

Optimum Time Coordination of Overcurrent Relays using Two Phase Simplex Method

Prashant P. Bedekar, Sudhir R. Bhide, and Vijay S. Kale

Abstract—Overcurrent (OC) relays are the major protection devices in a distribution system. The operating time of the OC relays are to be coordinated properly to avoid the mal-operation of the backup relays. The OC relay time coordination in ring fed distribution networks is a highly constrained optimization problem which can be stated as a linear programming problem (LPP). The purpose is to find an optimum relay setting to minimize the time of operation of relays and at the same time, to keep the relays properly coordinated to avoid the mal-operation of relays.

This paper presents two phase simplex method for optimum time coordination of OC relays. The method is based on the simplex algorithm which is used to find optimum solution of LPP. The method introduces artificial variables to get an initial basic feasible solution (IBFS). Artificial variables are removed using iterative process of first phase which minimizes the auxiliary objective function. The second phase minimizes the original objective function and gives the optimum time coordination of OC relays.

Keywords—Constrained optimization, LPP, Overcurrent relay coordination, Two-phase simplex method.

I. INTRODUCTION

OVER current relays are usually employed as backup protection. In distribution feeders, they play a more important role and there it may be the only protection provided [1,2]. The problem of coordinating protective relays consists of selecting their suitable settings such that their fundamental protective function is met under the requirements of sensitivity, selectivity, reliability, and speed [3,4]. If backup protections are not well coordinated, mal-operation can occur and, therefore, OC relay coordination is a major concern of power system protection [5]. Each protection relay in the power system needs to be coordinated with the relays protecting the adjacent equipment. The overall protection coordination is thus very complicated.

The OC relay coordination problem in distribution system can be defined as constrained optimization problem. The objective is to minimize the operating time of relay for fault at

any point. The problem can be defined as a LPP and can be solved using two phase simplex technique.

In two-phase simplex method artificial variables are introduced in the objective function to get an IBFS. Phase one minimizes the auxiliary objective function which is sum of all artificial variables. Phase two uses the optimum solution of phase one as the starting solution. Using the iterative process the optimum solution to the original objective function is obtained.

The two-phase simplex optimization method has been employed for optimum coordination of OC relays in this paper. Initially a simple radial system is taken. The detailed procedure for formulation of relay coordination problem is explained for this system and the detailed solution for this problem is presented. Then a parallel feeder system with five relays is taken and the optimum value of time multiplier setting (TMS) of each relay is obtained using two-phase simplex method.

II. COORDINATION OF OC RELAYS IN RING FED SYSTEM

As soon as the fault takes place it is sensed by both primary and backup protection. The primary protection is the first to operate as its operating time being less than that of the backup relay. If the operating time of primary relay is set to 0.1 s, the backup relay should wait for 0.1 s plus, a time equal to the operating time of circuit breaker (CB), associated with primary relay, plus the overshoot time of primary relay [1,6]. This is necessary for maintaining the selectivity of primary and backup relays. A ring main feeder system is shown in Fig. 1.

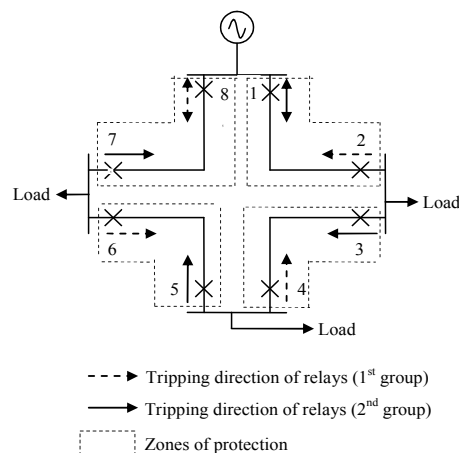


Fig. 1 A ring main feeder

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It allows supply to be maintained to the loads connected to all the buses, in spite of fault on any section. Relays 1 and 8 are non directional whereas all other relays (2, 3, 4, 5, 6 and 7) are directional OC relays. All directional relays have their tripping direction away from the concerned bus. For coordination purpose the relay operating times will be related as

$$T_{R8} > T_{R6} > T_{R4} > T_{R2}$$

$$\text{and } T_{R1} > T_{R3} > T_{R5} > T_{R7}$$

where, T_{Ri} indicates operating time of i^{th} relay.

The actual operating time for each relay can be decided considering the operating time of preceding relay, operating time of CB associated with preceding relay, and the overshoot time of the relay. As the size and complexity of the system goes on increasing it becomes more and more difficult to coordinate the relays.

