

# Optimum Design of Launching Nose during Incremental Launching Construction of Same-Span Continuous Bridge

Weifeng Wang, Hengbin Zheng, and Xianwei Zeng

**Abstract**—The launching nose plays an important role in the incremental launching construction. The parameters of the launching nose essentially affect the internal forces of the girder during the construction. The appropriate parameters can decrease the internal forces in the girder and save the material and reduce the cost. The simplified structural model, which is made with displacement method according to the characteristic of incremental launching construction and the variation rule of the internal forces, calculates and analyzes the effect of the length, the rigidity and weight of launch nose on the internal forces of girder during the incremental launching construction. The method, which can calculate the launching nose parameters for the optimum incremental launching construction, is achieved. This method is simple, reliable and easy for practical use.

**Keywords**—incremental launching, launching nose, optimum analysis, displacement method

## I. EFFECT OF NOSE GIRDER IN INCREMENTAL LAUNCHING CONSTRUCTION OF BRIDGE

IN the development of incremental launching construction technology, various methods have been used to increase bridge span. Among them, nose girder is an important one [1]. When Dr. Leonhardt was designing Caroni Bridge in 1961, he used a steel nose girder of 17-meter in length in the front of concrete main beam for the first time. Its aim was to decrease the internal force in the front-end of concrete main beam during the incremental launching construction, by means of reducing the maximum jib length of concrete main beam. As a result, the use of nose girder took a good effect on the incremental launching construction of Caroni Bridge. Since then, nose girder has become a technical standard in the incremental launching construction of bridge [3]. During the construction, such parameters as the length, the rigidity of nose girder and its weight per unit length have great influence on the stressed main beam [4]. Through selecting the value of nose girder

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parameters appropriately, the internal force of construction can be reduced, and then the material will be saved and the construction cost decreased.

## II. OPTIMUM ANALYSIS OF NOSE GIRDER IN INCREMENTAL LAUNCHING CONSTRUCTION OF BRIDGE

### A. Establishment of Simplified Calculation Model

Ignoring support sedimentation and the influence of shrinkage and creep of concrete, the simplified model of the same-span continuous bridge can be made, shown as Fig. 1[6].

Supposing the length of nose girder is  $l_n$ , the length of main beam per span is  $l$ , then their ratio is represented as a non-dimensional parameter  $\alpha$ , that is  $\alpha = l_n / l$ ; supposing the weight of nose girder per unit length is  $q_n$ , the weight of concrete main beam per unit length is  $q$ , then a non-dimensional parameter  $\beta$  is used to denote their ratio as  $\beta = q_n / q$ ; supposing the rigidity of nose girder is  $E_n I_n$ , the rigidity of concrete main beam is  $EI$ , then a non-dimensional parameter  $\gamma$  is used to denote their ratio as  $\gamma = E_n I_n / EI$ . Thus, the problem of optimum parameter of nose girder in incremental launching construction of bridge can be considered as how to determine the optimum combination of the three parameters.

In the simplified model, supposing in some work conditions, for example  $\alpha = 0.7$ ,  $\beta = 0.1$ ,  $\gamma = 0.2$ , the turn angle of all portion of the beam can be calculated when the main beam is in dead load, shown in Fig. 3.

Under this work condition, changing the constraint condition of supporting point D, the corresponding internal force in all portion of the beam can be shown in Fig. 4. It is found that whether calculating based on the original simplified Figure, or regarding the constraint condition of supporting point D as equivalent to consolidation or simple support, the influence on the internal force of main beam and nose girder in the front of A—B span is very small. As seen from Fig. 3, in the simplified model, the turn angle of all portion of the beam at supporting point D is approximately 0. At the same time, in the simplified model of Fig. 4, when regarding the constraint condition of supporting point D as equivalent to consolidation, the stress of

main beam is basically as same as that of non-equivalent. Evidently, in the simplified model, regarding the constraint condition of supporting point D as equivalent to consolidation takes nearly no effect on the stress of main beam front-end. Thus, in the following optimum analysis of nose girder, the constraint condition of supporting point D is always regarded as equivalent to consolidation.

