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Order Penetration Point Location using Fuzzy Quadratic Programming

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***Abstract*—**This paper addresses one of the most important issues have been considered in hybrid MTS/MTO production environments. To cope with the problem, a mathematical programming model is applied from a tactical point of view. The model is converted to a fuzzy goal programming model, because a degree of uncertainty is involved in hybrid MTS/MTO context. Finally, application of the proposed model in an industrial center is reported and the results prove the validity of the model.

***Keywords*—**Fuzzy sets theory, Hybrid MTS/MTO, Order penetration point, Quadratic programming.

I. INTRODUCTION

IN the face of global competition, a manufacturing firm's survival increasingly depends on how best it can design, manage and restructure its production system to deal with product diversity, improve delivery reliability and also reduce system costs [1]. To cope with these issues**,** manufacturing companies often use different production systems. These production systems can be classified into two major categories: Make-To-Stock (MTS) and Make-To-Order (MTO) based on market demands' response policy [2]. The main advantage of MTS system is the short delivery time, since the final products are already in stock even before the customer order enters [1]. In an MTO system, an order is fulfilled only when it enters the system. This kind of production systems have to supply a wide variety of products, usually in small quantities. Hence, ability to manufacture diverse orders and high production flexibility are the major advantages of this production system [2].

Recent years have shown a number of changes in companies' production policy and they are gradually moving more toward hybrid MTS/MTO production mode. In an MTS/MTO system, a portion of the production system operates as an MTS system and the remaining portion operates in an MTO mode [1]. A proper combination of MTO and MTS can exploit the advantages of both lower inventory and shorter delivery time. In such systems, some semi-finished products are maintained at one stocking point, so the production delay is just the time needed for the MTO stage to be accomplished. A schematic of this production mode is shown in Fig. 1.

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In the related literature, the stocking point is called Order Penetration Point (OPP) or Customer Order Decoupling Point (CODP) [3] in the production line. Generally speaking, OPP is a point in the manufacturing value chain for a product, where the product is linked to a specific customer order and also divides the manufacturing stages that are forecast-driven from those that are customer-order-driven [4].

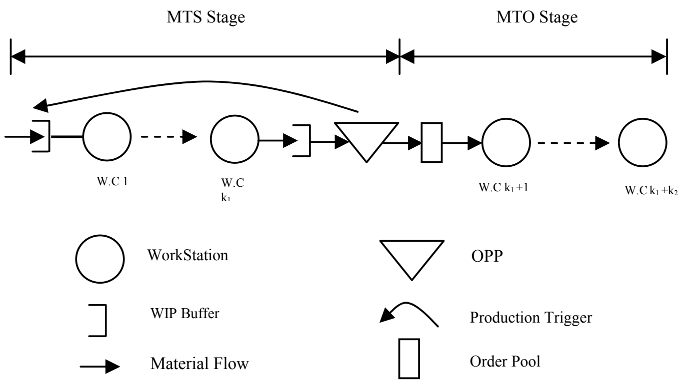

Fig. 1 Schematic of hybrid MTS/MTO production line

As it can be seen in Fig. 1, OPP determines nature of the activities before and after itself. Activities downward the OPP are performed upon what is forecasted about demands, customers' requirements etc. On the other hand, activities after the OPP are solely performed in the case of receiving orders from customers. Hence, OPP location is one of the most important decisions must be taken into consideration in hybrid MTS/MTO production environments. Incorrect location of OPP can yield higher holding cost and longer delivery lead time in the case of shifting forward and backward the OPP, respectively. To cope with determination of OPP location, a mathematical model is proposed in this paper. This paper intends to introduce fuzzy set theory [5] into the mathematical model to location OPP. Additionally, introduction of a tactical level is concentrated rather than operational issues. Section 2 reviews the related literature, while the proposed model is elaborated in Section 3. A case study is reported in Section 4 in which the proposed model is implemented. Finally, remarking conclusions are presented in Section 5.

II. Literature Review

Pure MTS was firstly studied in 1970s in [6] which introduced the concept. Their primary research started a growing trend toward different concerns, such as higher rate of production, forecasting demands, lower inventory level in MTS-based production systems. Some of the models have been proposed so far are [7]-[13]. Since production upon orders came into the production context, researches in the field of production planning began in 1990s and have continued so far. Readers are referred to some instances [14]-[18].