III. PROBLEM FORMULATION

The coordination problem of directional OC relays in a ring fed distribution systems, can be stated as an optimization problem, where the sum of the operating times of the relays of the system, for different fault points, is to be minimized [5,7,8],

$$\text{i.e., } \min z = \sum_{i=1}^m W_i \cdot t_{i,k} \quad (1)$$

where

m is the number of relays,

$t_{i,k}$ is the operating time of the relay R_i , for fault at k , and

W_i is weight assigned for operating time of the relay R_i

In distribution system since the lines are short and are of approximately equal length, equal weight (=1) is assigned for operating times of all the relays [5,9,10].

The objective of minimizing the total operating time of relays is to be achieved under three sets of constraints [5-10], as mentioned below.

A. Coordination Criteria

Fault is sensed by both primary as well as secondary relay simultaneously. To avoid mal-operation, the backup relay should takeover the tripping action only after primary relay fails to operate. If R_j is the primary relay for fault at k , and

R_i is backup relay for the same fault, then the coordination constraint can be stated as

$$t_{i,k} - t_{j,k} \geq \Delta t \quad (2)$$

where,

$t_{j,k}$ is the operating time of the primary relay R_j , for fault at k

$t_{i,k}$ is the operating time for the backup relay R_i , for the same fault (at k)

Δt is the coordination time interval (CTI)

B. Bounds on the relay setting and operating time

Constraint imposed because of restriction on the operating time of relays can be mathematically stated as

$$t_{i,\min} \leq t_{i,k} \leq t_{i,\max} \quad (3)$$

where,

$t_{i,\min}$ is the minimum operating time of relay at i for fault at any point

$t_{i,\max}$ is the maximum operating time of relay at i for fault at any point

The bounds on time multiplier setting (TMS) of relays can be stated as

$$TMS_{i,\min} \leq TMS_i \leq TMS_{i,\max} \quad (4)$$

where,

$TMS_{i,\min}$ is the minimum value of TMS of relay R_i

$TMS_{i,\max}$ is the maximum value of TMS of relay R_i

Instead of taking these two constraints (mentioned in equation 3 and 4) for each relay, one is taken and other, which is redundant, is not considered. This is explained in illustration I.

$TMS_{i,\min}$ and $TMS_{i,\max}$ are taken as 0.025 and 1.2 respectively [11].

C. Relay Characteristics

All relays are assumed to be identical and are assumed to have normal IDMT characteristic as [1,5,9,12] :

$$t_{op} = \frac{\lambda(TMS)}{(PSM)^\gamma - 1} \quad (5)$$

where,

t_{op} is relay operating time, and

PSM is plug setting multiplier.

For normal IDMT relay γ is 0.02 and λ is 0.14. As the pickup currents of the relays are pre determined from the system requirements, equation (5) becomes

$$t_{op} = a(TMS) \quad (6)$$

where,

$$a = \frac{\lambda}{(PSM)^\gamma - 1} \quad (7)$$

Making substitution from equation (6) in equation (1), the objective function becomes

$$\min z = \sum_{i=1}^m a_{i,k} (TMS)_i \quad (8)$$

where, $a_{i,k}$ is constant of relay R_i for fault at k .

Thus the relay characteristic constraint is incorporated in the objective function itself. The values of $a_{i,k}$ for relay R_i for different fault locations (k) are predetermined. Value of TMS for each relay is to be determined using two-phase simplex method.

IV. TWO PHASE SIMPLEX METHOD

After incorporating the relay characteristic in the objective function, the relay coordination problem have two set of constraints. Generally, the upper bound on the relay operating time need not be taken care of, because when optimum solution is obtained the operating time of relays do not exceed the upper bound. So, the remaining constraints are coordination criteria constraint and relay operating time constraint (lower bound). Both the set of constraints are inequalities of \geq type. To convert these constraints to equality type, non negative variable (surplus variable) is subtracted from left hand side. If surplus variables are taken as basics for the initial (starting) solution it will give infeasible solution as the coefficient of surplus variables is -1. Thus surplus variables can not become starting basic variables. In order to obtain an IBFS, artificial variable is added to the left hand side of the constraints [13-17].

Artificial variables have no meaning in a physical sense and are only used as a tool for generating an IBFS. Before the final solution is reached, all artificial variables must be dropped out form the solution. If at all an artificial variable becomes a basic variable in the final solution, its value must be zero [16,17].