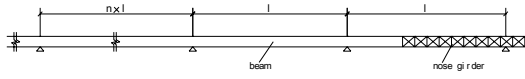


Fig. 1 The simplified analysis model of the nose girder



Fig. 2 The sketch of the simplified analysis model of the nose girder

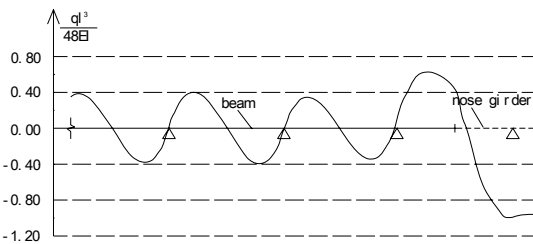


Fig. 3 The turn angle of all portion of the beam in the simplified model

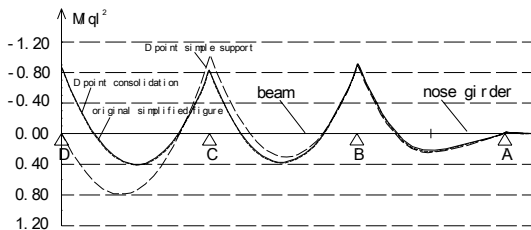


Fig. 4 The bending moment of all portion of the beam in the simplified model

As found from the stress characteristics of main beam during the incremental launching construction, the influence of variable nose girder parameters on the stress of main beam is primarily restricted in the first span of main beam [7]. Based on this, the process can be analyzed as below:

If making it as the work condition in which the front-end of concrete main beam is at supporting point B, the nose girder in the front of main beam gets its maximum jib length  $l_n$ .

With the advancement of the incremental launching process, the position of main beam often changes. Supposing the distance between the front-end of main beam and the supporting point B is  $x$ , when  $\lambda = x/l$ , then the bending moment at supporting point B can be represented as the function of  $\lambda$  and the parameters of main beam and nose girder  $\alpha, \beta, \gamma$ :  $M_b(\lambda, \alpha, \beta, \gamma)$ . Based on whether the front-end of nose girder can reach the front pier, the incremental launching process of

A-B span can be divided into two stages:

At the first stage, the definition domain of  $M_{b1}(\lambda, \alpha, \beta, \gamma)$  is  $0 \leq \lambda < 1 - \alpha$ . The simplified calculation diagram is shown as Fig. 5.

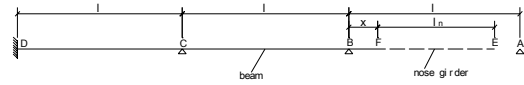


Fig. 5 The diagram of the first stage of incremental launching process

At the stage, the front-end of nose girder does not reach the support of front pier, so it is in the state of cantilever. The expression of bending moment of main beam at supporting point B can be displayed as:

$$M_{b1} = -[q_n l_n (0.5l_n + x) + 0.5qx^2] = -(0.5\alpha^2 \beta + \alpha\beta\lambda + 0.5\lambda^2)ql^2 \quad (1)$$

At the second stage, the definition domain of  $M_{b1}(\lambda, \alpha, \beta, \gamma)$  is  $1 - \alpha \leq \lambda < 1$ . Here, the nose girder is supported by the front pier. When the front-end of nose girder will reach supporting point A, through auxiliary equipments such as the lifting jack which bear the nose girder up, the elastic deformation of the nose girder front-end will recover, and the bearing force against the main beam will decrease the bending moment of main beam at supporting point B. As a result, the value of bending moment at supporting point B will get discontinuity [8]. When the nose girder reaches the pier, the supporting point A is the constraint of the structure, and the calculation can be conducted according to the simplified calculation diagram shown in Fig. 6.

