

# Optimum Radio Capacity Estimation of a Single-Cell Spread Spectrum MIMO System under Rayleigh Fading Conditions

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**Abstract**—In this paper, the problem of estimating the optimal radio capacity of a single-cell spread spectrum (SS) multiple-input-multiple-output (MIMO) system operating in a Rayleigh fading environment is examined. The optimisation between the radio capacity and the theoretically achievable average channel capacity (in the sense of information theory) per user of a MIMO single-cell SS system operating in a Rayleigh fading environment is presented. Then, the spectral efficiency is estimated in terms of the achievable average channel capacity per user, during the operation over a broadcast time-varying link, and leads to a simple novel-closed form expression for the optimal radio capacity value based on the maximization of the achieved spectral efficiency. Numerical results are presented to illustrate the proposed analysis.

**Keywords**—Channel capacity, MIMO systems, Radio capacity, Rayleigh fading, Spectral efficiency.

## I. INTRODUCTION

MIMO communication techniques have been an important area of focus for next-generation wireless systems because of their potential for high capacity, increased diversity, and interference suppression. The study of multi-user MIMO systems has emerged recently as an important research topic due to the high capacity improvement that they can offer over single-input-single-output (SISO) systems, [1,2].

The Shannon's capacity of a single-user time-invariant channel is defined as the maximum mutual information between the channel input and output. This maximum mutual information is shown by Shannon's capacity theorem to be the maximum data rate that can be transmitted over the channel with arbitrarily small error probability, [3]. Many practical MIMO techniques have been developed to capitalise on the theoretical capacity gains predicted by Shannon theory. An overview of the recent advances in this area and practical techniques along with their performance can be found in, [4]. In contrast to previous published works, in this work, a novel method based on the maximization of the achieved spectral efficiency, of estimating the optimal radio capacity of a single-cell spread spectrum MIMO multiple-user system is presented.

The spectral efficiency of the considered system is evaluated in terms of each user's achievable average channel capacity (in the Shannon sense). This average channel capacity formula would indeed provide the true channel capacity, if channel side information were available at the receiver, [5].

The final equation, theoretically derived, to the author's best knowledge, is the first time such expression has been exposed theoretically and in contrast to other previous reported works, here, we avoided the exact evaluation of the characteristic function (c.f) of the average channel capacity of MIMO system or the Monte Carlo simulation, [1,6-9]. Numerical results are presented to illustrate the proposed mathematical analysis. However, in a future work a simulation process must be described in order to compare with the theoretical results derived here.

It must be noticed that, in the followed analysis, a fixed number of simultaneously transmitting users is assumed. Although a dynamic user population is a reasonable assumption, the results derived in the paper, can be applied directly in a MIMO SS system with a variable number of users, considering that the number of users per cell  $K$ , represents the mean value of users per cell in a birth-death model describing the variable allocation of users, [10].

## II. SYSTEM'S MODEL

A spread spectrum MIMO communication system is described that consists of  $K$  simultaneously transmitting users, each of them transmitting in parallel, via  $N_T$  different transmitting antennas the same spread spectrum signal of bandwidth  $W_{ss}$ , after spreading the same signal of bandwidth  $W$  by the system's processing gain i.e.  $G_p = W_{ss}/W$ . Each system's user transmit a signal of bandwidth  $W_{ss}$  with average transmitted power  $P_i = P$ ,  $1 \leq i \leq K$ , without splitting the totally transmitted power among transmit antennas as described in [2]. Firstly, we denote as  $\Gamma_i$  the received signal-to-noise ratio (SNR), over the signal bandwidth  $W$ , in a non-fading additive white gaussian noise (AWGN) environment from the  $i$ -th,  $1 \leq i \leq K$ , user, i.e.:

$$\Gamma_i = P_i / N_0 W, \quad 1 \leq i \leq K \quad (1)$$

where  $N_0$  is the noise power spectral density. Without loss of generality, we consider the case where the number  $N_T$  of transmit and receive antennas  $N_R$  are equal.

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### III. TOTAL CHANNEL CAPACITY FOR OPERATION IN A NON-FADING AWGN ENVIRONMENT