Despite diverse and dense literature of pure MTS or pure MTO, there are only handful research papers on the subject of hybrid MTS/MTO. The papers on this subject can be divided into two major categories; the first concentrates on mathematical approaches mostly presented during 1990s; while the comprehensive managerial concepts have been studied since 2000. The latter often constitutes the qualitative decision making frameworks which do not usually present a model with specific outputs. Among the issues involved in hybrid MTS/MTO, OPP location can be recalled with the aforementioned importance in Section 1. Five papers focusing on MTS/MTO partitioning and OPP location are [4], [19]-[22]. The methodologies presented in [19] and [20] are based upon queue models in terms of mathematical concepts. These research papers consider issues in operational level as decision criteria based upon decisions are made. On the other hand, [4] and [21] consider conceptual models with qualitative decision making structures. Additionally in [22], authors adopted stochastic programming to model uncertain nature of hybrid MTS/MTO production environments.

III. Proposed Model

In this section, a mathematical programming model is proposed by which OPP location is determined from a tactical point of view. In other words, the following model is applied after order partitioning that decides on which products are processed upon MTS, which upon MTO, and which to be manufactured according to hybrid MTS/MTO. When order partitioning is performed, the OPP should be determined for every product family for a tactical planning period; i.e. neither long-term nor short-term planning. At this stage, the OPP for products which are decided to manufacture upon MTS, locates on the last planning point or alternative. In the case of MTO-based products, the OPP goes to the first manufacturing resource, while it must be exactly determined for hybrid MTS/MTO products, because the most remarking role of OPP location rises in the hybrid MTS/MTO, since OPP is pre-defined in two other production strategies. It must be noted that the output of the proposed model should be used in order to decide on operational issues for every item of products. Following presents the nomenclature of the model.

x_{ijt}: production volume of product j in work station i during period t
y_{ijt}: whether work station i is located after OPP for product j during period t
p_{ij}: processing time for product j in work station i
WIP_{ijt}: WIP of product j before work station i
$WIPC_{it}$: buffer capacity before work station i during period t
$\tilde{d}_{jt}$: demand of product j during period t
h_{ijt}: holding cost of product j in work station i during period t

The above introduced holding cost bears all kinds of costs shifting the OPP backward, such as risk of obsolescence or customization opportunities; i.e. all issues which move OPP backward are presented as the holding cost in the proposed model. With respect to the definition of variables and parameters, the proposed model is as followings.

$$Min \sum_{i,j,t} h_{ijt} \cdot WIP_{ijt} \cdot \left(1 - y_{ijt}\right) \tag{1}$$

$$Min \sum_{i,j,t} y_{ijt} \cdot p_{ij} \tag{2}$$

$$y_{ijt} \le y_{i+1,jt} \qquad \forall i, j, t \tag{3}$$

$$\sum_{i} y_{ijt} \ge 1 \qquad \forall j, t \tag{4}$$

$$\sum_{j} WIP_{ijt} \le WIPC_{it} \qquad \forall i, t \tag{5}$$

$$WIP_{ijt} \cdot \left(1 - y_{ijt}\right) - x_{i-1,jt} + x_{ijt} = 0 \qquad \forall i, j, t \tag{6}$$

$$x_{njt} = \tilde{d}_{jt} \qquad \forall j, t \tag{7}$$

$$x_{ijt}, WIP_{ijt} \ge 0 \qquad y_{ijt} \in \{0,1\} \qquad \forall i, j, t \tag{8}$$

Equations (1) and (2) minimize holding cost and lead-time, respectively. However, two objective functions are aggregated into one objective function, since the above objective functions are not independent. Equation (9) shows the aggregated objective function.

$$\begin{aligned} &Min \sum_{i,j,t} h_{ijt} \cdot WIP_{ijt} - y_{ijt} \cdot h_{ijt} \cdot WIP_{ijt} + y_{ijt} \cdot p_{ij} = \\ &Min \sum_{i,j,t} \left\{h_{ijt} \cdot WIP_{ijt} - y_{ijt} \cdot \left(h_{ijt} \cdot WIP_{ijt} + p_{ij}\right)\right\} \end{aligned} \tag{9}$$

Moreover, by defining symmetric fuzzy number for demand of product j during period t, $\tilde{d}_{jt}$, as indicated in (10), (7) is replaced by the two followings (12) and (13) [23]. d_{jt} and Δ present the core value and the spread of the fuzzy number, respectively. To apply the equivalence of the fuzzy (7), objective function (11) is required to be considered.

$$\mu_{x_{jnt}}(x)=\begin{cases} 0 & x_{njt} \le d_{jt} - \Delta \\ \dfrac{x_{njt} - d_{jt} + \Delta}{\Delta} & d_{jt} - \Delta \le x_{njt} \le d_{jt} \\ \dfrac{x_{njt} - d_{jt}}{\Delta} & d_{jt} \le x_{njt} \le d_{jt} + \Delta \\ 0 & x_{njt} \ge d_{jt} + \Delta \end{cases} \tag{10}$$

$$Max\ \alpha \tag{11}$$

$$x_{njt} \ge d_{jt} - (1-\alpha)\Delta \qquad \forall j,t \tag{12}$$

$$x_{njt} \le d_{jt} + (1-\alpha)\Delta \qquad \forall j,t \tag{13}$$

With respect to (9)-(12), the proposed model is equivalent to the model indicated by (13)-(21).