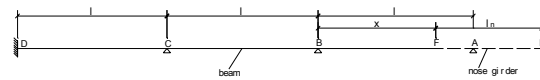


Fig. 6 The diagram of the second stage of incremental launching process

According to the displacement method, the balanced equation of the each node of the structure is shown as follows:

$$A : \frac{4E_n I_n}{l-x} \theta_A + \frac{2E_n I_n}{l-x} \theta_F + \frac{6E_n I_n}{(l-x)^2} \Delta_F \quad (2)$$

$$= \frac{1}{2} q_n (l_n + x - l)^2 - \frac{1}{12} q_n (l - x)^2$$

$$F : \frac{2E_n I_n}{l-x} \theta_A + \left( \frac{4E_n I_n}{l-x} + \frac{4EI}{x} \right) \theta_F + \left[ \frac{6E_n I_n}{(l-x)^2} - \frac{6EI}{x^2} \right] \Delta_F + \frac{2EI}{x} \theta_B \quad (3)$$

$$= \frac{1}{12} q_n (l-x)^2 - \frac{1}{12} q_n x^2$$

$$\frac{6E_n I_n}{(l-x)^2} \theta_A - \left[ \frac{12E_n I_n}{(l-x)^3} + \frac{12EI}{x^3} \right] \Delta_F$$

$$+ \left[ \frac{6E_n I_n}{(l-x)^2} - \frac{6EI}{x^2} \right] \theta_F - \frac{6EI}{x^2} \theta_B \quad (4)$$

$$= -\left[ \frac{qx}{2} + \frac{q_n(l-x)}{2} \right]$$

$$\begin{aligned}
 B: \quad & \frac{2EI}{x}\theta_F - \frac{6EI}{x^2}\Delta_F \\
 & + \left(\frac{4EI}{x} + \frac{4EI}{l}\right)\theta_B + \frac{2EI}{l}\theta_C \\
 & = \frac{1}{12}q(x^2 - l^2)
 \end{aligned} \tag{5}$$

$$C: \quad \frac{2EI}{l}\theta_B + \frac{8EI}{l}\theta_C + \frac{2EI}{l}\theta_D = 0 \tag{6}$$

$$D: \quad \frac{2EI}{l}\theta_C + \frac{4EI}{l}\theta_D = P_D \tag{7}$$

It is know from the edge-restraint condition that

$$\theta_D = 0 \tag{8}$$

Solving the equations from (2) to (8) jointly, the bending moment of main beam  $M_{b2}$  at the second stage of incremental launching process can be expressed as below:

$$M_{b2} = \frac{C_1 + C_2 + C_3 + C_4}{C_5} ql^2 \tag{9}$$

In this equation, each coefficient can be expressed as below:

$$\begin{aligned}
 C_1 = & \beta(\alpha + \lambda - 1)^2 \left[ \left( \frac{1}{2} - \frac{\lambda}{3} \right) \lambda^2 \right. \\
 & \left. + \frac{1}{\gamma} \left( \frac{1}{6} - \frac{1}{2} \lambda^2 + \frac{1}{3} \lambda^3 \right) \right]
 \end{aligned} \tag{10}$$

$$C_2 = -\frac{1}{24} \left( \frac{2 + 3\sqrt{3}}{3 + 2\sqrt{3}} \right) \tag{11}$$

$$\begin{aligned}
 C_3 = & -\frac{1}{2} \left\{ -\frac{1}{3} \lambda^3 + \frac{1}{4} \lambda^4 + [\beta(1 - \lambda)^2 \right. \\
 & \left. + \lambda(2 - \lambda)] \left[ \frac{1}{2} \lambda^2 - \frac{1}{3} \lambda^3 \right] \right\}
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 C_4 = & -\frac{1}{24\gamma} [4\lambda^3(1 - \lambda)^3 \\
 & + \beta(1 - 10\lambda^2 + 20\lambda^3 - 15\lambda^4 + 4\lambda^5)]
 \end{aligned} \tag{13}$$

$$C_5 = \frac{3 + 2\sqrt{3}}{12 + 6\sqrt{3}} + \frac{1}{3} \left[ 1 + \left( \frac{1}{\gamma} - 1 \right) (1 - \lambda)^3 \right] \tag{14}$$

From (1) and (9), it is found that  $M_b$  is the direct proportion of  $ql^2$ , and it has functional relationship with the ratio of each parameter of nose girder and the corresponding parameters of concrete main beam  $\alpha, \beta, \gamma$ . If a set of parameters  $\{\alpha, \beta, \gamma\}$  are determined, the changing process of the bending moment at supporting point B can be made when  $x$  changes from 0 to  $l$ . Then the influence of each parameter on the bending moment of main beam at supporting point B can be discussed qualitatively in the incremental launching process.