The problem to estimate the total channel capacity  $C_T$  available to all  $K$  users of the previous described SS MIMO system, when operating in a non-fading AWGN environment is examined. In such an environment, the channel capacity  $C_{i,t}$  assigned to each user  $i$ , i.e. a single user's conditional channel capacity in the Shannon sense, when arbitrarily complex coding and delay is applied, will be expressed in the form, [3]:

$$C_{i,t} = W_{ss} \log_2(1 + \Gamma_{i,ss,t}) = W_{ss} \log_2 \left( 1 + \frac{P_i}{N_0 W_{ss} + P_{MAI}} \right), 1 \leq i \leq K \quad (2)$$

where the suffices ' $t$ ' indicates the  $t$ -th,  $t=1, \dots, N_T$ , transmitting/receive antenna, ' $ss$ ' indicates the SS system and  $\Gamma_{i,ss,t}$  is the spread signal-to-interference plus noise ratio (SINR) received from the  $t$ -th,  $t=1, \dots, N_T$ , receiving antenna of the  $i$ -th,  $1 \leq i \leq K$ , user and  $P_{MAI}$  is the Gaussian distributed total multiple-access interference (MAI) power expressed as:

$$P_{MAI} = \sum_{t=1}^{N_T} \sum_{j \neq i}^K P_{j,t} = N_T (K-1) P_i, 1 \leq i \leq K, 1 \leq t \leq N_T \quad (3)$$

without including interfering transmitted power from other transmit antennas of the same user assumed, in the presented analysis, negligible. Then, according to eq.(3),  $\Gamma_{i,ss,t}$  will then be:

$$\Gamma_{i,ss,t} = \frac{P_i}{N_0 W_{ss} + N_T (K-1) P_i} \quad (4)$$

equivalently expressed as:

$$\Gamma_{i,ss,t} = \frac{\Gamma_i}{G_p + N_T (K-1) \Gamma_i} \quad (5)$$

Then, the total channel capacity  $C_T$  available to all  $K$  users and for all the  $N_T (=N_R)$  transmitting antennas, will be equal to the sum of the corresponding individual rates, i.e.,:

$$\begin{aligned} C_T &= \sum_{t=1}^{N_T} \sum_{i=1}^K C_{i,t} = K \sum_{t=1}^{N_T} W_{ss} \log_2(1 + \Gamma_{i,ss,t}) = \\ &= K N_T W_{ss} \log_2 \left( 1 + \frac{\Gamma_i}{G_p + N_T (K-1) \Gamma_i} \right) = \\ &= N_T W_{ss} \log_2 \left( 1 + \frac{K \Gamma_i}{G_p + N_T (K-1) \Gamma_i} \right) \end{aligned} \quad (6)$$

due to the fact that, in practice,  $\Gamma_{i,ss,t}$  is well below unity (in linear scale), [11].

### IV. OPTIMAL MIMO RADIO CAPACITY IN A RAYLEIGH FADING ENVIRONMENT

We consider now the previously described SS MIMO system operating in a Rayleigh fading environment. The Rayleigh fading channel assigned to each transmitting user is modelled as a tapped delay line, [12]. In addition, we assume that all the associated spread spectrum signals are received by an optimum maximal-ratio combiner (MRC) RAKE receiver. In particular, each user's MRC RAKE receiver has  $M_C$  taps corresponding to  $M_C$  resolvable signal paths, on the condition that the transmitted signal bandwidth  $W_{ss}$  is much greater than the coherence bandwidth  $W_{coh}$  of the fading channel, [12,13], given by:

$$M_C = [W_{ss} T_m] + 1 \quad (7)$$

where  $T_m$  is the total multipath delay spread of the Rayleigh fading channel and  $[.]$  returns the largest integer less than, or equal to, its argument. Although the number of resolvable paths  $M_C$  may be a random number, it is bounded by eq.(7). Assuming that for all  $K$  users, all the  $M_C$  branches are selected and combined, the average received SINR  $\langle \gamma_{i,l,ss} \rangle$  in the  $l$ -th,  $l=1, \dots, M_C$ , branch of the corresponding  $i$ -th,  $1 \leq i \leq K$ , MRC RAKE receiver, resulting from the signals of all  $K$  simultaneously transmitting users and all  $N_T$  antennas, can be written as:

$$\langle \gamma_{i,l,ss} \rangle = \sum_{t=1}^{N_T} E \left[ \sum_{l=1}^{M_C} \gamma_{l,i,ss} \right] = \sum_{t=1}^{N_T} E [\gamma_{l,1,ss} + \dots + \gamma_{l,K,ss}]_t \quad (8)$$

where  $\langle . \rangle = E[.]$  indicates the average value. Since the  $K$  received signals are independent identically distributed (i.i.d.) random variables, each of which corresponds to a SINR that follows a chi-square distribution with two degrees of freedom, we can write that:

$$\langle \gamma_{i,l,ss} \rangle = N_T K \Gamma_{i,ss,t} \quad (9)$$

Then, the probability density function (p.d.f.) of the combined SINR  $\gamma_{i,l,ss}$  at each MRC RAKE receiver's output will be given by, [13,14],:

$$p(\gamma_{i,l,ss}) = \frac{1}{(M_C - 1)! \langle \gamma_{i,l,ss} \rangle^{M_C}} \exp \left( -\frac{\gamma_{i,l,ss}}{\langle \gamma_{i,l,ss} \rangle} \right), 1 \leq i \leq K \quad (10)$$

where  $\langle \gamma_{i,l,ss} \rangle$  is given by eq.(9).