A. Phase I

In the first phase of this method IBFS to the original problem is found out. For this all the artificial variables are driven to zero. To do this an auxiliary objective function (W) is defined which is the sum of all artificial variables. This function is minimized subject to the given constraints to get a basic feasible solution of the LPP. In phase I, the iterations are stopped as soon as the value of auxiliary objective function becomes zero. There is no need to continue till the optimality is reached, if the auxiliary function becomes zero earlier. At the end of phase I, one of the two cases arise-

- i) Min $W > 0$, and at least one artificial variable appears in the basis with a positive value. In this case the LPP does not posses any feasible solution. The procedure is terminated.
- ii) Min $W = 0$, and no artificial variable appears in the basis then basic feasible solution to the problem is obtained. Proceed for phase II.

B. Phase II

The second phase finds the optimum solution of the original objective function. In this phase the optimum solution of phase I is taken as the starting solution. The actual costs are assigned to the objective variables. The simplex algorithm is then applied to find the optimum solution.

If at the end of phase I, min $W = 0$, and one or more artificial variable appears in the basis at zero level then basic feasible solution to the problem is obtained. In this case, care should be taken that this artificial variable never becomes positive in phase II.

V. RESULTS AND DISCUSSION

Coordination of OC relays is basically an LPP, in which the aim is to find out the optimum value of TMS for all relays,

hence TMS of the relays are taken as variables. The optimum values of TMS will ensure optimum time of operation of relays.

Out of the three sets of constraints, the relay characteristic constraint is already incorporated in the objective function. The bounds on TMS and the coordination criteria are included in the problem as constraints. In case of optimum time coordination of OC relays the objective function will always be of minimization type and all the constraints will be inequality constraints of \geq type. The constraints are converted to equality type by subtracting non-negative surplus variable from the left hand side of each constraint. To get an IBFS non-negative artificial variable is added to the left hand side of each constraint. The IBFS have all the artificial variables as basics and all the original variables (TMS for all relays) and the surplus variables as non-basics.

Two-phase simplex method was applied for optimum coordination of OC relays. A program was written in MATLAB for the same. The program was successfully tested for various cases, out of which two cases are presented in this paper. Detail calculations are given for formation of objective function and constraints, and for finding the optimum solution in illustration I. Similar calculations were performed in illustration II.

A. Illustration I

To test the algorithm, initially a simple radial system, shown in Fig. 2, was considered.

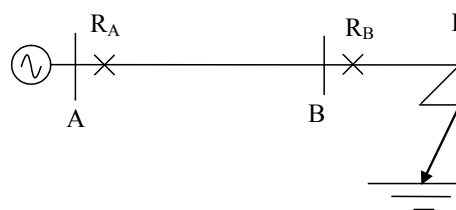


Fig. 2 A simple radial system

The maximum fault current just beyond bus A and bus B are 4000 A and 3000 A respectively, the plug setting of both the

TABLE I
CALCULATION OF a_i CONSTANTS FOR RELAYS

SN	Fault position	Relay	
		R_A	R_B
1.	Just beyond bus A	0.14	---
		$(13.33)^{0.02} - 1$ = 2.63	
2.	Just beyond bus B	0.14	0.14
		$(10)^{0.02} - 1$ = 2.97	$(30)^{0.02} - 1$ = 2.00

relays is 1, the CT ratio for R_A is 300:1 and for R_B is 100:1. Minimum operating time for each relay was considered as 0.2

second and the CTI was taken as 0.57 s. Calculation of value of a_i (mentioned in equation 7) for relays is shown in Table I.

The fault current in relay coil (relay R_A) for fault just beyond bus A is 13.33. As the plug setting for relay R_A is 1, PSM in this case is 13.33. The system considered here is radial; hence, fault just beyond bus A will not be sensed by relay R_B . Similarly 10 and 30 are the current in coils of relay R_A and R_B respectively, for fault just beyond bus B. Again as the plug setting of both relays is 1, the PSM values are 10 and 30 respectively, in this case.

Considering x_1 and x_2 as *TMS* of relay R_A and R_B respectively, the problem can be stated as –

$$\min z = 2.63x_1 + 2x_2 \tag{9}$$

$$\text{subject to } 2.97x_1 - 2x_2 \geq 0.57 \tag{10}$$

$$2.63x_1 \geq 0.2 \tag{11}$$

$$\text{and } 2x_2 \geq 0.2 \tag{12}$$

TABLE II
VARIABLES ENTERING AS AND DEPARTING FROM BASIC (PHASE I)

Iteration Count	Variable entering as basic	Variable departing from basic
1	x_1	A_2
2	s_2	A_1
3	x_2	A_3

This is an LPP. Equation (10) gives the coordination criteria constraint and the lower bound on the relay operating time is given by equation (9) and (10). The upper limit of *TMS* of for both relays is taken as 1.2. As the optimum value of *TMS* comes out to be less than upper bound, the constraints due to upper bound are not considered.