*B. Calculation of Optimum  $\alpha, \beta$*

Firstly, supposing  $\alpha = 0.8, \beta = 0.1, \gamma$  is given different values, when substituting them to (1) and (9), the change of bending moment value  $M_b$  at supporting point B can be

obtained in the whole process of incremental launching construction, shown in Fig. 7.

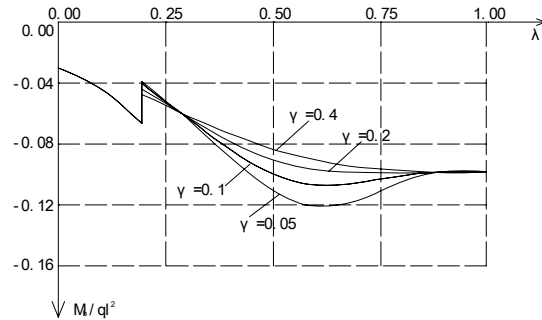


Fig. 7 When  $\alpha = 0.8, \beta = 0.1$ , the bending moment of B point in the incremental launching process of the A—B span

After the nose girder reaches the pier, the bending moment value at supporting point B will change according to the principle of multi-power function. For different values of  $\gamma$ , with the progress of incremental launching, the changing principle of  $M_b$  will be different; when the incremental launching at the stage is finished,  $M_b$  will get a certain value, then substituting  $\lambda = 1$  to (9), the following equation can be obtained:

$$M_{b2}(\lambda = 1) = (0.134\alpha^2\beta - 0.106)ql^2 \tag{15}$$

As seen from Fig. 7, the negative bending moment of main beam at supporting point B at the end of the first stage is smaller than that at the end of the second stage. It shows that the length of the nose girder is so long that the negative moment can only get its maximum value at the second stage of incremental launching process. Evidently, such a nose girder is not economical.

Thus, the length of the nose girder should be reduced. Supposing  $\alpha = 0.5, \beta = 0.1, \gamma = 0.1$ , and substituting them to (1) and (9), the changing process of negative bending moment of main beam at supporting point B in the whole incremental launching process can be obtained as shown in Fig. 8.

As seen from Fig. 8, when the nose girder reaches the pier, the negative bending moment at supporting point B is much larger than that when the second stage is finished. It shows that the length of the nose girder is too short, resulting in the fact that the main beam jib is too long in the incremental launching process. Consequently, the negative bending moment value of main beam at supporting point B at the end of the first stage is much larger than that at the second stage. Giving this value to the length of nose girder, the maximum negative bending moment of main beam cannot be reduced effectively in the incremental launching process. So, the length of nose girder is not enough here.

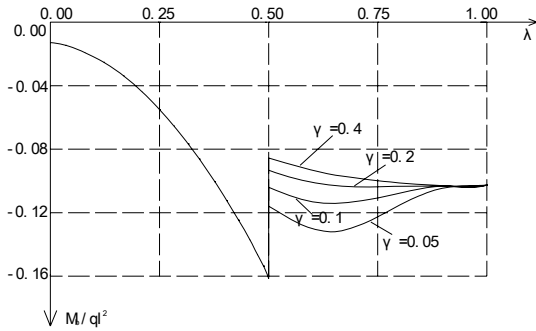


Fig. 8 When  $\alpha = 0.5$ ,  $\beta = 0.1$ , the bending moment of B point in the incremental launching process of the A—B span