$$Min \sum_{i,j,t} \{h_{ijt} \cdot WIP_{ijt} - y_{ijt} \cdot (h_{ijt} \cdot WIP_{ijt} + p_{ij})\} - \alpha \tag{14}$$

$$y_{ijt} \le y_{i+1,jt} \qquad \forall i,j,t \tag{15}$$

$$\sum_i y_{ijt} \ge 1 \qquad \forall j,t \tag{16}$$

$$\sum_j WIP_{ijt} \le WIPC_{it} \qquad \forall i,t \tag{17}$$

$$WIP_{ijt} \cdot (1 - y_{ijt}) - x_{i-1,jt} + x_{ijt} = 0 \qquad \forall i,j,t \tag{18}$$

$$x_{njt} \ge d_{jt} - (1-\alpha)\Delta \qquad \forall j,t \tag{19}$$

$$x_{njt} \le d_{jt} + (1-\alpha)\Delta \qquad \forall j,t \tag{20}$$

$$x_{ijt}, WIP_{ijt} \ge 0 \qquad y_{ijt} \in \{0,1\} \qquad \forall i,j,t \tag{21}$$

IV. Case Study

In this section, a real case study is briefly reported to show how the proposed model is implemented in practice. The case study is a part of greater production planning structure design in manufacturing company. The case company is a supplier of electric components of one of the most leading audio and video manufacturer in Iran of which the report herein focuses on one of its production lines. Since the company is also an authorized manufacturer of one the famous world-wide manufacturers, line reconfigurations and product varieties are usual which leads to have high level of obsolescence. Hence, it was decided that the production line activates upon hybrid MTS/MTO production strategy. After order partitioning has been decided, it is time to locate the OPP for the products processed in that line. To apply the model, we required a group of experts from the company, including supervisors of engineering, marketing, R&D, manufacturing and procurement departments to help us gain parameters of the model. Some parameters, like buffer capacity of workstations and their processing times were directly requested from engineering. However, holding costs with the aforementioned descriptions and demands of products during any planning periods are gained from the group of experts involved. The model is applied for five products during three two-season periods in a seven-station production line. For instance, data relevant to demands of products during planning periods are presented in Table I.

TABLE I
CORE VALUES OF FUZZY DEMANDS

Product	Period 1	Period 2	Period 3
1	450	400	350
2	300	500	370
3	380	390	390
4	410	400	340
5	475	400	365

In the mentioned production line, buffer capacity of each workstation is 1500 units of products, while tolerance of fuzzy demands is allowed 40 units. With respect to the above data, the results for OPP location of each product family are obtained as shown Table II.

TABLE II
OPP LOCATION OF PRODUCTS

Product	Time period	OPP location
1	1	Workstation 4
	2	Workstation 4
	3	Workstation 3
2	1	Workstation 6
	2	Workstation 6
	3	Workstation 6
3	1	Workstation 6
	2	Workstation 5
	3	Workstation 5
4	1	Workstation 5
	2	Workstation 5
	3	Workstation 4
5	1	Workstation 4
	2	Workstation 3
	3	Workstation 2

The results in Table II are obtained with $\Delta = 0.87$. This value of Δ reveals that 87 percent of satisfaction is obtained about the demands. The value of Δ can be representative of satisfaction of demands of hybrid products.

V. Conclusion and Future Research Directions

Ever growing global competitive markets has emerged great attention from practitioners and academicians toward customers' requirements. Based upon the fact, MTS-based production systems have been gradually replaced with MTO ones. Afterward, combining two introduced strategies as hybrid MTS/MTO production strategy came into existence to benefit from both MTS and MTO concepts. One of the most promising issues in hybrid contexts is OPP location by which the nature of activities behind and after the point is determined. In this paper, a fuzzy quadratic programming model is developed to tackle OPP location from a tactical point of view. The uncertainty of coming orders in a hybrid production environment is modeled through fuzzy numbers. Finally, a brief report of a real case on which the proposed model is applied is presented.

As future research direction, application of fuzzy stochastic programming is highly recommended, because coming orders for hybrid MTS/MTO products embedded uncertainty due to variation and ambiguity. Moreover, introduction of other fuzzy parameters can make the model better-fitted. Also, considering more comprehensive aspects from manufacturing process can make the initial idea behind the proposed model much more interesting and applicable. These aspects include maintenance and machine break-down, outsourcing decisions, overtime and idol time decisions, etc.