We now estimate the average total channel capacity  $\langle C_T \rangle$  available to all  $K$  users, under the previously described MRC RAKE reception. Following eq.(6), the total channel capacity  $C_T$  is averaged over the pdf of the combined SINR  $\gamma_{i,l,ss}$  at the MRC RAKE receiver output, so that:

$$\langle C_T \rangle = N_T W_{ss} \int_0^\infty \log_2(1 + \gamma_{i,l,ss}) p(\gamma_{i,l,ss}) d\gamma_{i,l,ss} \quad (11)$$

and taking into account eq.(10):

$$\langle C_T \rangle = N_T W_{ss} \int_0^\infty \log_2(I + \gamma_{i,ss}) \frac{I}{(M_C - I)! \langle \gamma_{i,ss} \rangle^{M_C}} \gamma_{i,ss}^{M_C - I} \exp\left(-\frac{\gamma_{i,ss}}{\langle \gamma_{i,ss} \rangle}\right) d\gamma_{i,ss} \quad (12)$$

Clearly, this capacity estimation indicates the average channel capacity that appears at the MRC RAKE receiver output in the form of the average best recovered data rate from all  $K$  active transmitting users, [13]. Therefore, we can write that:

$$\langle C_{i,t} \rangle = \frac{\langle C_T \rangle}{K} = \frac{N_T W_{ss}}{K} \int_0^\infty \log_2(I + \gamma_{i,ss}) \frac{I}{(M_C - I)! \langle \gamma_{i,ss} \rangle^{M_C}} \gamma_{i,ss}^{M_C - I} \exp\left(-\frac{\gamma_{i,ss}}{\langle \gamma_{i,ss} \rangle}\right) d\gamma_{i,ss} \quad (13)$$

from which, the average channel capacity per user,  $\langle C_{i,t} \rangle$ , normalised over the system bandwidth  $W_{ss}$ , will be given by:

$$\frac{\langle C_{i,t} \rangle}{W_{ss}} = \frac{\langle C_T \rangle}{W_{ss} K} = \frac{N_T}{K} \int_0^\infty \log_2(I + \gamma) \frac{I}{(M_C - I)! \langle \gamma \rangle^{M_C}} \gamma^{M_C - I} \exp\left(-\frac{\gamma}{\langle \gamma \rangle}\right) d\gamma \quad (14)$$

where  $\langle \gamma_{i,ss} \rangle$ , used in eq.(11), has been changed now to  $\langle \gamma \rangle$ . Assuming that the MIP has equal path strengths on the average, the SINR after path-diversity applied, in each of the  $K$  users,  $\langle \gamma_{i,pt,ss} \rangle = \langle \gamma_{pt} \rangle = N_T K \Gamma_{i,ss,t}$ , will be given by, [15]:

$$\langle \gamma_{i,pt,ss} \rangle = M_C \langle \gamma_{i,ss} \rangle = M_C N_T K \Gamma_{i,ss,t} \quad (15)$$

where the new suffice 'pt' refers to the path-diversity reception applied. However, in general, the multipath-intensity profile (MIP) in a Rayleigh fading environment is exponential, but, here, MIP is assumed discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average. Then, combining eqs (14) and (15), the average channel capacity per user  $\langle C_{i,t} \rangle$ , normalized over the total system's bandwidth  $W_{ss}$  when operating in a Rayleigh fading environment and expressed in (bits/sec/Hz), is found to be:

$$\frac{\langle C_{i,t} \rangle}{W_{ss}} = \frac{\langle C_T \rangle}{W_{ss} K} = \frac{N_T}{K} \int_0^\infty \log_2(I + \gamma_{pt}) \frac{I}{(M_C - I)! \langle \gamma_{pt} \rangle^{M_C}} \gamma_{pt}^{M_C - I} \exp\left(-\frac{\gamma_{pt}}{\langle \gamma_{pt} \rangle}\right) d\gamma_{pt} \quad (16)$$

We now address the problem to finding the number of users  $K$  which maximizes the average channel capacity per user  $\langle C_{i,t} \rangle$ ,  $K = K_{op}$ , given an average transmit power constraint and keeping the total allocated system's bandwidth  $W_{ss}$  constant (the subscript 'op' refers to the optimal value). Then, the problem of finding the optimal  $K_{op}$ , can then be stated as follows:

$$\max \int_0^\infty \log_2(I + \gamma_{pt}) \frac{(\gamma_{pt})^{M_C - I}}{(M_C - I)! \langle \gamma_{pt} \rangle^{M_C}} \exp\left(-\frac{\gamma_{pt}}{\langle \gamma_{pt} \rangle}\right) d\gamma_{pt} \quad (17)$$

