCFs under-predicting $\leq$ 25%, i.e. [%P/R < 1.0] $\leq$ 25%, and if % SCFs considerably under-predicting $\leq$ 5%, i.e. [%P/R < 0.8] $\leq$ 5%, then accept the equation. If, in addition, the percentage SCFs considerably over-predicting $\leq$ 50%, i.e. [%P/R > 1.5]

$\geq$ 50%, then the equation is regarded as generally conservative.

- If the acceptance criteria is nearly met i.e. 25% < [%P/R < 1.0] $\leq$ 30%, and/or 5% < [%P/R < 0.8] $\leq$ 7.5%, then the equation is regarded as borderline and engineering judgment must be used to determine acceptance or rejection. Otherwise reject the equation as it is too optimistic.

In view of the fact that for a mean fit equation, there is always a large percentage of under-prediction, the requirement for joint under-prediction, i.e. P/R < 1.0, can be completely removed in the assessment of parametric equations [24]. Assessment results according to the UK DoE criteria are tabulated in Table II. It can be seen that all the proposed equations except from (6) and (11), satisfy the criteria and consequently are accepted according to the UK DoE [23].

Equations (6) and (11) require revision to satisfy the criteria. SCF values obtained by these equations are multiplied by individual coefficients in such a way that resulted SCF satisfies the UK DoE criteria. This idea can be expressed as follows:

$$\text{Design Factor} = SCF_{Des.}\ /\ SCF_{Eq.}\qquad(13)$$

where values of $SCF_{Eq.}$ are calculated from the proposed equations and the values of $SCF_{Des.}$ are expected to satisfy the UK DoE acceptance criteria.

Multiple comparative analyses were carried out to determine the optimum values of design factors. The results showed that the optimum design factors are 1.32 and 1.03 for (6) and (11), respectively. Hence, the following equations should be used for design purposes:

$$SCF_{Des.} = 1.32 \times SCF_{Eq.\ (6)}$$
(Inner saddle; 2nd loading condition)   (14)

$$SCF_{Des.} = 1.03 \times SCF_{Eq.\ (11)}$$
(Heel; 1st loading condition)   (15)

$$SCF_{Des.} = 1.00 \times SCF_{Other\ Eqs.}\qquad(16)$$

TABLE II
RESULTS OF EQUATION ASSESSMENT ACCORDING TO UK DOE [25] ACCEPTANCE CRITERIA

Position	Load case	Equation	Conditions		Overall status
			%P/R < 0.8	%P/R > 1.5	
Inner saddle	1st loading condition	(5)	3.7 % < 5 % OK.	0 % < 50 % OK.	accepted
	2nd loading condition	(6)	17 % > 5 %	0 % < 50 % OK.	requires revision
Outer saddle	1st loading condition	(7)	3.7 % < 5 % OK.	1.2 % < 50 % OK.	accepted
	2nd loading condition	(8)	4.9 % < 5 % OK.	0 % < 50 % OK.	accepted
Toe	1st loading condition	(9)	2.5 % < 5 % OK.	0 % < 50 % OK.	accepted
	2nd loading condition	(10)	0 % < 5 % OK.	0 % < 50 % OK.	accepted
Heel	1st loading condition	(11)	8.6 % > 5 %	0 % < 50 % OK.	requires revision
	2nd loading condition	(12)	4.9 % < 5 % OK.	0 % < 50 % OK.	accepted

C. Verification using the Experimental Data

Table III presents the results of validating the proposed equations at the inner and outer saddle positions under the 1st loading condition using the data from a strain gauged acrylic model test. The source of the experimental data is the HSE OTH 91 353 report prepared by Lloyd's Register [16] in which a comprehensive experimental database of SCFs for acrylic complex joints including multi-planar and overlapped K- and KT-joints has been presented.

This report covers only the value of SCF at the chord inner and outer saddle positions. As can be seen in Table III, there is a good agreement between the predictions of the proposed equations and the experimental measurements. It must be noted that since the weld profile is not included in an acrylic specimen, the SCFs obtained from the acrylic model tests are typically 5-10% bigger than the realistic values in the steel tubular joints.
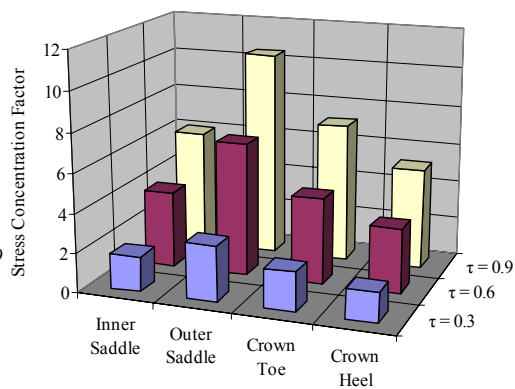
TABLE III
RESULTS OF VALIDATING THE PROPOSED EQUATIONS AT THE INNER AND OUTER SADDLE POSITIONS USING THE DATA FROM A STRAIN GAUGED ACRYLIC MODEL TEST

Geometrical parameters	SCF value at the inner saddle			SCF value at the outer saddle		
	Experimental [18]	Eq. (5)	Difference [a]	Experimental [18]	Eq. (7)	Difference [a]
D = 150 mm , θ = 45 , τ = 0.6 , β = 0.5 , γ = 12	9.64	8.19	17.7%	7.60	7.51	1.2%