Based on the above analysis, it is found that the bending moment value of main beam at supporting point B is irrelevant to the rigidity of the nose girder at the end of the first and second stage. However, it is related to the ratio of nose girder length and bridge span, as well as the ratio of nose girder and main beam in terms of the weight per unit length. Through selecting the appropriate value of parameter  $\alpha$  and  $\beta$ , the bending moment value of the main beam in the two work conditions can be equal. Also, by selecting the appropriate value of nose girder rigidity, the maximum negative bending moment value of main beam at supporting point B can be equal to the negative bending moment value at the end of the first and second stage in the incremental launching process of A-B span. So, the optimum parameter of nose girder can be obtained when the maximum negative bending moment of main beam at supporting point B in the maximum jib length is equal to the negative bending moment at the end of second stage of incremental launching process. Then,  $\lambda = 1 - \alpha$  which is the condition when the maximum jib length of main beam is achieved, can be substituted to (1)

$$M_{b1}(\lambda = 1 - \alpha) = [0.5\alpha^2\beta + \alpha\beta(1 - \alpha) + 0.5(1 - \alpha)^2]ql^2 \quad (16)$$

$$M_{b1}(\lambda = 1 - \alpha) = M_{b2}(\lambda = 1) \quad (17)$$

From (15) and (16), it is found that in the two work conditions the bending moment value of main beam at supporting point B is related to the ratio of nose girder length and span called  $\alpha$ , the ratio of the weight of nose girder and main beam per unit length called  $\beta$ , but irrelevant to the ratio of nose girder rigidity and main beam called  $\gamma$ . When solving (17), it is found:

$$\alpha_{best} = \frac{\sqrt{\beta^2 - 1.42318\beta + 0.212} + \beta - 1}{0.732\beta - 1} \quad (18)$$

According to (18), a graph that describes relationship of the optimum nose girder length and the weight of nose girder per unit length in the incremental launching process can be made.

As seen from Fig. 9, if (18) is meet, the value of  $\beta$  should range from 0 to 0.18. That is to say, the weight of nose girder per unit length can not be higher than 0.18 times of the weight of main beam per unit length. To a given value of  $\beta$ , the

corresponding optimum parameter of nose girder length  $\alpha$  can be calculated through (18). In Fig. 9, it is found if (17) is meet, with the increasing of  $\beta$ , the corresponding  $\alpha$  will also increase. Substituting (18) to (16), (19) can be obtained. Based on the above, a graph that describes relationship between the maximum negative bending moment of main beam at supporting point B and the value of  $\beta$  in the incremental launching process of A—B span can be shown in Fig. 12.

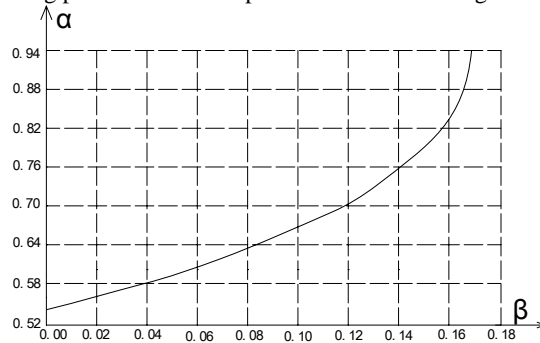


Fig. 9 The relationship between the optimized length and optimized density of the nose-deck in the incremental launching process

$$M_b = [-0.106 + \frac{0.124\beta(-1 + \beta + \sqrt{0.212 - 1.42318\beta + \beta^2})^2}{(-1 + 0.732\beta)^2}]ql^2 \quad (19)$$

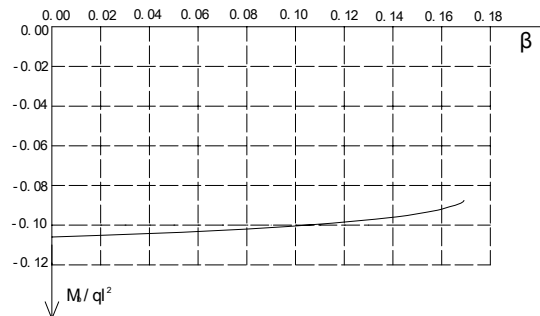


Fig. 10 The relation between the value of  $\beta$  and the max bending moment of B point in the incremental launching process