The combined average spread SINR after diversity reception i.e.,  $\langle \gamma_{i,pt,ss} \rangle = M_C N_T K \Gamma_{i,ss,t}$  that maximizes eq.(17), equals to 6 dB, [16, 17], i.e.,

$$\langle \gamma_{i,pt,ss} \rangle = M_C N_T K \Gamma_{i,ss,t} = 10^{0.6} \quad (18)$$

using directly eq.(15). Combining eq.(5) and eq.(18), the optimal  $K_{op}$ , can be found directly, as following i.e.:

$$K_{op} = \frac{3.9(G_p - \Gamma_i N_T)}{N_T \Gamma_i (M_C - 3.9)} \quad (19)$$

The optimal number of users  $K_{op}$ , given by eq.(19), is plotted in Fig. 1 as a function of the SNR  $\Gamma_i$  (expressed in dB) where, in addition, the following values are assumed: (i) totally constant allocated system's bandwidth:  $W_{ss} = 10$  MHz, (ii) signal bandwidth:  $W = 30$  KHz, (iii) number of transmit (receive) antennas:  $N_T = 2$ , (iv) total multipath spread of the Rayleigh fading channel:  $T_m = 3$   $\mu$ sec. In addition, in Fig. 2, the optimal  $K_{op}$  is plotted as a function of the number of transmit (receive) antennas:  $N_T$ , for  $\Gamma_i = 0$  dB (indicative value),  $W_{ss} = 10$  MHz,  $W = 30$  KHz and  $T_m = 3$   $\mu$ sec. However, an integer value for the number  $N_T$  of transmit (receive) antennas is selected in practice. As it can be seen directly from Fig. 1 and Fig. 2, the optimal number of users  $K_{op}$ , of a SS MIMO system, is decreased as the SNR  $\Gamma_i$  over the signal bandwidth  $W$  or the number of transmit  $N_T$  are increased, indicating that a complicated signal processing scheme must be applied in order to reduce the MAI power that appears, in a such system, since the total MAI power reduces the system's radio capacity achievable.

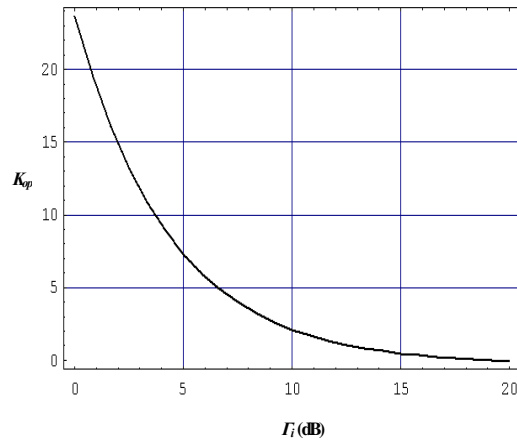


Fig. 1 Optimal radio capacity  $K_{op}$  of a single-cell SS MIMO system versus the average received SNR  $\Gamma_i$  (expressed in dB) in a Rayleigh fading environment

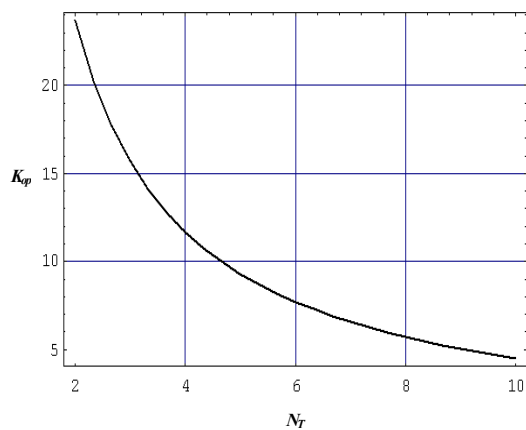


Fig. 2 Optimal radio capacity  $K_{op}$  of a single-cell SS MIMO system versus the number of transmits antennas  $N_T$  in a Rayleigh fading environment

## V. CONCLUSION

In the presented work, the optimal radio capacity of a single-cell SS MIMO system, operating in a Rayleigh fading environment, that maximises the achieved spectral efficiency, in terms of the average channel capacity available to each active user, with additional path-diversity reception, is estimated. A mathematical novel, simple and general expression for the optimal number of users is derived theoretically, which indicates finally the need for the application of an appropriate signal processing scheme in order to reduce the total MAI power and then to increase system's radio capacity. However, a simulation process and the future extension of this work for the multi-cell case are needed in order to compare with the theoretical results derived here and for the application of these results to a practical scenario.

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