[a] Difference = (Experimental SCF / SCF predicted by proposed Eq.) – 1.0
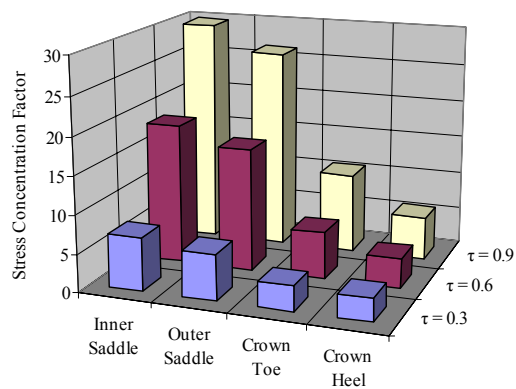
(a) θ = 45˚, γ = 18, β = 0.4, 1st loading condition   (b) θ = 45˚, γ = 18, β = 0.4, 2nd loading condition
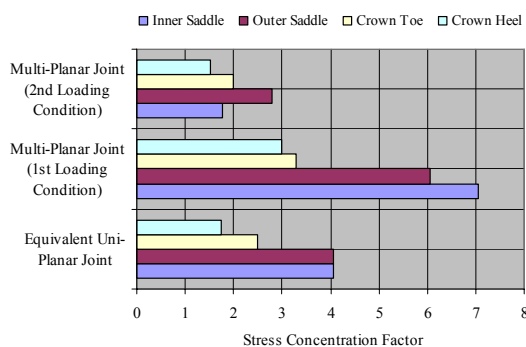
Fig. 8 Comparison of the SCFs at the toe, heel, inner saddle and outer saddle positions
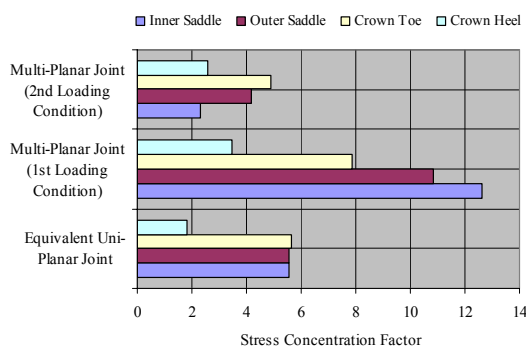
(a) θ = 30˚, τ = 0.6, γ = 18, β = 0.4   (b) θ = 45˚, τ = 0.3, γ = 18, β = 0.4

Fig. 9 Comparison of the SCFs in uni- and multi-planar joints

VI. CONCLUSIONS

In the present paper, the results of parametric FE stress analyses were used to present general remarks on the effect of geometrical parameters on the SCF values at the inner saddle, outer saddle, toe, and heel positions on the outer brace of the two-planar tubular DKT-joints under two different axial load cases. Thereafter, based on the results of FE models and using the nonlinear regression analysis, a new set of SCF design equations was established for the fatigue design of multi-planar DKT-joints under axial loads.

The detailed and quantitative results of parametric study which were extensively discussed in the text are not repeated here for the sake of brevity and only a summary of general remarks is presented:

Under the 1st loading condition, for small values of the γ and τ (say $\gamma = 12$ and $\tau = 0.3$), increasing the β from 0.3 to 0.5 leads to decrease of SCFs at both inner and outer saddle positions in the joints with small values of θ (say $\theta = 30°$). However, such increase in the β results in the increase of SCFs at these positions in the joints having big θ values (say $\theta = 60°$). For intermediate values of θ (say $\theta = 45°$), SCF change in these two positions due to the increase of the β follows this pattern: $SCF_{\beta=0.4} > SCF_{\beta=0.3}$; $SCF_{\beta=0.5} < SCF_{\beta=0.4}$. Under the 2nd loading condition, increase of the β leads to decrease of SCFs at the inner and outer saddles but increase of SCF values at the toe and heel positions. Under both loading conditions, increase of the γ results in increase of SCF values at the inner and outer saddle positions. Magnitude of the increase in these SCFs becomes larger as the τ increases. Under the 1st loading condition, the change of the SCFs at the toe and heel positions due to the increase of the γ does not follow a regular pattern for different geometrical parameters. On the contrary, increase of γ leads to increase in the SCFs at the toe and heel positions under the 2nd loading condition. Under both loading conditions, increase of the τ results in increase of SCF values at all four considered positions: inner saddle, outer saddle, toe, and heel. Under both loading conditions, increase of the θ from 30° to 60° leads to increase of SCF values at inner and outer saddle positions. Under both loading conditions, magnitude of effect of different geometrical parameters on the SCFs follows this order: (Effect of τ > Effect of γ > Effect of θ > Effect of β). Highly remarkable differences exist between the SCF values in a multi-planar DKT-joint and the corresponding SCFs in an equivalent uni-planar KT-joint having the same geometrical properties. It can be clearly concluded from this observation that using the equations proposed for uni-planar KT-connections to compute the SCFs in multi-planar DKT-joints will lead to considerably either under-predicting or over-predicting results. Hence it is necessary to develop SCF formulae specially designed for multi-planar DKT-joints. Good results of equation assessment according to UK DoE acceptance criteria, high values of correlation coefficients, and the good agreement between the predictions of proposed equations and the experimental data guaranty the accuracy of the equations. Hence, the developed equations can reliably be used for fatigue design of offshore structures.

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