It is found in Fig. 12, when giving different value to  $\beta$  in its range, the corresponding maximum negative bending moment value of main beam at supporting point B in the whole incremental launching process of span A-B is almost unchanged. That is to say, when  $\alpha$  and  $\beta$  value meet (17), the corresponding maximum bending moment almost remains the same. But when giving  $\beta$  a relatively larger value, the corresponding  $\alpha$  value will increase. It requires the nose girder to be heavy and long. In this sense, the cost of construction and application of the nose girder will increase considerably, which is an improper way in construction. When giving  $\beta$  a small value, the corresponding  $\alpha$  will decrease, which implies the nose girder will be light and short. But if the girder is very light,

restricted by the production materials, it is hard for the nose girder to carry adequate degree of rigidity. If the nose girder lacks rigidity, in the second stage of incremental launching process, the maximum negative bending moment value at supporting point B will be much larger than that at the end of the two incremental launching stages [10]. In practical construction, the value of  $\beta$  generally ranges from 0.08 to 0.12, and the corresponding  $\alpha$  value from 0.63 to 0.71.

C. Calculation of Optimum  $\gamma$

In (18), if  $\beta = 0.1$ , the optimum length parameter of nose girder can be obtained as:  $\alpha_{best}(\beta = 0.1) = 0.665$ . When it is substituted to (1) and (9), selecting different values of  $\gamma$ , the changing process of negative bending moment of main beam at supporting point B can be made in the incremental launching process corresponded by different degrees of nose girder rigidity, under the condition of the optimum joint of nose girder length and its weight per unit length, as shown in Fig. 11:

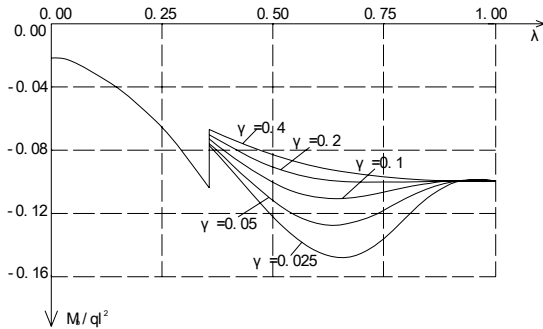


Fig. 11 When  $\alpha = 0.665$ ,  $\beta = 0.1$ , the bending moment of B point in the incremental launching process of the A—B span

It is found from the Fig. , under the condition of the optimum joint of parameters of nose girder length and its weight per unit length, if the degree of nose girder rigidity is too low, at the second stage of the incremental launching process, the maximum negative bending moment value of main beam at supporting point B will be larger than that at the end of first stage of the process. Therefore, the smallest rigidity value that the nose girder needs should be determined at present, so that the maximum negative bending moment value of main beam at supporting point B at the second stage of the incremental launching process will be equal to that at the first stage of process.

Thus, in the condition that  $\alpha, \beta$  are determined already, the optimized  $\gamma$  value can be obtained when the maximum negative bending moment value of main beam at supporting point B at the second stage of the incremental launching process is just equal to the negative bending moment value when the main beam has its longest jib, ( i.g. , when  $x = l - l_n$  ).

$$M_{b2}(\min) = M_{b1}(\lambda = 1 - \alpha) \tag{20}$$

When the determined  $\alpha$  and  $\beta$  value are substituted to (9), the negative bending moment value  $M_{b2}(\lambda, \gamma)$  at supporting

point B represented as  $\gamma$  at the second stage of the incremental launching process can be obtained. Calculating the corresponding  $M_{b2}(\min)$  when  $\gamma$  is given different values by using the displacement method, the optimum  $\gamma$  value can be obtained, in the condition that  $\alpha$  and  $\beta$  are known. When  $\alpha = 0.665, \beta = 0.1$ , the optimum  $\gamma$  value is:  $\gamma = 0.186$ .