References

[1] Y. Mu, "Design of hybrid Make-to-Stock (MTS) - Make-to-Order (MTO) manufacturing system," M.Sc. Thesis, The University of Minnesota, 2001.

[2] M. Ebadian, M. Rabbani, F. Jolai, S. A. Torabi, and R. Tavakkoli-Moghaddam, "A new decision-making structure for the order entry stage in make-to-order environments," *Int. J. Prod. Econ.*, vol. 111, no. 8, pp. 351-367, 2008.

[3] S. Hoekstra,and J. Romme, *Integrated logistics structures: developing customer oriented goods flow*, McGraw-Hill, London, UK, 1992.

[4] J. Olhager, "Strategic positioning of order penetration point," *Int. J. Prod. Econ.*, vol. 85, pp. 319-329, 2003.

[5] L. A. Zadeh, "Fuzzy sets," *Inf. Control.*, vol. 8, pp. 338-353, 1965.

[6] A. C. Hax, and H. C. Meal, "Hierarchical integration of production planning and scheduling," MIT working paper, 1973.

[7] G. R. Bitran, and A. C. Hax, "On the design of hierarchical production planning system," *Decis. Sci.*, vol. 8, no. 1, pp. 28-55, 1977.

[8] G. R. Bitran, and A. C. Hax, "Disaggregation and resource allocation using convex knapsack problems with bounded variables," *Manage. Sci.*, vol. 27, no. 4, pp. 431-441, 1981.

[9] J. J. Golovin, "Hierarchical integration of production planning and control," Ph.D. Thesis, MIT, Sloan School of Management, Massachusetts, 1975.

[10] H. Gabbay, *In optimal aggregation and disaggregation in hierarchical planning*, Boston, Martin Nijhof, pp. 95-106, 1979.

[11] G. R. Bitran, E. A. Haas, and A. C. Hax, "Hierarchical production planning: a single stage system," *Oper. Res.*, vol. 29, no. 4, pp. 717-743, 1981.

[12] S. Ray, Y. Gerchak, and E. M. Jewkes, "Joint pricing and inventory policies for make-to-stock products with deterministic price-sensitive demand," *Int. J. Prod. Econ.*, vol. 97, pp. 143-158, 2005.

[13] Z. Jemaï, and F. Karaesman, "The influence of demand variability on the performance of a make-to-stock queue," *Eur. J. Oper. Res.*, vol. 164, pp. 195-205, 2005.

[14] S.-D. Lee, and S.-H. Ho, "Buffer sizing in manufacturing production systems with complex routings," *Int. J. Compt. Integr. Manuf.*, vol. 15, no. 5, pp. 440-452, 2002.

[15] Q.-M. He, E. M. Jewkes, and J. Buzacott, "Optimal and near-optimal inventory control policies for a make-to-order inventory-production system," *Eur. J. Oper. Res.*, vol. 141, pp. 113-132, 2002.

[16] C. Thierry, J. Lamothes, and V. Galvagnon, "Re-planning support system for make-to-order production with reserved resources," *Int. J. Prod. Res.*, vol. 42, no. 23, pp. 4993-5008, 2004.

[17] S. A. Slotnick, and M. J. Sobel, "Manufacturing lead-time rules: Customer retention versus tardiness costs," *Eur. J. Oper. Res.*, Article in press, 2004.

[18] A. Cakravastia, and K. Takahashi, "Integrated model for supplier selection and negotiation in a make-to-oeder environment," *Int. J. Prod. Res.*, vol. 42, no. 21, pp. 4457-4474, 2004.

[19] T. M. Williams, "Special products and uncertainty in production/ inventory systems," *Eur. J. Oper. Res.,* vol. 15, pp. 46-54, 1984.

[20] A. Arreola-Risa, and G. A. DeCroix, "Make-to-order versus make-to-stock in a production-inventory system with general production times," *IIE Trans. (Inst. Ind. Eng.)*, vol. 30, no. 8, pp. 705-713, 1998.

[21] D. P.van Donk, "Make to stock or make to order: the decoupling point in the food processing industries," *Int. J. Prod. Econ.*, vol. 96, no. 3, pp. 297-306, 2001.

[22] S. Rajagopalan, "Make-to-order or make-to-stock: Model and application," *Manage. Sci.*, vol. 48, no. 2, 241-256, 2002.

[23] H. J. Zimmermann, *Fuzzy set theory and its applications*, Third edition, Kluwer Academic Publishers, 1996.