The problem is rewritten by converting the constraints to equality type with the help of surplus variables and adding artificial variables. The surplus variables are taken as s_1, s_2, s_3 and artificial variables as A_1, A_2, A_3 . In phase I, A_1, A_2, A_3 are taken as starting basics and the auxiliary objective function is

$$\min W = A_1 + A_2 + A_3 \tag{13}$$

The auxiliary objective function is minimized in three iterations in phase I. The variables departing from basics and entering as basics in different iterations is shown in Table II.

After getting $\min W = 0$, phase-II is started. In this illustration Phase-II leads to optimum solution in single iteration. The optimum solution is

$$TMS_1 = x_1 = 0.2592 \quad TMS_2 = x_2 = 0.10$$

B. Illustration II

In this case a single-end-fed, distribution system with

parallel-feeder, with five OC relays (as shown in Fig. 3) was considered. Three different fault points were considered. The load currents during the fault were assumed to be negligible.

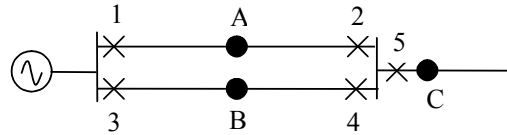


Fig. 3 A single-end-fed, distribution system with parallel-feeders

The primary-backup relationships of relays for the three fault

TABLE III
PRIMARY-BACKUP RELATIONSHIPS OF RELAYS

Fault point	Primary relay	Backup relay
A	1 2	-- 3
B	3 4	-- 1
C	5	1 and 3

points are given in Table III and the CT ratios and plug settings of relays are given in Table IV.

TABLE IV
CT RATIOS AND PLUG SETTING OF RELAYS

Relay	CT ratio (A / A)	Plug setting
1	300 / 1	1
2	300 / 1	1
3	300 / 1	1
4	300 / 1	1
5	100 / 1	1

The current seen by the relays and the a_i constant (for different fault points) is shown in Table V. The optimization problem is formed in the same way as explained in illustration I.

In this case there are five variables (*TMS* of five relays), five constraints due to bounds on relay operating time (or bounds on *TMS* of relays) and four constraints due to coordination criteria. Thus the total number of constraints is nine. Value of CTI is taken as 0.2 s and minimum operating time of relay is taken as 0.1 s. The optimum values of *TMS* obtained are as under (the subscripts indicate the relay number) –

$$\begin{aligned} TMS_1 &= 0.069 & TMS_2 &= 0.025 \\ TMS_3 &= 0.069 & TMS_4 &= 0.025 \\ TMS_5 &= 0.0499 \end{aligned}$$

TABLE V
CURRENT SEEN BY THE RELAYS AND a_i CONSTANTS

Fault point		Relays				
		1	2	3	4	5
A	I_{relay}	9.059	3.019	3.019	--	--
	a_i	3.106	6.265	6.265	--	--
B	I_{relay}	3.019	--	9.059	3.019	--
	a_i	6.265	--	3.106	6.265	--
C	I_{relay}	4.876	--	4.876	--	29.25
	a_i	4.348	--	4.348	--	2.004

The above values ensure that the relays will operate in minimum possible time for fault at any point in the system and will also maintain the coordination. It can be seen that the time taken by relay 1 to operate is minimum for fault at point A (0.2144 s) and will take more time for fault at point B (0.4323 s) and C (0.3001 s). This is desirable, because for fault at point A, relay 1 is first to operate, whereas for fault at points B and C, relay 4 and relay 5 respectively, should get first chance to operate. If they fail to operate then only relay 1 should takeover tripping action.

VI. CONCLUSION

Two-phase simplex method for optimum time coordination of overcurrent relays in distribution system is presented in this paper. The optimum relay coordination problem is basically a highly constrained optimization problem. Formation of this problem as an LPP is explained in this paper. A program has been developed in MATLAB for finding the optimum time coordination of relays using two-phase simplex method. The program can be used for optimum time coordination of relays in a system with any number of relay and any number of primary-backup relationships. The program has been successfully tested for various systems, including multi loop systems and was found to give satisfactory results in all the cases.

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