In the incremental launching process of A—B span, the maximum positive bending moment occurs in the first span of concrete main beam. From the above (2) to (8), the maximum positive bending moment  $M_{ab}(\max)$  can be calculated in the incremental launching process as shown below:

$$M_{ab}(\max) = (D_1 + D_2 + D_3)ql^2 \tag{21}$$

Among it:

$$D_1 = \left( \frac{C_1 + C_2 + C_3 + C_4}{C_5} \right) \tag{22}$$

$$+ \frac{\lambda^2}{2} + \alpha\beta\lambda + \frac{\alpha^2\beta}{2}(1 - \lambda - \alpha\beta)$$

$$D_2 = \frac{1}{2} \left( \frac{C_1 + C_2 + C_3 + C_4}{C_5} \right) \tag{23}$$

$$+ \frac{\lambda^2}{2} + \alpha\beta\lambda + \frac{\alpha^2\beta}{2}$$

$$D_3 = \frac{1}{2} \alpha^2 \beta^2 - \frac{1}{2} \alpha^2 \beta \tag{24}$$

Expressions of  $C_1, C_2, C_3, C_4, C_5$  are shown in (10) to (14).

When  $\alpha = 0.665, \beta = 0.1$  are substituted to (21), giving different values to  $\gamma$ , the changing process of the maximum positive bending moment of the first span in the incremental launching process of A-B span can be made, shown as Fig. 12.

It is found from the calculation result that different  $\gamma$  value has influence on  $M_{ab}(\max)$  in the incremental launching process, but takes no effect on  $M_{ab}(\max)$  when the process of A-B span ends. The maximum value of  $M_{ab}(\max)$  is obtained at the end of second stage of the incremental launching process. That is to say,  $\gamma$  has no influence on the maximum positive bending moment value of the first span in the incremental launching process, while  $\alpha$  and  $\beta$  have only a little influence on the maximum value of  $M_{ab}(\max)$  in the incremental launching process of A-B span. Thus, the parameter value of nose girder cannot determine the maximum positive bending moment value, and the maximum positive bending moment value of the first span in the incremental launching process is primarily decided by the dimension of concrete main beam. Thus, in the optimum analysis of nose girder, the maximum positive bending moment is not decisive in selecting each parameter value. Rather, the optimum result of nose girder analysis is determined by the maximum negative bending moment at supporting point B in the incremental launching process.

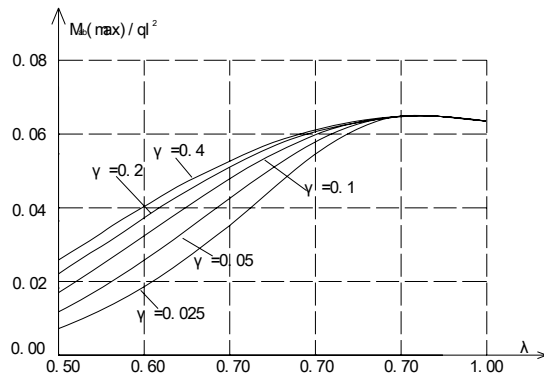


Fig. 12 When  $\alpha = 0.665$ ,  $\beta = 0.1$ , the bending moment of the first span in the incremental launching process of the A—B span

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### III. CONCLUSION

- 1) In the study, the calculation method of the optimum nose girder parameter value in the continuous incremental launching construction of bridge is achieved, through the calculation and analysis of the length, the rigidity and the weight per unit length of nose girder, as well as their effect on the internal forces of main beam during the incremental launching construction. This method is simple, reliable and easy for practical use.
- 2) When  $\beta = 0.1$ , the corresponding optimum  $\alpha = 0.665$ ,  $\gamma = 0.186$ .
- 3) In the optimum analysis of nose girder, the maximum positive bending moment of the first span is not decisive in selecting each parameter value of nose girder. Rather, the optimum result of nose girder analysis is determined by the maximum negative bending moment at supporting point B in the incremental launching process.

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